Parametric time-domain identification of multiple-input systems using decoupled output signals

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SUMMARY

Civil engineering structures are often subjected to multidirectional actions such as earthquake ground motion, which lead to complex structural responses. The contributions from the latter multidirectional actions to the response are highly coupled, leading to a MIMO system identification problem. Compared with single-input, multiple-output (SIMO) system identification, MIMO problems are more computationally complex and error prone. In this paper, a new system identification strategy is proposed for civil engineering structures with multiple inputs that induce strong coupling in the response. The proposed solution comprises converting the MIMO problem into separate SIMO problems, decoupling the outputs by extracting the contribution from the respective input signals to the outputs. To this end, a QR factorization-based decoupling method is employed, and its performance is examined. Three factors, which affect the accuracy of the decoupling result, including memory length, input correlation, and system damping, are investigated. Additionally, a system identification method that combines the autoregressive model with exogenous input (ARX) and the Eigensystem Realization Algorithm (ERA) is proposed. The associated extended modal amplitude coherence and modal phase collinearity are used to delineate the structural and noise modes in the fitted ARX model. The efficacy of the ARX-ERA method is then demonstrated through identification of the modal properties of a highway overcrossing bridge. Copyright © 2013 John Wiley & Sons, Ltd.

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1. INTRODUCTION

System identification is a process of building a mathematical model of a dynamic system based on measured data. In civil engineering applications, system identification has served as an essential tool, which delivers realistic structural models to facilitate a wide range of practical problems such as design verification, structural health monitoring, damage detection, structural control, and others. Various algorithms have been developed in the past decades to identify linear models of engineering structures. These methods can generally be classified into two categories, namely nonparametric and parametric methods, depending on whether a parameterized model of the underlying system with a finite-dimensional parameter vector is assumed. Parametric methods choose the mathematical model structure, such as time series model, transfer function, frequency response function, and state-space model, and estimate the model parameters for best fit, such as the Eigensystem Realization Algorithm (ERA) [1, 2], Stochastic Subspace Identification [3, 4], Prediction Error Method [5], and
Polyreference Least Squares Complex Frequency Domain Method [6]. Nonparametric methods, on the other hand, do not specify a priori model structure but instead determine the model structure directly from data. Examples of nonparametric methods include the peaking-picking method [7–9] and the frequency domain decomposition method [10, 11]. Note that although ERA and SSI are parametric methods, their inputs are sometimes derived through nonparametric methods. Additional techniques were also developed to account for the cases when only output data can be measured, including the Natural Excitation Technique [12, 13] and the Random Decrement Technique [14–16]. More comprehensive review of system identification methods can be found in the literature [17, 18].

Despite the numerous advances in system identification techniques, challenges still remain in their applications to civil engineering structures, such as bridges, buildings, and towers. These structures typically have numerous degrees-of-freedom (DOFs) and large physical dimensions. They are often subjected to complex loadings such as earthquake excitations and hence exhibit complex responses. For example, earthquake excitations are intrinsically multidimensional, creating inputs in different directions, which are applied to the structural system simultaneously. Moreover, the structures may respond to the earthquake excitations in a complex way such that the effects of these inputs on the structural responses are coupled, that is, the measured response contains the contributions from more than one input. System identification of these structures is therefore treated as a MIMO problem if the inputs and the coupled output measurements are to be used.

Compared with Single-Input, Multiple-Output (SIMO) system identification problems, MIMO problems are more challenging, regardless of the use of parametric or nonparametric methods. In the case of parametric methods, the existence of multiple inputs implies the estimation of more parameters. For nonparametric methods, estimating impulse or frequency response functions requires the inversion of the input correlation or cross power spectrum matrices. For experimental modal analysis performed in a controlled manner such as in the laboratory, the inputs can be selected carefully so that broadband and uncorrelated input excitations are used. MIMO approach can achieve accurate result under these conditions. However, earthquake inputs typically have restricted bandwidth and are not uncorrelated, the input correlation or cross power spectrum matrices can be ill-conditioned and the inversions are prone to error. Therefore, to reduce computational complexity and improve the accuracy of results, decoupling a MIMO problem into a number of SIMO problems is desirable.

The key issue in decoupling a MIMO problem for system identification is to isolate the contribution of each input to a specific output. In regression analysis, this idea has also been used to select the optimal set of regressors [19–24]. For example, to select the centers (regressors) for the RBF neural networks, Chen et al. [22] proposed an Orthogonal Least Squares method to decouple the desired output into a linear combination of a set of orthogonal basis vectors. This method allows calculation of the individual contributions of the output energy from each basis vector. However, the drawback of this approach is that the coefficients of the linear combination are obtained from a least squares solution, introducing error in the decoupled result. A variation of the Orthogonal Least Squares method was then proposed by Westwick et al. [25] based on QR factorization. In the latter approach, the factorization is performed for the matrix formed by not only the inputs but also the desired output. The individual contribution to the output from each input is thus obtained directly after the factorization. Nonetheless, applications of these methods have been focused on identifying the relative contribution of an individual input to the output energy, by which the less significant inputs can be removed from regression analysis or system identification. In contrast, the approach advocated herein is to decouple the outputs and then perform system identification directly.

The non-stationary nature and short duration of earthquake excitation pose another challenge to system identification of civil engineering structures [26, 27]. Nonparametric methods are generally more efficient, but often less accurate than parametric methods. Although averaging can help smooth the data and reduce noise, a relatively larger amount of stationary data is generally needed. Seismic records are non-stationary transient signals with relatively short duration; therefore, parametric methods, such as the autoregressive exogenous (ARX) model adopted in this study, are more often applied to identify the modal properties in these situations [28–30]. Because of unavoidable noise in the measured data, the order of the ARX model is usually set much higher than twice the number of positive frequency poles to represent the measured response, introducing
Considering a MIMO LTI system with subjected to earthquake excitations. The proposed system employed to eliminate the noise modes in the identification strategy with output decoupling; and (2) a method to eliminate the noise modes from the identified ARX model. In their approach, the probable modes are firstly determined by the first earthquake, and the posterior PDF is formed. In the result of the next earthquake, the mode that maximizes the posterior PDF is considered as the corresponding structural mode. This method, however, is designed for the case when multiple records are available and is not suitable for a single identification result. Ji et al. [35] used damping ratio and Modal Phase Collinearity (MPC) combined with the stabilization diagram to select structural modes from the identified ARX model. However, the MPC has limited sensitivity to noise level, and therefore the noise modes may not be distinguished by this single rule.

The goal of this study is to develop an effective linear system identification strategy for civil engineering structures subject to multiple inputs, which lead to highly coupled outputs. In this study, the structural systems are assumed to be linear and their properties are time invariant. Two critical issues have been investigated and solutions have been proposed, including the following: (1) simplification of MIMO system identification to SIMO problems through a new system identification strategy with output decoupling; and (2) a method to eliminate the noise modes from the identified ARX model. The first goal was achieved using the QR factorization-based method developed by Westwick et al. [25]. Three parameters affecting the performance of the method for application to civil engineering structures were investigated, including length of the memory of the system used in the formulation, correlation between the input signals, and the level of damping of the structure. The second goal was realized with a technique that combines the ARX method with ERA. The inclusion of the Extended Modal Amplitude Coherence (EMAC) and the MPC are then employed to eliminate the noise modes in the identified ARX model. Finally, the effectiveness of the proposed system identification strategy is demonstrated for a highway overcrossing bridge subjected to earthquake excitations.

2. DECOUPLING OF MIMO SYSTEMS

Considering a MIMO LTI system with \( n \) inputs \((x_1, x_2, \ldots, x_n)\) and \( m \) outputs \((y_1, y_2, \ldots, y_m)\), in discrete time domain, the response of each output can be expressed as a linear combination of delayed copies of the input signals given by

\[
y_j(t) = \sum_{i=1}^{n} \sum_{\tau=0}^{M-1} h_{ji}(\tau)x_i(t - \tau) + \omega(t), \quad j = 1, 2, \ldots, m
\]

(1)

in which \( h_{ji} \) is the Impulse Response Function from input \( x_i \) to output \( y_j \). \( M \) is the memory length of the system, and \( \omega(t) \) is a term to account for the noise in the output measurement as well as the effect of unmeasured disturbances. The memory length \( (M) \) is a finite number if the underlying system is a Finite Impulse Response (FIR) system and is infinity for an Infinite Impulse Response (IIR) system.

In the aforementioned MIMO system, the measured output \( y_j \) may contain contribution from any of the \( n \) inputs if the corresponding Impulse Response Function is not zero. As discussed in the previous section, having coupled multiple inputs will complicate the system identification problem not only because more computational power is required, but also its accuracy may decrease. To identify the component in the output, which can be attributed solely to a specific input, Westwick et al. [25] proposed a method based on QR factorization. To find the contribution of the input \( x_k \) to the output \( y_m \), two matrices are first constructed

\[
M_1 = [X_1, \ldots, X_{k-1}, X_{k+1}, \ldots, X_n]
\]

\[
M_2 = X_k
\]

(2)

where \( X_i (i = 1, 2, \ldots, k, \ldots, n) \) are block matrices formed by delayed copies of the input \( x_i \) with \( T \) elements.
The matrix $M_1$ contains the inputs of the structure whose contribution to the output $y_p$ are not of interest, while $M_2$ contains the input of interest to the output $y_p$. $X_i$ is formed by delayed copies of the input because the outputs are linear combination of the delayed copies of the input signals, as shown in Eq. (1).

The next step is to combine the aforementioned two matrices with the output vector to be decoupled and then apply QR factorization to the combined matrix, as shown in Eq. (4). The input signals are orthogonalized and the desired output can be projected onto these orthogonalized vectors.

$$[M_1 \ M_2 \ \bar{y}_p] = QR = [Q_1 \ Q_2 \ \bar{q}_y]$$

(4)

Here, $Q_1$ and $Q_2$ have the same dimensions as $M_1$ and $M_2$, respectively. The projection of each input and the output to a new set of orthogonalized vectors can be obtained as follows,

$$M_1 = Q_1 R_{11}$$
$$M_2 = Q_1 R_{12} + Q_2 R_{22}$$

(5)

As can be seen from Eq. (6), the three terms in the equation are orthogonal to each other because all columns in $Q$ are orthogonal. Therefore, the second term $Q_2 \bar{r}_{2y}$ is orthogonal to $Q_1$ and hence $M_1$. Thus, $Q_2 \bar{r}_{2y}$ can be considered as the component in the output $\bar{y}_p$, which is attributed solely to the input $X_i$. Note that the columns in $M_1$ can be arranged in different orders. In the QR factorization result shown in Eq. (4), $Q_2$ and $\bar{r}_{2y}$ will be independent to the order of columns in $M_1$, because $M_2$ is placed to the right of $M_1$. Therefore, the decoupling result $Q_2 \bar{r}_{2y}$ is independent to the way $M_1$ is formed.

The next step in Westwick et al. [25] is the computation of the mean squared value of the second term in Eq. (6). This step ranks the importance of each input, allowing insignificant inputs to be removed subsequently. However, the interests in the application of this method herein is to use the term related to the decoupled output to convert the MIMO problem into a SIMO one. The aforementioned procedure can be applied in a sequential manner to all output measurements such that the contributions from a specific input to these outputs can be extracted and then used to identify the SIMO systems. In this regard, the accuracy of the decoupled output is essential to system identification. In the following sections, three factors that have significant impact on the accuracy of the decoupling result will be investigated, including the memory length used in Eq. (3), the correlation of the inputs, and the system damping ratio.

2.1. **FIR and IIR systems**

Two types of dynamic systems are considered including a FIR system and an IIR system. The reason for investigating both types of systems is that the original algorithm was developed for FIR systems, but most civil engineering structures belong to IIR systems. For illustrative purposes, systems having three DOFs are considered. As shown in Figure 1a, the FIR system consists of three 30th order FIR low-pass filters, which have different stopband and passband frequencies. Each of the filters takes a band limited Gaussian white noise as input and the corresponding outputs are then mixed with a $3 \times 3$ mixing matrix to generate three coupled outputs. The IIR system depicted in
Figure 1b is realized by a spring-mass system, which also takes three band limited Gaussian white noise inputs and generates three coupled outputs. 5% damping is considered in the IIR system. The inputs and outputs are acceleration signals for both systems. The simulation parameters, for both systems, consist of 30 s in duration and a sampling frequency of 100 Hz.

2.2. Effect of the memory length

As indicated in Eq. (1), the memory length $M$ controls the duration of the past input, which can affect the current output. Therefore, for FIR systems, which have a finite memory length, the number of columns in Eq. (3) is equal to $M$. However, for IIR systems, the memory length is infinite, and in such cases, the selection of $M$ for the input matrices in Eq. (3) becomes less apparent. If the memory length is too small, the constructed input matrices may not provide adequate information for the orthogonalized matrices in Eq. (6) to fully reconstruct the target output. If the selected memory length is too large, resulting in a large matrix in Eq. (4), the QR decomposition will be more computationally demanding and error prone.

To investigate the effect of the memory length on the accuracy of the decoupled responses for both FIR and IIR systems, numerical simulations are carried out on the basis of the aforementioned two models. Three uncorrelated band limited Gaussian white noise inputs are generated and then fed into the two systems. The effect of correlation between inputs is not considered here but will be discussed in the next section. The coupled output signals are recorded for decoupling analysis. By changing the memory length $M$, the QR decomposition-based method introduced in the preceding section is applied to all three outputs sequentially to exact contributions from the three inputs, resulting in nine sets of decoupled outputs. The accuracy of the decoupled output is represented by an error factor $J$ [36], defined by

$$J_{kp} = \frac{1}{\sum_{i=1}^{T} \left( y_{kp}^\text{ref}(i) - y_{kp}^\text{dec}(i) \right)^2} \sum_{i=1}^{T} \left( y_{kp}^\text{ref}(i) \right)^2, \quad k = 1, 2, 3, \quad p = 1, 2, 3 \quad (7)$$

The superscript ‘ref’ denotes the reference signals and ‘dec’ means the decoupled outputs. $k$ and $p$ are the indices of the input and output, respectively. For example, $y_{12}$ denotes the contribution from the Input 1 to the Output 2 and $J_{12}$ is the corresponding error factor.

Figure 2 shows the change of $J$ values as the memory length increases. The minimum $J$ values and the corresponding optimal memory lengths are listed in Table I. First of all, for the FIR system, when the memory length reaches a threshold ($M=17$ in this case), the error of each decoupled output
decrease dramatically and becomes relatively stable beyond that point. The minimum errors are found
be very small (less than 0.015), indicating high accuracy achieved in the decoupled outputs. A
sample result for the FIR system is plotted in Figure 3a,b, where the reference signals are obtained
by simulations with a single input. The decoupled contribution from Input 1 to Output 1 matches
very well with the reference signal. However, the minimum errors are found when
$M < 31$, which is the
filter length or the theoretical memory length, possibly because of numerical
error in QR decomposition when dealing with large matrices.

For the IIR system, no clear threshold is observed as the $J$ values change more slowly when
varying the memory length. Unlike the FIR system, the IIR system takes infinite time for the
system dynamics to die out, or at least it takes much longer time to decrease to a low level when
damping is small; therefore, the optimal memory lengths are all larger than the FIR system and
are associated with larger uncertainty. The overall accuracy of the decoupled outputs is not as
high as the FIR system, because of the fact that the QR decomposition has to deal with much
larger matrices and the larger uncertainties from the optimal memory length. However, the
decoupled outputs still possess reasonable accuracy, especially for the diagonal terms where the
input and the output share the same DOF (i.e., $J_{11}$, $J_{22}$, and $J_{33}$ all less than 0.1). Sample
results for $J_{11}$ shown in Figure 3c,d demonstrate a good match between the decoupled response
and reference signal.

2.3. Effect of the input correlation

The effectiveness of the decoupling method has been demonstrated with uncorrelated inputs; however,
the correlation of the inputs can affect the accuracy of the result. If a system is subjected to two
identical inputs (fully correlated), distinguishing between the contributions from these two inputs to
the output will be difficult, if not impossible. Assessment of the influence of the input correlation
gives a broader picture of the decoupling approach.

Two aspects of the input correlation are investigated including global correlation and local
correlation; both are quantified by a correlation coefficient. The global correlation coefficient
between two discrete signals $x_1(i)$ and $x_2(i)$ is computed using the entire records and is defined as

![Figure 2. Accuracy of decoupled outputs for different memory lengths for FIR system (left) and IIR system (right).](image)

| Table I. Minimum errors of the decoupled outputs and the corresponding memory lengths. |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
|                                | $J_{11}$ | $J_{12}$ | $J_{13}$ | $J_{21}$ | $J_{22}$ | $J_{23}$ | $J_{31}$ | $J_{32}$ | $J_{33}$ |
| FIR                            | 0.0119   | 0.0119   | 0.0119   | 0.0122   | 0.0121   | 0.0125   | 0.0130   | 0.0130   | 0.0130   |
| IIR                            | 0.0941   | 0.1567   | 0.1845   | 0.1353   | 0.0958   | 0.1341   | 0.1381   | 0.1338   | 0.093    |
|                                | $M_{11}$ | $M_{12}$ | $M_{13}$ | $M_{21}$ | $M_{22}$ | $M_{23}$ | $M_{31}$ | $M_{32}$ | $M_{33}$ |
| FIR                            | 25       | 25       | 25       | 23       | 22       | 23       | 24       | 24       | 24       |
| IIR                            | 100      | 140      | 180      | 160      | 100      | 160      | 200      | 160      | 80       |

FIR, Finite Impulse Response; IIR, Infinite Impulse Response.
where $N$ is the length of a signal. $\bar{x}_1$ and $\bar{x}_2$ are the mean values of $x_1$ and $x_2$, respectively. The local correlation coefficient is calculated within a sliding window to track the correlation between two signals over time, and is therefore defined as

$$
\rho = \frac{\sum_{i=1}^{N} [x_1(i) - \bar{x}_1][x_2(i) - \bar{x}_2]}{\sqrt{\sum_{i=1}^{N} [x_1(i) - \bar{x}_1]^2 \sum_{i=1}^{N} [x_2(i) - \bar{x}_2]^2}}
$$

(8)

where $N$ is the length of a signal. $\bar{x}_1$ and $\bar{x}_2$ are the mean values of $x_1$ and $x_2$, respectively. The local correlation coefficient is calculated within a sliding window to track the correlation between two signals over time, and is therefore defined as

$$
\rho_w(t) = \frac{\sum_{i=t}^{t+w} [x_1(i) - \bar{x}_{1,t}][x_2(i) - \bar{x}_{2,t}]}{\sqrt{\sum_{i=t}^{t+w} [x_1(i) - \bar{x}_{1,t}]^2 \sum_{i=t}^{t+w} [x_2(i) - \bar{x}_{2,t}]^2}}
$$

(9)

where $w$ is window length. $\bar{x}_{1,t}$ and $\bar{x}_{2,t}$ are the mean values of $x_1$ and $x_2$ within the sliding window at time $t$, respectively.

To generate two inputs, which are globally uncorrelated but have varying correlation over time, two earthquake records measured at the same location but in perpendicular directions are selected. These two records are orthogonalized to make them globally uncorrelated, but the local correlation is still time-varying. Figure 4 shows the time history of the orthogonalized records, as well as the change of correlation angle over time. Note that correlation coefficient is cosine of the angle between two time series vectors. The window length is 5 s when calculating the local correlation coefficients. These two inputs are then used as the Input 1 and Input 2 to the FIR system shown in Figure 1a and Input 3 is set to zero. Figure 5a shows the change of $J_{11}$, which has a minimum value of 0.00956 when the memory length equals to 23. Such a high accuracy indicates that the performance of the decoupling approach is not affected by the local correlation of the inputs. Therefore, high accuracy can be achieved as long as the inputs are globally uncorrelated.

Knowing that the local correlation of inputs is not affecting the performance of the decoupling approach, the effect of the global correlation is then further investigated. A series of pairs of band limited white noise inputs are generated with global correlation angles ranging from 0 to 90°. Similarly, the two inputs are applied to the first two DOFs of both the FIR and the IIR systems. The changes of $J_{11}$ are plotted in Figure 6, showing clearly that the accuracy of the decoupled output increases when the input correlation decreases. At 90° angle, the result reflects the conclusion from

![Figure 3. Comparison between the decoupled and reference outputs. (a) and (b): $J_{11}$ of the FIR system; (c) and (d): $J_{11}$ of the IIR system.](image-url)
the previous section, namely, the FIR system has higher accuracy in the result than the IIR system. One important observation for the IIR system is that when the angle between the two inputs is larger than 80°, the error in the decoupled output is lower than 0.1. Because all civil engineering structures are IIR systems, this observation can serve as a rough guideline to assess whether the decoupling approach is applicable or not to a specific problem.

2.4. Effect of damping

The third factor that affects the performance of the decoupling approach is the system damping. As explained in previous section, the IIR system requires a much longer time to have its system dynamics decay, and hence the required memory length is much larger than for the FIR system. The fact that QR decomposition has to deal with much larger matrix introduces numerical errors into the result; therefore, the accuracy of the decoupled output is lower than the FIR system. System damping affects directly the decay time and hence the optimal memory length, therefore affecting the accuracy of the decoupled output. Using the IIR system in Figure 1b, the damping ratio of the IIR system is changed from 5 to 90%. As shown in Figure 7, the error of the decoupled output

Figure 4. Two orthogonalized earthquake inputs (a) with time-varying correlation (b).

Figure 5. Decoupling result with the orthogonalized earthquake inputs: (a) change of error with different memory lengths; (b) decoupled output at the optimal memory length.

Figure 6. Effect of the global correlation between inputs for the FIR (left) and the IIR (right) systems.
decreases dramatically as the system damping increases. With 20% damping in the IIR system, a comparable accuracy to the FIR system is achieved.

3. ARX-ERA METHOD FOR SYSTEM IDENTIFICATION

As mentioned previously, earthquake records are non-stationary signals and have relatively short duration; therefore, parametric time-domain methods such as time series models are often used. The autoregressive model with exogenous input (ARX) method is adopted herein because of its good performance in such situations, as reported in the literature [28–30].

Consider an LTI system with input $x(t)$ and output $y(t)$, the input-output relationship can be described in the following linear difference equation (Ljung, 1999):

$$ y(t) = b_1 x(t) + b_2 x(t-1) + \cdots + b_m x(t-n_b+1) + a_1 y(t-1) + \cdots + a_n y(t-n_a) + e(t) \quad (10) $$

The output coefficients $a_i$ and input coefficients $b_i$ are the so-called autoregressive and moving average coefficients, respectively. $n_a$ is the number of poles and $n_b$ is the number of zeros plus one. $e(t)$ is a white noise term representing the stochastic system disturbance. In a more compact form, Eq. (10) can be written as in Eq. (11), where $A(q)$ and $B(q)$ are polynomials with respect to the backward shift operator $q^{-1}$.

$$ A(q)y(t) = B(q)x(t) + e(t) $$

$$ A(q) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n_a} $$

$$ B(q) = b_1 + b_2 q^{-1} + \cdots + b_m q^{-n_b+1} \quad (11) $$

Eq. (11) is the well-known ARX. AR refers to the auto-regressive part $A(q)y(t)$ and $X$ to the extra input part $B(q)x(t)$. For the SIMO case, the output coefficient $A(q)$ in the ARX model becomes a matrix defined in Eq. (12), in which $m$ is the number of outputs.

$$ A(q) = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
a_{11} q^{-1} & a_{12} q^{-1} & \cdots & a_{1m} q^{-1} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} q^{-n_a} & a_{n2} q^{-n_a} & \cdots & a_{nm} q^{-n_a}
\end{bmatrix} \quad (12) $$

The coefficients of the ARX model can be solved by minimizing the errors or residuals $e(t)$ in Eq. (11) by the least squares method. In the multiple-output case, the norm of the error is minimized (Ljung, 1999), which is

\[
V(\theta, \hat{y}, \hat{x}) = \frac{1}{T} \sum_{t=1}^{T} (y(t) - \hat{y}(t))^2
\]

\[
\theta = [a_1, \ldots, a_n, b_1, \ldots, b_n]^T
\]

\[
\hat{\theta} = \arg \min_{\theta} V(\theta, \hat{y}, \hat{x})
\]

where \(\hat{y}\) and \(\hat{x}\) are measured outputs and inputs in the time interval \(t \in [0, T]\), and \(\hat{\theta}\) is the solution of the least squares problem, which gives the coefficients of the ARX model.

To minimize estimation error, especially when the signal-to-noise ratio is low, the ARX model order is usually set higher than the actual model; therefore, the fitted model may contain more poles and zeros in the transfer function than the actual structure with a certain sampling frequency. To distinguish effectively the structural modes with the noise modes, two noise mode indicators associated with the ERA, namely, the MPC and the EMAC are used (see Juang and Pappa [1] and Pappa and Elliott [37] for more details). Herein, the impulse response function extracted from the fitted ARX model is used as input to the ERA; subsequently, the identified modes with EMAC and MPC values higher than the predetermined thresholds are considered as candidates of structural modes. Then, the stabilization diagram is constructed by increasing the number of retained singular values in the ERA and plotting all the candidates of structural modes in one chart. Based on the idea that a structural mode should exist in systems with various orders, a mode that is identified in at least five realizations of different orders is considered stable and reliable. Among them, the ones with the highest EMAC values are selected as the confirmed modes.

Modal Phase Colinearity has been previously proposed to work with ARX model without introducing ERA method [35], because MPC can be calculated directly with the identified mode shapes. However, there are several drawbacks for using MPC only. First of all, MPC is not applicable for SISO problems because no mode shapes can be identified with only one input and one output. In other words, the MPC values are always equal to 100%. Second, MPC is not as sensitive as EMAC to noise modes. Relying on MPC only may not filter out all noise modes. As an illustration, three impulse response functions \((h_{12}, h_{22}, h_{32})\) of the IIR system shown in Figure 1b with 1% damping are obtained and added with 5% Gaussian white noise. ERA is applied to these impulse response functions and the candidate structural modes are identified by applying either MPC or EMAC; hence, two stabilization diagrams are plotted as shown in Figure 8. The solid dots in each stabilization diagram are the final confirmed modes, which have the highest EMAC values. Figure 8a shows the result after filtering the noise modes by setting the MPC threshold to 90%; however, the stabilization diagram is not as clean as the one which has EMAC threshold equal to 20%. In fact, after adding 5% noise to the impulse response functions, the MPC values associated with noise modes are still very high, whereas the EMAC values drop down significantly. This simulation shows that combining EMAC and MPC can lead to a more effective method to filter out noise modes from the fitted ARX mode.

4. APPLICATION TO A HIGHWAY OVERCROSSING BRIDGE

The Meloland Road Overcrossing (MRO) Bridge is located over Interstate 8, about 10 mi to the east of El Centro, California. The bridge is a reinforced concrete box-girder bridge with two spans monolithically connected to a single pier. Each span is 104 ft (31.70 m) and the deck is 34 ft (10.36 m) wide. The center pier is about 21 ft (6.40 m) high from the foundation and has a circular cross section with the diameter of 5 ft (1.52 m). Figure 8 shows the elevation, cross section, and plan view of the MRO Bridge. In April 1978, 26 accelerometers were deployed on the bridge deck, foundation, embankment, and free field. In December 1991, the sensor array was augmented by six additional accelerometers. Figure 9 illustrates the sensor locations and directions. A number of earthquake events were captured because the bridge was instrumented. The recorded seismic ground motions and structural responses have been studied extensively in the past. These studies include soil-structural interaction analysis [38–40], seismic analysis [41–43], and system identification [36, 30, 44], among others. In the following system identification analysis, the data measured during the
Calexico Area Earthquake of November 20, 2008 is utilized. As one of the most recent recorded earthquakes, this one has a small PGA (PGA = 0.007 g) and hence is believed to induce only linear response of the bridge-embankment system.

Inputs and outputs are first identified from the measurement channels. For structures subjected to three-directional ground excitation, the equations of motion (EOMs) are commonly formulated in relative coordinates such that the input term on the right-hand side of the EOM is the product between the mass and the ground acceleration, as shown in the succeeding text:

\[
(a) \quad \text{Stabilization diagram with MPC threshold} = 90\% \\
(b) \quad \text{Stabilization diagram with EMAC threshold} = 20\% 
\]

Figure 8. Sensitivities of Extended Modal Amplitude Coherence and Modal Phase Collinearity to noise modes (5% noise level).

Figure 9. Meloland Road Overcrossing Bridge configuration and sensor instrumentation.

Calexico Area Earthquake of November 20, 2008 is utilized. As one of the most recent recorded earthquakes, this one has a small PGA (PGA = 0.007 g) and hence is believed to induce only linear response of the bridge-embankment system.

Inputs and outputs are first identified from the measurement channels. For structures subjected to three-directional ground excitation, the equations of motion (EOMs) are commonly formulated in relative coordinates such that the input term on the right-hand side of the EOM is the product between the mass and the ground acceleration, as shown in the succeeding text:

To validate the effectiveness of the proposed decoupling method and the ARX-ERA system identification approach, a linear finite element (FE) model of the MRO Bridge is built in MATLAB (The mathworks, Inc., Natick, Massachusetts, United States) based on the as-built drawings of the bridge. The FE model is depicted in Figure 10. A lumped mass model is adopted for the bridge. The embankment-abutment system is modeled as two linear springs attached with the embankment mass, one in the lateral direction and the other in the vertical direction. The model has in total 350 DOFs. Static condensation is then performed to condense out the rotational DOFs, leaving 150 effective DOFs in the model. Damping (5%) is added to the first two modes of the system by assuming proportional damping. Newmark-β method with γ = ½ and β = ¼ is selected to perform linear time-stepping integration to calculate dynamic response of the model under earthquake inputs. The measured free field records from channels 24 and 14 during the 2008 Calexico Area Earthquake are used to excite the FE model in the lateral and vertical directions simultaneously to generate coupled response in the vertical direction. Meanwhile, the responses because of only the vertical ground motion (channel 14) are also simulated, which serve as the ground motion inputs in the vertical direction because of only the vertical ground motion (channel 14) are also simulated, which serve as the vertical response. The lateral mode of the bridge are the lowest frequency modes and hence are of the most interest. The lateral mode has low frequency because the lateral movement of the deck is essentially driven by the movement of the embankments, which has low stiffness in the lateral direction. Therefore, data from channel 24 and 14 are considered as inputs to the lateral and vertical systems, respectively. However, further scrutiny of the bridge indicates that although the vertical ground motion contributes only to the vertical response, the lateral ground motion affects not only the lateral responses but also the vertical responses because of the rotation of the bridge deck in response to the lateral excitation. In this regard, system identification is a SIMO problem in the lateral direction, but a MIMO problem in the vertical direction if the measured data is used as it is. To convert the MIMO problem in the vertical direction into a SIMO problem, the vertical measurements on the bridge deck should be decoupled using the approach proposed in the previous sections.

In what follows, the proposed decoupling method and system identification strategy are first validated through numerically simulated structural response in vertical direction and then applied to identify the modal parameters of the bridge in both lateral and vertical directions using measured data.

4.1. Numerical validation of the proposed method

To validate the effectiveness of the proposed decoupling method and the ARX-ERA system identification approach, a linear finite element (FE) model of the MRO Bridge is built in MATLAB (The mathworks, Inc., Natick, Massachusetts, United States) based on the as-built drawings of the bridge. The FE model is depicted in Figure 10. A lumped mass model is adopted for the bridge. The embankment-abutment system is modeled as two linear springs attached with the embankment mass, one in the lateral direction and the other in the vertical direction. The model has in total 350 DOFs. Static condensation is then performed to condense out the rotational DOFs, leaving 150 effective DOFs in the model. Damping (5%) is added to the first two modes of the system by assuming proportional damping. Newmark-β method with γ = ½ and β = ¼ is selected to perform linear time-stepping integration to calculate dynamic response of the model under earthquake inputs. The measured free field records from channels 24 and 14 during the 2008 Calexico Area Earthquake are used to excite the FE model in the lateral and vertical directions simultaneously to generate coupled response in the vertical direction. Meanwhile, the responses because of only the vertical ground motion (channel 14) are also simulated, which serve as the vertical response. The lateral mode of the bridge are the lowest frequency modes and hence are of the most interest. The lateral mode has low frequency because the lateral movement of the deck is essentially driven by the movement of the embankments, which has low stiffness in the lateral direction. Therefore, data from channel 24 and 14 are considered as inputs to the lateral and vertical systems, respectively. However, further scrutiny of the bridge indicates that although the vertical ground motion contributes only to the vertical response, the lateral ground motion affects not only the lateral responses but also the vertical responses because of the rotation of the bridge deck in response to the lateral excitation. In this regard, system identification is a SIMO problem in the lateral direction, but a MIMO problem in the vertical direction if the measured data is used as it is. To convert the MIMO problem in the vertical direction into a SIMO problem, the vertical measurements on the bridge deck should be decoupled using the approach proposed in the previous sections.

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reference signals for assessing the error of the decoupled result. All measured input and output data are downsampled from 100 to 20 Hz to focus on the frequency range of interest. Before downsampling, digital anti-aliasing filters were applied to remove the frequency contents beyond the Nyquist frequency (10 Hz).

4.1.1. Decoupling outputs in vertical direction. To decouple the vertical responses, as discussed in Section 2.2, the optimal memory lengths need to be known at various locations on the bridge. The simulated vertical responses at the locations of channel 21, 22, and 29, each corresponds to the locations of the pier top, mid-span and abutment, respectively, are used to determine the corresponding optimal memory lengths. The procedure applied in Section 2.2 is repeated to search for the optimal memory lengths. Table II lists the optimal memory lengths and the minimum errors associated with the decoupled responses at the three locations. The angle between the two ground motion inputs is 89°, indicating low correlation between the inputs. Therefore the errors in the decoupled outputs are expected to be very low based on the discussion in Section 2.3. These small errors have validated the effectiveness of the proposed decoupling strategy based on the simulated data.

The optimal memory lengths are then applied to decouple the simulated vertical deck responses corresponding to the locations of the ten sensor channels, including two pier top channels (17 and 21), four abutment channels (6, 19, 28, and 29), and four mid-span channels (16, 18, 22, and 20). Figure 10 shows the deck responses from the two mid-span channels (16 and 18) before and after decoupling. Before decoupling, these responses contain contributions from both the vertical and lateral ground motion inputs. The lateral input introduces different response to these two locations of the deck in the vertical direction, which make the total responses look quite different (Figure 11a). However, after decoupling, the contributions from the lateral input are effectively removed, leaving only the contributions from the vertical input. Therefore, the decoupled responses are almost identical to each other (Figure 11b).

4.1.2. System identification with the decoupled outputs. With the decoupled outputs, which contain only the contribution from the vertical ground input, system identification in the vertical direction can be treated as a SIMO problem. A SIMO ARX model is therefore adopted. Channel 14 is considered as the input and the ten decoupled channels are used as the outputs. \( n_a \) and \( n_b \) are determined through a trial-and-error process. The ARX model utilizes the same set of poles and zeros to describe both the system dynamics and noise properties. If the values of \( n_a \) and \( n_b \) are too small, the resulting model may not have enough poles and zeros to represent the system dynamics; on other hand, overfitting may occur if the values are too large. Herein, in the identification of

<table>
<thead>
<tr>
<th>Location</th>
<th>Pier top</th>
<th>Mid-span</th>
<th>Abutment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal ( M )</td>
<td>19</td>
<td>47</td>
<td>44</td>
</tr>
<tr>
<td>( J_{\text{min}} )</td>
<td>0.098</td>
<td>0.080</td>
<td>0.079</td>
</tr>
</tbody>
</table>
vertical modes, \( n_a \) and \( n_b \) are respectively set to 7 and 3, resulting in a model with 70 poles and 2 zeros. The transfer functions of the ARX model are plotted in Figure 12a.

In the vertical direction, the FE model of the bridge has two modes, which are both symmetric bending modes, as shown in Figure 12b. The embankment masses move in phase with the deck in the first mode but out-of-phase in the second mode. The two modal frequencies of the FE model are 4.17 Hz and 5.03 Hz, respectively, as listed in Table III. Apparently, within the range of the sampling frequency, which is 20 Hz after downsampling, only four of the poles correspond to true system dynamics, whereas the rest are noise modes. The EMAC and MPC associated with the ERA method are then used to automatically filter out these noise modes and find the true modal properties, as shown in the stabilization diagram in Figure 11a. In this case, the threshold values of EMAC and MPC are set to 20 and 90%, respectively. Both vertical bending modes are picked up by the stabilization diagram with high accuracy, with natural frequency errors less than 0.5%. The clear stabilization diagram, with only true modes retained in the plot, indicates the effectiveness of the combined usage of EMAC and MPC in filtering the noise modes in the fitted ARX model. Meanwhile, both identified mode shapes match with the FE model very well as indicated by the Model Assurance Criteria (MAC) values [51]. The results are summarized in Table III. This section has demonstrated that the proposed decoupling approach and the ARX-ERA method are effective for system identification using highly coupled output signals.

4.2. Decoupling and system identification with measured structural response

System identification is carried out independently in the lateral and vertical directions using measured data. In the lateral direction, as mentioned previously, the lateral responses are induced only from the lateral ground motion and hence a SIMO ARX model is used, with channel 14 as the input and channels 11, 3, 5, 7, 9, 13, and 26 as the outputs. Note that channels 11 and 26, which measure the embankment responses are included herein, because the bridge and the embankments are treated as an integral system subject to a single uniform ground excitation. In the identification of lateral modes, \( n_a \) and \( n_b \) are set to 2 and 3, respectively, resulting in a model with 14 poles and 2 zeros. In this case, the threshold values of EMAC and MPC are set to 20 and 90%, respectively. As shown in Figure 14a, a lateral bending mode at 3.13 Hz associated with 4.9% damping is identified. The mode shape is depicted in Figure 14b.
In the vertical direction, the procedure described in Section 4.1.1 and the optimal memory lengths listed in Table II are applied to decouple the measured structural responses from the ten vertical sensor channels. Figure 13 shows the deck responses from the four mid-span channels before and after decoupling. Similar to Figure 11, one indication of the effectiveness of the decoupling method is that these records become quite similar to each other after the components contributed from the lateral input have been removed. With the decoupled vertical outputs, a SIMO ARX model is used to fit the input-output data. $n_a$ and $n_b$ are also determined as 2 and 3, respectively, resulting in 20 poles and 2 zeros in this case. The stabilization diagram is shown in Figure 15a, in which the EMAC and MPC thresholds are again set to 20 and 90%, respectively. Again, a clear stabilization diagram is obtained with all noise modes filtered out effectively using the proposed method. One asymmetric bending mode at 3.13 Hz and one symmetric bending mode at 4.51 Hz are identified as shown in Figure 14b. The associated damping ratios are 3.3 and 2.8%, respectively.

Comparing between the results with the simulated (Figure 12) and measured responses (Figure 15), one can observe that both cases have identified the first symmetric bending mode, which has the embankment mass moving in phase with the bridge deck. However, in the latter case, the second symmetric bending mode is not identified, whereas an additional asymmetric bending mode is identified. The difference can be attributed to the discrepancy between the FE model and the real bridge. As indicated previously, the FE model is built on the basis of the as-built drawing and many parameters such as the embankment mass and the spring stiffness can have some uncertainties. For example, the stiffness of the embankment could be higher than the FE model such that the second

![Figure 13. The vertical deck responses before (a) and after (b) decoupling using measured data.](image)

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE Model</th>
<th>Identified</th>
<th>Error</th>
<th>Mode shape (MAC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.1671</td>
<td>4.1488</td>
<td>-0.44%</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>5.0373</td>
<td>5.0269</td>
<td>-0.21%</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

FE, finite element; MAC, Model Assurance Criteria.

![Figure 14. Identification result in the vertical direction: (a) stabilization diagram; (b) modal frequencies and mode shapes.](image)
symmetric bending mode exists in much higher frequency range for the real bridge. In addition, the 3.23 Hz asymmetric bending mode indicates that, unlike the FE model, the bridge is not perfectly symmetric, because otherwise, the asymmetric mode could not be excited by the uniform vertical ground excitation. Note that the asymmetric mode does not have identical shapes along the two sides of the bridge deck in the longitudinal direction, which supports the speculation that the bridge is not symmetric. In contrast, the 4.51 Hz symmetric bending mode has quite similar shape along the two sides of the deck. One possible reason could be that during earthquakes, the center pier was not providing identical bending stiffness to the deck in the longitudinal direction because of uneven support at the bottom of the pier.

5. CONCLUSIONS

In this paper, a new linear system identification strategy is proposed for civil engineering structures subjected to multiple earthquake inputs that lead to strong coupling in the response signals. The structural systems considered in this study are assumed linear and their properties are time invariant. By decoupling the outputs, the contribution from each individual input can be separated. This separation leads to the conversion of the MIMO system identification into SIMO problems, which are computationally less complex and have potential to yield more accurate results. The accuracy of the decoupling approach has been comprehensively examined by focusing on three critical factors: the memory length, the input correlation, and the system damping. By selecting the appropriate memory length, both FIR and IIR systems can be identified with high accuracy in the decoupled outputs. The global correlation of the inputs, rather than the local correlation, affects the accuracy of decoupled outputs. Increasing the global correlation of inputs will decrease the accuracy. For the IIR system with 5% damping, when the input angle is larger than 80°, satisfactory result can be achieved. Finally, systems with higher damping can yield higher accuracy in the result.

A parametric system identification method, which combines the ARX and the ERA methods has been proposed for seismic measurements. The noise mode indicators associated with the ERA method, including the EMAC and MPC, are utilized to filter out the noise modes in the fitted ARX model, enabling accurate and reliable system identification with ARX. The combined usage of EMAC and MPC has been found beneficial because of the lack of sensitivity of the MPC to noise modes compared with the EMAC.

Finally, the effectiveness of the proposed system identification strategy approach has been demonstrated using the Meloland Road Overcrossing Bridge using both simulated and measured seismic responses. The dynamic modes in lateral direction is directly identified as a SIMO problem using the proposed ARX-ERA method, whereas in the vertical direction, the decoupling strategy was first applied to remove the contribution from the lateral input to the vertical outputs, allowing SIMO identification in the vertical direction. One lateral bending mode and two vertical bending modes were successfully identified.
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