Distortion of pulsed signals in carbon nanotube interconnects

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Abstract

This paper investigates the distortion of DC and radio frequency (RF) pulsed signals in carbon nanotube interconnects. A modified transmission line model for single-walled carbon nanotubes is employed for the simulation. Comparisons with the conventional AlCu interconnect are performed.

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1. Introduction

Due to the ever increasing operating frequency and levels of integration, interconnect lines have become one of the most crucial issues for integrated circuit (IC) designs [1]. The performance of an IC is strongly dependent on the current carrying capacity and the parasitics of the interconnects. It has been emphasized by the International Technology Roadmap for Semiconductors (ITRS) that there is a great need for reliable, high speed interconnects for future technologies [2]. The conventional metallic interconnect materials are highly susceptible to electro-migration at high current densities and less reliable when the device dimensions are scaled down. Hence, the single-walled carbon nanotubes (SWCNTs) are considered as potential candidates [2,3].

Any pulsed signal traveling along interconnect lines gets distorted especially when the operating frequency approaches gigahertz [4,5]. In order to understand how well the SWCNTs can serve as interconnects for the ICs, a knowledge of the distortion information is important. However, up to now the relevant studies are quite limited. The objective of this work is to examine the distorted shape of DC and radio frequency (RF) pulses having square and Gaussian envelope characteristics.

The paper is organized as follows: Section 2 introduces the transmission line model for the SWCNTs. Section 3 interprets the computations of the signal distortion in details. In Section 4, the simulation results are presented. Signal distortions in a SWCNT are compared with those in a conventional metallic interconnect. Finally, the paper is summarized and concluded in Section 5.

2. Transmission line model for SWCNTs

Single-lined SWCNTs have been modeled for RF applications in [6] using a LC transmission line model. Coupled SWCNTs are characterized as RLC transmission line models in [2,3]. In [2,3], the insufficiency of the model introduced in [6] is pointed out. Based on [2,3], resistance should be included for characterization. Hence, we modified the model introduced in [6]. The modified model is shown in Fig. 1. The calculations of the RLC components (\( L_K \), \( R \), \( C_Q \) and \( C_{ES} \)) are discussed in [2,3,6].

2.1. Resistance (\( R \))

The resistance is a function of the bias voltage \( V_b \). For \( V_b < 0.16 \text{V} \) [2,3], the equation for calculating \( R \), is presented in Eq. (1). The resistance \( R \) is denoted as \( R_{\text{low}} \).
Fig. 1. Segment diagram of transmission line model for SWCNTs.

to represent the low bias voltage

$$R_{\text{low}} = \left( \frac{h}{4e^2} \right) \Theta \left( \frac{l}{\lambda_{\text{low}}} \right),$$ (1)

where $h (\approx 6.626 \times 10^{-34} \text{m}^2\text{kg}/\text{s})$ is Plank’s constant, $e$ ($\approx 1.602 \times 10^{-19} \text{C}$) is the electronic charge, $l$ is the length of the SWCNT and $\lambda_{\text{low}}$ is the mean free path (mfp) of the acoustic phonons under this circumstance ($\approx 1.6 \mu\text{m}$). The equation of the function $\Theta$ is presented in

$$\Theta(x) = \begin{cases} 
1, & x < 1, \\
n, & \text{elsewhere}. 
\end{cases}$$ (2)

For $V_b \geq 0.16 \text{V} [2, 3]$, the value of $R$ should be calculated using Eq. (3). $R_{\text{high}}$ is used here to represent the high bias voltage.

$$R_{\text{high}} = \left[ \frac{0.16}{V_b} \left( \frac{1}{R_{\text{low}}} - \frac{1}{R_{\text{diff}}} \right) + \frac{1}{R_{\text{diff}}} \right]^{-1},$$ (3)

where $R_{\text{diff}}$ is the differential resistance of the SWCNT when $V_b \geq 0.16 \text{V}$

$$R_{\text{diff}} = \frac{dV}{dt} = \left( \frac{h}{4e^2} \right) \Theta \left( \frac{l}{\lambda_{\text{high}}} \right),$$ (4)

$$\lambda_{\text{high}} = \frac{\lambda_{\text{op}}\lambda_{\text{zo}}}{\lambda_{\text{op}} + \lambda_{\text{zo}}},$$

$\lambda_{\text{op}}$ ($\approx 200 \text{nm}$) is the mfp of optical phonon. $\lambda_{\text{zo}}$ ($\approx 30 \text{nm}$) is the mfp of zone boundary phonon, the scattering of which becomes dominant at higher biases.

2.2. Inductance (L)

The kinetic inductance per unit length (p.u.l) can be expressed as

$$L_k = \frac{h}{2e^2v_F},$$ (5)

where $v_F (\approx 8 \times 10^5 \text{m/s})$ is the Fermi velocity in graphite, so that numerically

$$L_k = 16(\text{nH}/\mu\text{m}).$$ (6)

Since the kinetic inductance always dominates the magnetic inductance ($L_m \approx \mu \text{H}/\lambda \text{m}$) [6], the magnetic inductance, which is in series with the kinetic inductance is omitted in the model. Therefore

$$L = L_m + L_k \approx L_k = 16(\text{nH}/\mu\text{m}).$$ (7)

2.3. Capacitances ($C_Q$ and $C_{ES}$)

The p.u.l. quantum capacitance is shown in

$$C_Q = \frac{2e^2}{h v_F}.$$ (8)

Numerically, it is

$$C_Q = 100(\text{aF}/\mu\text{m}).$$ (9)

The p.u.l. electrostatic capacitance between a wire and a ground plane

$$C_{ES} = \frac{2\pi \epsilon}{\cosh^{-1}(2H/d)},$$ (10)

where $\epsilon$ is the permittivity between the wire and the ground, $H$ is the distance from the SWCNT to the ground plane and $d$ is the diameter of the SWCNT. When the distance to the ground plane is less than the tube length, it can be approximated numerically as [6]

$$C_{ES} = 50(\text{aF}/\mu\text{m}).$$ (11)

3. Computations of signal distortion

3.1. Signal transform

In time domain, the voltage pulse at a reference point in the CNT is represented as

$$v(t, z) = \begin{cases} 
v(t), & -\frac{T}{2} \leq t \leq \frac{T}{2}, \\
0, & \text{elsewhere}. 
\end{cases}$$ (12)

The corresponding signal in the frequency domain can be described in

$$V(\omega, z = 0) = \int_{-T/2}^{T/2} v(t, z = 0)e^{-j\omega t} dt,$$ (13)

where $V(\omega)$ and $V(t)$ are a transform pair. For a square pulse, $-T/2 \leq t \leq T/2$ defines the pulselength, while for a Gaussian pulse, the time range $-\infty \leq t \leq \infty$ is required for the characterization.

At a particular distance of $l$, the pulse in the frequency domain is derived in

$$V(\omega, z = l) = V(\omega, z = 0)e^{-j\omega l},$$ (14)

where $\gamma$ is the propagation constant of the transmission line as described in

$$\gamma = \sqrt{j\omega C_{total}(R + j\omega L)},$$ (15)

where

$$C_{total} = \frac{C_Q C_{ES}}{C_Q + C_{ES}}.$$ (16)
Applying the inverse transform of Eq. (13), the time domain representation of the pulse at \( z = l \) can be derived as

\[
\nu(t, l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega, z = l) e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega, z = 0) e^{i\omega t - \gamma(\omega) l} d\omega.
\] (17)

The time domain representation of a Gaussian pulse is

\[
f(t) = A \exp\left[-\left(\frac{t}{a}\right)^2\right],
\] (18)

where \( A \) is the amplitude and \( a \) is the pulse half-duration at \( 1/e \) points.

The corresponding frequency domain interpretation is presented in Eq. (19), according to the work of Veghte and Balanis [5]

\[
F(\omega) = \frac{A\sqrt{\pi}}{a} \exp\left[-\left(\frac{a\omega}{2}\right)^2\right],
\] (19)

where \( 2a \) is the 3 dB pulsewidth.

For a square pulse, the time domain representation is shown in

\[
f(t) = \begin{cases} A, & -t_0 \leq t \leq t_0, \\ 0, & \text{elsewhere}. \end{cases}
\] (20)
The corresponding Fourier transform is presented in Eq. (21) according to [5]

\[ F(\omega) = 2A \frac{\sin(t_0\omega)}{\omega}, \]  
(21)

where \(2t_0\) is the pulsewidth.

A square RF pulse with a time domain representation of

\[ f(t) = \begin{cases} 
A \cos \omega_0 t, & -t_0 \leq t \leq t_0, \\
0 & \text{elsewhere}.
\end{cases} \]  
(22)

Its Fourier transform is written as Eq. (23), according to [5]

\[ F(\omega) = A \left\{ \frac{\sin[t_0(\omega - \omega_0)]}{(\omega - \omega_0)} + \frac{\sin[t_0(\omega + \omega_0)]}{(\omega + \omega_0)} \right\}. \]  
(23)

3.2. Integration

The numerical integration introduced in [5] is employed herein for the integral of Eq. (17).

Then Eq. (17) can be simplified into

\[ v(t, l) \equiv \frac{1}{2\pi} V(\omega_i, z = 0)e^{j\omega_i t - \gamma(\omega_i)l}\Delta\omega_i, \]  
(24)

where \(N\) is the number of divisions in the frequency spectrum and \(\Delta\omega_i = 2\zeta/\tau/N\). \(\tau\) is the width of the pulse and \(\zeta\) is a constant which depends on the waveshape. For a Gaussian pulse, a \(\zeta\) with a value of 20 is sufficient, while for a square pulse, a value around 500 is required.

Eq. (24) can be easily programmed as \(V(\omega, z = 0)\) for the Gaussian pulse, the square pulse and the square RF pulse are obtained as shown in Eqs. (19), (21) and (23) respectively.

4. Simulation results

The distortion of a rectangular DC pulse, a Gaussian DC pulse and a Square RF pulse traveling 1 mm along a SWCNT interconnect are examined. The pulsewidths of both the rectangular DC pulse and the Gaussian DC pulse are 0.1 ns. The pulsewidth of the square RF pulse is 2 ns

<table>
<thead>
<tr>
<th>Table 1</th>
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<tbody>
<tr>
<td>Component values of transmission line model</td>
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<table>
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<tr>
<th></th>
<th>(R) ((\Omega)/(\mu)m)</th>
<th>(L) (nH/(\mu)m)</th>
<th>(C_{\text{total}}) (aF/(\mu)m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWCNT ((V_b &lt; 0.16, \text{V}))</td>
<td>4</td>
<td>16</td>
<td>33.33</td>
</tr>
<tr>
<td>SWCNY ((V_b \geq 0.16, \text{V}))</td>
<td>0.36</td>
<td>16</td>
<td>33.33</td>
</tr>
<tr>
<td>AlCu</td>
<td>(~10^{-2})</td>
<td>(~10^{-4})</td>
<td>300</td>
</tr>
</tbody>
</table>

Fig. 6. Distortion of a square RF pulse at a distance \(l = 1\, \text{mm}\) \((V_b = 1.8\, \text{V})\).
while the carrier frequency is 20 GHz. A conventional AlCu interconnect with a width of 5.3 \mu m and a thickness of 0.5 \mu m \cite{7} is used as a reference. As stated in Section 2.1, the resistance of the SWCNT is a function of the bias voltage. Two representative voltages, i.e. 1.8 and 0.1 V, which are above and below the critical voltage (0.16 V), respectively, are employed.

Comparisons shown in Figs. 2–5 demonstrate that the signals traveling along the SWCNT have longer delay time while the attenuation and distortion are less compared with AlCu interconnects. As for the signal transferring in the same SWCNT, the distortion is affected by the bias voltage. When the bias voltage is less than 0.16 V, the distortion is worse. However, the delay is not affected by the bias voltage. These phenomena can be explained with the help of Eqs. (15) and (17), while Eq. (15) is rewritten as Eq. (25) and (17) is reinterpreted as Eq. (28).

$$\gamma = \sqrt{-\omega^2 C_{total} L + j \omega RC_{total}} = \alpha + j \beta, \quad (25)$$

where

$$\alpha \approx \frac{R}{2} \sqrt{\frac{C_{total}}{L}} \quad (26)$$

and

$$\beta \approx \frac{\omega}{2} \sqrt{L/C_{total}}. \quad (27)$$

$$v(t, l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega, l = 0) e^{-l/\sqrt{\omega^2 + R^2}} e^{j(\omega t - \beta)} d\omega. \quad (28)$$

The values of $R$, $L$ and $C_{total}$, which are the components of the transmission line model, are listed in Table 1. Since $R$ and $L$ of the AlCu interconnect are frequency dependent \cite{7}, the ranges are given in the table instead of the exact values. (Figs. 6 and 7).

As it is known that the amplitude of the signal is affected by $\alpha$, while the delay is determined by $\beta$. The larger the values, the worse the distortion and delay are. With the help of Eq. (26) and Table 1, it is not difficult to get to the conclusion that the signal in AlCu interconnect suffers the worst distortion and signal in the SWCNT with bias voltage greater than or equal to 0.16 V suffers less. On the other hand, using Eq. (27) and Table 1, it is obvious that the signal in AlCu interconnect has less delay time, while the delay in SWCNT is not affected by the bias voltage.

5. Conclusion

The distortion of DC and RF pulses as they propagate along a SWCNT is investigated. A modified transmission
line model is used for the simulation. Comparisons with distortion in conventional AlCu interconnects are performed. It is observed that the signals traveling along the SWCNT have longer delay time while the attenuation and distortion are less compared with AlCu interconnects. The SWCNT has different properties with variations of the biased voltages. The attenuation is less for stronger biased signals ($V_b \geq 0.16\, V$) while the delay time is not affected. Mathematical analyzes explaining those phenomena are also provided.

References


