Coverage and Rate Trends in Moderate and High Bandwidth 5G Networks

Mandar N. Kulkarni, Timothy A. Thomas, Frederick W. Vook, Amitava Ghosh, Eugene Visotsky

Abstract—Higher frequency bands (>6 GHz) look promising to meet the proposed 5G data rates, given the large amount of available spectrum in these bands. However, a rigorous understanding of some fundamental tradeoffs like network densification, sectorization, and bandwidths has only begun to be investigated at millimeter wave (mmW) bands. In this work, we investigate the coverage and rate performance of cellular networks with sectorized access points (APs) operating at high frequency bands, using tools of stochastic geometry. We observe that sectorizing the APs can significantly improve the data rates and thus can be used in conjunction with network densification, in order to achieve the 5G data rate requirements. However, the increased data rates come at the expense of increased interference in the network. We investigate the interference effects on a typical moderate (200 MHz) bandwidth network at 28 GHz and a high (2 GHz) bandwidth network at 72 GHz carrier frequency, with 4 sector APs and validate the trends observed with the help of detailed system-level simulations using METIS-like scenarios.

I. INTRODUCTION

Recently, there has been a growing interest to investigate the possibility of using millimeter wave (mmW) and centimeter wave (cmW) frequency bands for next generation cellular communication [1]–[4]. The results of recent link level experiments, system simulations and analytical studies in [3]–[7] have highlighted that highly directional transmission and dense deployment of access points (APs) are key enabling features of cellular networks operating at these high frequency bands. The large amount of spectrum available in these bands could make it possible to design a dense yet noise-limited cellular network [7], [8], and thus the expected 5G requirements (greater than 10 Gbps peak rates and 100 Mbps cell-edge rates [4]) could be met without the need of complicated interference mitigation or other MIMO techniques.

At mmW frequencies, a promising way to achieve highly directional transmission is through the use of arrays of patch antennas [9]. The use of multiple such patch antenna arrays per AP is necessary for providing omni-directional coverage, thus making sectorization a natural choice for mmW APs. Additionally, it could be desirable to have two of these antennas per sector, where the two would have orthogonal polarizations relative to each other (e.g., one horizontally polarized and the other vertically polarized) [3], [4]. The reason for dual polarization is two-fold; first, it enables an approximate doubling of the peak data rate through two-stream SU-MIMO (assuming the user device also has two polarizations available) and second, it also improves robustness to random polarizations at the user device.

Although directional transmission helps to overcome the path losses at the mmW frequency bands, system capacity evaluations in [4], [6]–[8] show the importance of having a sufficiently dense network in order to achieve high data rates in the order of Gbps. Densifying the network boosts data rates by: (a) reducing the average number of users connected to each AP and (b) decreasing the average path loss over the user-AP service links. Sectorizing the APs would distribute the users connected to an AP over different sectors and thus, we would expect gain in data rates with sectorization as well [10]. Since leasing sites for deploying APs is a costly affair for cellular operators, sectorization can be a good way to boost data rates with comparatively lower AP densities in order to meet the 5G data rate requirements.

In this paper, we propose an analytical framework for evaluating the coverage and rate performance of a mmW cellular network with sectorized APs, operating in downlink. We observe that sectorization can definitely improve data rates but at the expense of increased interference, which might render moderate bandwidth 5G networks to be interference-limited. We validate our observations with the help of detailed system-level simulations [11].

II. SYSTEM MODEL

Consider a mmW cellular network operating in downlink at frequency \( f_c \) and with bandwidth \( B \) in a X-Y plane centred at an outdoor user, whose performance is to be evaluated. The APs are assumed to be distributed as a homogeneous Poisson point process (PPP) \( \Phi \) with density \( \lambda \) [12]. It is assumed that the user at origin can see two types of APs based on the propagation channel between the user and the AP - line of sight (LOS) and non line of sight (NLOS). Let us denote the point processes governing the locations of these two type of APs by \( \Phi_L \) and \( \Phi_N \), respectively. It is assumed that each of these two point processes are obtained from \( \Phi \) by independent thinning with probabilities \( p_L(x) \) and \( p_N(x) = 1 - p_L(x) \), respectively, where \( x \) is the distance of an AP from the origin. Thus, the densities of \( \Phi_L \) and \( \Phi_N \) are given by \( \lambda_L(x) = \lambda p_L(x) \) and \( \lambda_N(x) = \lambda p_N(x) \), respectively [13].

Each AP is assumed to provide omni-directional coverage with the help of K sector beams. Each sector has an angular spread of \( 2\pi/K \) radians and uses two sets of antenna arrays, one with horizontal and other with vertical polarization, that are responsible for providing directional coverage to the users that lie in the sector. We assume that the two polarization

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streams are completely orthogonal to each other, and there is no cross-talk. We, thus, model the two-stream transmission as two single-stream transmissions with transmit power per stream equal to half the total transmit power of each sector. Users are assumed to have two co-located omni-directional antennas (one for each stream). Henceforth in this section, as well as in section III, we will focus on transmissions across a single stream, unless specified otherwise.

Let \( X_e, Y_e \) denote the X-Y co-ordinate axes shifted to a location \( \epsilon \in \Phi \). All angles measured with respect to (w.r.t.) \( X_e, Y_e \) axes are assumed to be in radians, modulo \( 2\pi \) and measured counter-clockwise w.r.t. positive \( X_e \) axis. Let \( \theta_e \) denote the angle w.r.t. \( X_e, Y_e \), along which a beam of the AP at \( \epsilon \) should be directed for offering minimum path loss to the user at origin of X-Y axes. In this work, we assume that \( \theta_e \) points along the line joining the AP and the user. Thus,

\[
\hat{\theta}_e = (\pi + \text{arg}(\epsilon)) \mod 2\pi,
\]

where \( \text{arg}(\epsilon) \) is the argument of \( \epsilon \) in the X-Y plane.

Let \( S_{e,i} \) denote the sector \( [2\pi(i - 1)/K, 2\pi i/K] \) in the \( X_e, Y_e \) plane, where \( i = 1, 2, \ldots, K \). The power received by the user under consideration from the antenna array of sector \( S_{e,i} \) is given by

\[
P_r(\theta_{e,i}, \hat{\theta}_e, \epsilon) = P_t H_e G(\theta_{e,i}, \hat{\theta}_e) L(\epsilon)/C_0,
\]

where \( P_t \) is the transmit power per data stream (half of the transmit power allocated to each sector), \( H_e \) is the small scale fading (assumed to be independently and identically distributed to an exponential random variable with unit mean), \( G \) is the antenna array gain, \( \theta_{e,i} \) is the angle w.r.t. the \( X_e, Y_e \) plane at which the beam in sector \( S_{e,i} \) is directed, \( C_0 \) is the free space path loss at reference distance 1 meter, the path gain \( L(\epsilon) = ||\epsilon||^{-\alpha_L} \) if the link is LOS, \( L(\epsilon) = ||\epsilon||^{-\alpha_N} \) if link is NLOS. Here \( \alpha_L \) and \( \alpha_N \) are the path loss exponents for LOS and NLOS links, respectively. For the sake of simplifying the analysis, we ignore the log-normal shadow fading (which, however, can be incorporated as done in [7]) and also ignore the Rician nature of the channel in LOS conditions.

The antenna array gain is modeled on the lines of [6], [7] as

\[
G(\theta_{e,i}, \hat{\theta}_e) = \begin{cases}
M & \text{if } |\theta_{e,i} - \hat{\theta}_e| < \frac{\pi}{2} \text{ or } 2\pi - |\theta_{e,i} - \hat{\theta}_e| < \frac{\pi}{2} \\
m & \text{otherwise,}
\end{cases}
\]

where \( M \) is the half-power beamwidth of the AP antenna arrays in the azimuth plane. The event \( G = M \) denotes that the beam of sector \( S_{e,i} \) is perfectly aligned with the user. The front to back ratio is given by \( \text{FBR} = M/m \). It is assumed that \( \Delta_n < 2\pi/K \).

This is a sufficient condition to ensure that the beams are narrow enough such that at most two beams of an AP can perfectly align with a user at the same time. A sector beam is assumed to be always active, irrespective of whether it is serving a user or not.

A user is assumed to associate with the sector providing largest \( L(\epsilon) \). Let \( \epsilon^* \in \Phi \) and \( i^* \in \{1, 2, \ldots, K\} \) be such that the antenna array of sector \( S_{e,i^*} \) serves the user under consideration. It is assumed that \( \theta_{e,i^*} = \hat{\theta}_e \). For all interfering sectors, \( \theta_{e,i} \) is independently and uniformly distributed in \([2\pi(i - 1)/K, 2\pi i/K]\), where \( \epsilon \in \Phi \) and \( i \in \{1, 2, \ldots, K\} \).

Let us denote the signal to noise ratio, signal to interference ratio and the signal to interference plus noise ratio by SINR, SIR and SINR, respectively. The SINR of the user under consideration is given as,

\[
\text{SINR} = \frac{P_r(\theta_{e,i^*}, \hat{\theta}_e, \epsilon^*)}{I_L + I_N + I_S + \sigma^2},
\]

where \( I_L \) and \( I_N \) denote the interference from LOS and NLOS APs (excluding the AP at \( \epsilon^* \)), respectively. \( I_S \) denotes interference from the remaining \( K - 1 \) sectors of the serving AP and \( \sigma^2 \) denotes the noise power. Note that \( \sigma^2 \text{ dB } = -174 \text{ dBm/Hz } + 10\log_{10}(B) + \text{NF} \), where NF is the noise figure in dB. Here,

\[
I_S = \sum_{i=1,\ldots,K;i\neq i^*} P_r(\theta_{e,i}, \hat{\theta}_e, \epsilon),
\]

\[
I_j = \sum_{\epsilon \in \Phi,j \neq \epsilon^*} \sum_{i=1}^K P_r(\theta_{e,i}, \hat{\theta}_e, \epsilon),
\]

where \( j \in \{L, N\} \).

The net downlink rate (combining both polarization streams) achieved by a user connected to a sector serving \( N \) other users is modeled as [14]

\[
\text{Rate} = \frac{2 B}{N + 1} \log_2 (1 + \text{SINR}).
\]

In the next section, we find the complementary cumulative distribution function (CCDF) of SINR and rate. Due to space constraints, detailed proofs are skipped, although a high-level idea of proof is provided.

### III. SINR and Rate Coverage

#### A. Laplace transform of interference from other APs

Let us tag each interfering AP by the type \( j \in \{L, N\} \) that represents whether it is LOS/NLOS w.r.t. the user at the origin of the X-Y plane. Specifically, \( j = L \) for LOS and \( j = N \) for NLOS. Similarly, let us tag the AP to which the user is connected by the type \( q \in \{L, N\} \).

**Lemma 1.** Given that the user is connected to an AP of type \( q \) at a distance \( r \), the laplace transform of the interference from \( j \) type of APs is given by

\[
L_j^q(t, r) = \exp \left( \int_0^\infty \frac{1}{1 + C_0 P_t v \gamma} \Psi \right),
\]

where \( \Psi \) is given by

\[
\Psi = \begin{cases}
\left( 2M + (K - 2)m \right) w. p. \left( \frac{K \Delta_n}{2\pi} \right)^3, & \text{for } K > 1, \\
M + (K - 1)m & \text{w. p. } \frac{K \Delta_n}{2\pi}, & \text{for } K = 1,
\end{cases}
\]

and for \( K = 1, \)

\[
\Psi = \left\{ \begin{array}{ll}
M & \text{w. p. } \frac{\Delta_n}{2\pi} \\
m & \text{otherwise,}
\end{array} \right.
\]
Proof highlights: Define $\psi(\theta_j, \theta_{e,1}, \ldots, \theta_{e,K}) = \sum_{i=1}^{K} G(\theta_{e,i}, \theta_j)$. Given that $\theta_j = \theta$ the distribution of this random variable can be found as a function of $\theta$ using the fact that for all interfering sectors, $\theta_{e,i}$ are uniformly distributed in $[2\pi(i-1)/K, 2\pi i/K)$ and independent of $\theta_{e,k}$ for $k \neq i$.

The Laplace transform of interference field $I_j$, given that the user is connected to an AP of type $q$ is given by
\[
L_q^I(t, r) = \mathbb{E} \left[ e^{-tI_j} | ||\epsilon^*|| = r \right], \quad \text{serving AP is type } q.
\]

The above equation can be simplified using the formula for Laplace functional of an independently marked PPP [13]. Note that $H_\epsilon$ is an independent mark of each $\epsilon \in \Phi \setminus \epsilon^*$. It can be shown that $\psi(\theta_e)$ is also an independent mark of $\epsilon \in \Phi \setminus \epsilon^*$.

B. SINR coverage

**Theorem 1.** The downlink SINR distribution of a typical user in a mmW cellular network with $K$ sector APs, is given by
\[
\mathbb{P}(\text{SINR} > \tau) = \sum_{q \in \{1, \ldots, K\}} \int_0^\infty \mathbb{E}_{\Psi_S} \left[ \exp(-\gamma_q(\Psi_S, r)\sigma^2) \right. 
\times L_q^I \left. (\gamma_q(\Psi_S, r), r) L_q^I (\gamma_q(\Psi_S, r), r) \right] f_q(r) \mathcal{Q}(\rho_q^2(r)) dr,
\]
where $\overline{\Psi} = \bar{\Psi}$ if $q = N$ and vice versa. Here,
\[
Q_q(r) = \exp(-2\pi \int_0^r v\lambda_{\overline{\Psi}}(v) dv)
\]
\[
f_q(r) = 2\pi r \lambda_{\overline{\Psi}}(r) \exp(-2\pi \int_0^r v\lambda_{\overline{\Psi}}(v) dv)
\]
and
\[
\gamma_q(\Psi_S, r) = \begin{cases} \frac{C_0}{r} & \text{if } M - \Psi_S r > 0, \\ \infty & \text{otherwise}. \end{cases}
\]
For $K > 1$,
\[
\Psi_S = \begin{cases} M + (K-2)m & \text{w. p. } \frac{1}{4} \left(\frac{K-2}{2\pi}\right)^2, \\ (K-1)m & \text{otherwise}. \end{cases}
\]
and for $K = 1$, $\Psi_S = 0$.

Proof of this theorem is on similar lines as Theorem 3 in [6]. The expressions for $Q_q(r)$ and $f_q(r)$ can be further simplified if expressions of $p_L(x)$ and $p_N(x)$ are known.

C. Rate coverage

**Lemma 2.** Assuming that the users are distributed as a homogeneous PPP with density $\lambda_u$ per km², the probability mass function of the number of other users ($N$) associated with the sector serving the user under consideration is approximated by
\[
p_N(n) = \frac{3.5^{3.5}}{n!} \frac{\Gamma(n+4.5)}{\Gamma(3.5)} \left( \frac{\lambda_u}{K\lambda} \right)^n \left( 3.5 + \frac{\lambda_u}{K\lambda} \right)^{(-n-4.5)},
\]

Proof highlights: Since the user association rule in this work is stationary [15], the mean association area of a sector in a K-sector AP can be shown to be equal to $1/K\lambda$. A gamma distribution approximation is done as per [7] in order to arrive at the above result.

**Theorem 2.** Assuming that a signal cannot be decoded below SINR threshold $\tau_0$, the rate coverage of a typical user in a mmW cellular network with two-stream transmissions is given by [7]
\[
\mathbb{P}(\text{Rate} > \tau_r) = \sum_{n=0}^\infty p_N(n) \mathbb{P}(\text{SINR} > \max \left(2\left(\frac{n+1}{\pi} + \frac{\tau_r}{\eta}\right) - 1, \tau_0\right)).
\]

In the next section, we describe the setup for detailed system-level simulations using METIS-like scenarios [16]. Note that a full description of the system-level simulations is given in [11] and we provide a brief summary here.

IV. SYSTEM SIMULATION SETUP

The simulation layouts are based on [16] and are shown in Fig. 1. AP locations are marked with an x and the red area indicates the region over which the user locations are averaged. The total area is a $1161 \times 1656$ m² grid which contains a $3 \times 3$ repetition of the same building blocks as shown in red in the figure. 8400 users are distributed uniformly along sidewalks of the roads and the park in 10 drops. Two AP layouts are considered, the first, layout A, has 144 APs placed at the intersection of two streets and the second, layout B, adds a second AP at each intersection to improve the chance of a LOS link being available to each user. A LOS probability is assigned to each user based on two blocking mechanisms. The first is whether a building is in between the AP and the user and the second models other blocking such as foliage, vehicles, and people as described in [11]. The second blocking mechanism is assigned independent at each AP regardless of how they are to each other. All APs have 4 sectors and there are two sets of antenna arrays per sector, one with horizontal polarization and the second with vertical polarization. The user devices have two omni-directional antennas, one with horizontal polarization and the other with vertical polarization.
increasing the number of sectors to 

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_t$</td>
<td>27.8 dBm/sector/stream</td>
<td>$NF$</td>
<td>10 dB</td>
</tr>
<tr>
<td>$C_0$</td>
<td>$\frac{34f_c}{(T \times 10^8)^2}$</td>
<td>$FBR$</td>
<td>30 dB</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>2</td>
<td>$\alpha_N$</td>
<td>3.3</td>
</tr>
<tr>
<td>$\Delta_n$</td>
<td>$260$</td>
<td>$M$</td>
<td>16 dB</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0.1012$</td>
<td>$\eta$</td>
<td>$0.932$</td>
</tr>
<tr>
<td>$a$</td>
<td>$0.0078$</td>
<td>$b$</td>
<td>0.1</td>
</tr>
<tr>
<td>$c$</td>
<td>0.8</td>
<td>$\tau_0$</td>
<td>-10 dB</td>
</tr>
</tbody>
</table>

V. NUMERICAL RESULTS

In this section, we discuss the numerical results. The 28 GHz band is somewhat fragmented, making contiguous allocations of greater than 500 MHz bandwidth difficult unless we resort to carrier aggregation [17]. At 72 GHz, however, it is feasible to allocate contiguous bandwidths in the order of GHz [4], [17]. In this work, we focus on a moderate bandwidth network, with $B = 200$ MHz, operating at $f_c = 28$ GHz and a high bandwidth network, with $B = 2$ GHz, at $f_c = 72$ GHz.

We use the following blockage model in order to compare with the system simulations in Section IV,

$$p_b(x) = \begin{cases} 
\exp(-\beta x + \eta)(1 - \min(ax + b, c)) & \text{if } x < \eta/\beta \\
1 - \min(ax + b, c) & \text{otherwise},
\end{cases}$$

(17)

where $\beta, \eta, a, b, c$ are some constants. The exponential decay term, inspired from [18], captures the blockage effects due to buildings, whereas the other term accounts for blockages due to foliages [11]. Table I specifies the system parameters used for generating the analytical results. Path loss parameters and the values of $a, b, c$ are based on [4]. The region under consideration has $\chi = 60.67\%$ of area under buildings, average area of buildings is $\mu = 8641.4$ m$^2$ and average perimeter of a building is $\nu = 384$ m. We estimate $\beta, \eta$ based on [18], that is $\beta = \nu\eta/\pi\mu$ and $\eta = -\ln(1 - \chi)$. An analytical equivalent of layout A has $\lambda = 75$ APs per km$^2$ and $\lambda_0 = 437$ users per km$^2$. Similarly, $\lambda = 150$ APs per km$^2$ for layout B.

Although the parameters in Table I are tailored to fit the more elaborate system simulations, it is important to note that the comparison between the simulation and analytical results is mainly for the purpose of showing similar trends. The different numerical estimates of $\text{SINR}$ coverage with the analysis and system-simulations can be attributed to the following assumptions in the analysis: (a) users and APs are distributed as PPP (gives conservative estimates of $\text{SINR}$ distribution [12]), (b) use of a blockage model instead of actual buildings and (c) the antenna beam pattern and propagation channel is simplified.

Fig. 2 shows the impact of sectorization on rate coverage of the network operating at 28 GHz. It can observed that increasing K from 1 to 4 significantly boosts the data rates. The increase in data rates is expected as the number of users connected per sector goes down, on an average, with the increasing number of sectors. However, note that further increasing the number of sectors to $K = 8$ does not give
any gain over \( K = 4 \). This can be explained as a result of the fact that due to finite user density, the gain in data rates as a result of decreasing load per sector would saturate with increasing number of sectors. Coupled with this there is also increased interference in the network, which reduces the SINR coverage. Unlike increasing the AP density, which not only increases the interference power but also decreases path loss to a user from the serving AP, increasing the number of sectors per AP does not decrease path loss to the serving AP and thus degrades the SINR coverage.

Although not shown here due to space constraints, similar trends can be observed for the high bandwidth network at 72 GHz. However, the degradation in SINR performance with \( K \) would be milder in this case. This can be explained from Fig. 3. It can be observed that both the system-level simulations and the analysis show that the high bandwidth network with 4 sector APs is pre-dominantly noise-limited. We can see that even in the ultra-dense case when \( \lambda_o = 150 \) APs/km\(^2\), that is about 600 outdoor sectors/km\(^2\) (greater than \( \lambda_o = 437 \) users/km\(^2\)), the gap in SINR and SIR distributions is within 5 dB threshold. Due to the noise-limited behavior, it can be observed that increasing the AP density improves the SINR coverage. In contrast to this, the analytical and simulation results in Fig. 4 show that the SINR coverage of the moderate bandwidth network at 28 GHz does not improve significantly when AP density increases from 75 APs/km\(^2\) to 150 APs/km\(^2\). Also the distribution of SIR \( \approx \) SINR. Thus, it would be fair to characterize the moderate bandwidth network under consideration to be interference-limited.

VI. CONCLUSION AND FUTURE WORK

To the best of authors’ knowledge, this is the first work to incorporate sectorized APs in the coverage and rate analysis of mmW cellular networks. The coverage and rate trends demonstrate that sectorization, which comes as a natural choice through use of patch antennas for enabling highly directional transmission, significantly boosts the data rates at the expense of increased interference in the network, which might even render moderate bandwidth networks to be interference-limited. Investigating the impact of sectorization on backhauling would be an avenue of future work.

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