Low Complexity Quantization Codebooks for CoMP

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Abstract—Coordinated multipoint (CoMP) is an interference mitigation technique in LTE release 10, which exploits base station (BS) cooperation to improve throughput for cell-edge users. An important new feature present in CoMP is that variable numbers of BSs can service a given user. This poses a new problem for beamforming that is not present in single-cell operation: the quantization codebook must support a variable dimension codebooks, with the dimension corresponding to the number of BSs employed. This is a problem that has not appeared in previous releases of LTE. In this paper, we propose a low complexity structured codebook that has linear complexity in both the codebook size and dimension. As such, the variable dimension codebooks are readily accommodated and the codebook can be constructed online as the number of transmitting BSs varies. We also propose a new method to store optimal structured codebooks—in the sense of the Grassmannian criterion—of variable dimension by exploiting properties of the combinatorial designs known as cyclic difference sets. Although the size of the optimal codebooks is limited, our method reduces storage requirements as subsets of the same parameters are used to construct the codebook for each dimension. We show via simulations that our low complexity codebook construction performs comparably with the standard Fourier codebook obtained using an exhaustive search, with only linear complexity in both size and dimension.

I. INTRODUCTION

Coordinated multipoint (CoMP) has been proposed as an interference mitigation technique in LTE release 10 [1]. In CoMP with joint processing, base stations (BSs) perform distributed beamforming, which significantly improves the throughput of cell-edge users. This is achieved by sharing each user’s data via backhaul links [2,3].

As in single-cell networks, channel state information (CSI) is required for beamforming. This is achieved by using a low-rate feedback link from each user to the BSs [4]. An effective approach to CSI feedback in frequency division duplex (FDD) networks is to employ codebook-based quantization [4–9], which is currently standardized for single-cell networks. In codebook-based quantization, a set of quantized channel vectors are stored at each user. The user then chooses the codeword that best quantizes the actual channel vector.

There are two key differences between quantization for single-cell networks and CoMP. The first is that CSI feedback is potentially required for a large number of links due to a large number of cooperating BSs. The second is that a variable number of BSs can transmit to each user, requiring variable codebook dimension.

Current approaches to the problem of quantization codebook design for CoMP have focused on random constructions. Random approaches are undesirable as both storage and search requirements are prohibitive in contrast with structured approaches such as the Fourier codebook [7]. In [10], a general codebook construction criterion based on the chordal distance was proposed for network MIMO exploiting per-cell codebooks. This technique decomposed the problem of globally determining the optimal codebook for all BSs to a joint codebook composed of codewords obtained by only considering individual cells. Unfortunately, specific constructions were not considered and analysis was performed only for random constructions. In [11] and [12], bit allocations for different channels and codeword selection algorithms were proposed, respectively, when per-cell codebooks are employed. As in [10], the problem of constructing structured codebooks was not considered.

In this paper, we propose a low-complexity structured codebook construction for a CoMP network. In particular, we overcome the problem of prohibitive complexity of variable dimension Fourier codebook construction, which preserves the high-performance of the Fourier (or Grassmannian) codebook [7]. Our algorithm to construct variable dimension codebooks has a complexity linear in both codebook size and dimension (which corresponds to the total number of antennas used by the transmitting BSs). This is a significant reduction compared with the exhaustive search technique, which has exponential complexity. Our construction is based on a new recursive construction that alleviates the problem of computing exponential sums, which are the main difficulty in analyzing and designing optimal Fourier codebooks.

Next, we propose a new storage method for Fourier codebooks, which are also equiangular tight frames (ETFs)—an optimal class of codebooks. We show that ETFs have good performance in large dimensions and exploit this, as yet unused, property to develop our storage method. A key feature of our approach is that only a single set of coefficients are required to construct Fourier codebooks when varying numbers of BSs can transmit. This means that the storage requirements are comparable to codebooks used in LTE, while providing for optimality.

We show via simulations that our low-complexity construction performs comparably with the exhaustive search, with linear complexity in the size of the codebook and dimension of the codewords. This means that codebooks of varying size and dimension can be constructed online as there are changes over time of the number of coordinated BSs or signal-to-interference and noise ratios (SINR) of the links.
II. System Model

Consider a CoMP network, with $K$ BSs and a single-antenna user. BS $i$ has a total of $N_i$ transmit antennas, and is at a distance $R_i$ from the user. We assume that the data for the user and the CSI from the BSs to the user is sent to a central unit (CU) via error-free and zero-latency backhaul links. The channel is block fading and the network operates using FDD, which means that reciprocity in the channel cannot be exploited in sharing CSI between the user and each BS.

In each time slot, the user that is serviced by the coordinated BSs perfectly estimates the channel between itself and each BS. The user then forms a channel vector

$$\mathbf{h} = [\mathbf{h}_1^T, \ldots, \mathbf{h}_d^T]^T,$$

where $\mathbf{h}_i$ is the channel between the user and BS $i$, and $d = \sum_i N_i$ is the total number of antennas employed by all the coordinated BSs in the time slot. The user then computes the channel shape

$$\tilde{\mathbf{h}} = \mathbf{h} / \|\mathbf{h}\|.$$  

At the user and the CU, a codebook consisting of $d$-dimensional quantization codewords $\mathbf{F} = [\mathbf{f}_1, \ldots, \mathbf{f}_N]$, $\|\mathbf{f}_i\|^2 = 1$, $i = 1, 2, \ldots, N$, of size $N$ is stored. Once the user has estimated the channel vector, it computes the inner product between each codeword and the channel shape to determine which codeword index to feedback to the CU. In particular,

$$i^* = \arg \max_{i=1,\ldots,N} |\mathbf{h}^* \mathbf{f}_i|^2,$$

is the codeword index fed back to the CU. The coefficients of the codeword $i^*$ are then shared with each BS to perform beamforming.

The throughput using distributed beamforming is given by

$$R = \log_2 \left( 1 + \frac{\|\mathbf{h}\| \mathbf{P} \mathbf{f}_{i^*}^\dagger \|^2}{N_0} \right),$$

where $\mathbf{P} = \text{diag}\{P_1, \ldots, P_d\}$ is the transmit power matrix and $N_0$ is the noise power. Note that (4) has a similar form to single-cell MISO transmission. The difference lies in the fact that the average channel gains in $\mathbf{h}$ may be different for each element $h_i$ due to the fact that the distance between the user and each BS may vary. The transmit signal-to-noise ratio (SNR) for BS $i$ is given by $P_i/N_0$.

III. Codebook Design

In practical wireless systems, random quantization codebooks are infeasible due to high storage and search complexity. In order to alleviate these problems, structured codebooks are required. Structured codebook construction has been extensively studied for single-cell networks (see [4–7, 9] and the references therein). In this section, we show that codebook design criteria to maximize the throughput for single-cells can in fact be directly translated to CoMP networks and as such the Fourier codebook is a desirable choice.

We first observe that CoMP networks employ a CU to compute the beamforming vector in a centralized fashion. As such, CoMP networks can be regarded as a generalization of standard single-cell beamforming. Moreover, since we only require the channel shape in order to perform beamforming, there is no difference in the computation of the beamforming vector for CoMP networks and single-cell networks that only employ the channel shape. As a result, we can exploit the design criteria for quantization codebooks employed in single-cell networks.

A common design criterion for quantization codebooks in the MIMO broadcast channel is the Grassmannian criterion [7], which minimizes the chordal distance between codewords. This is given by

$$\min \max_{i \neq j} |\mathbf{f}_i^\dagger \mathbf{f}_j|^2.$$  

Unfortunately, the optimal codebook using the Grassmannian criterion is not known or unstructured for most codebook sizes.

In [5], a coarser criterion known as the expected square correlation (ESC) was proposed for the MIMO broadcast channel with zero-forcing (ZF) precoding. As the beamforming is used to obtain the throughput in (4) is a special case of ZF, the ESC can be used to find classes of codebooks with high throughput. The Grassmannian criterion can then be applied to optimize over the class of codebooks, which is significantly easier than the original problem of optimizing over all possible codebooks. The ESC criterion is given by

$$\max_{\mathbf{F}} \mathbb{E} \left[ 1 - |\mathbf{f}_i^\dagger \mathbf{f}_j|^2 \right].$$

A key property of the ESC criterion is that it is minimized by codebooks with group structure. That is, the codebooks constructed as

$$\mathbf{F} = [\mathbf{U}_1 \phi, \ldots, \mathbf{U}_N \phi],$$

where $\{\mathbf{U}_i\}_{i=1}^N$ is an algebraic group of unitary matrices and $\phi \in \mathbb{C}^d$ with $|\phi|^2 = 1$. An important class of ESC-minimizing codebooks are known as Fourier (or Grassmannian) codebooks, where the group of unitary matrices forms a cyclic group. The Fourier codebook can be easily constructed using the direct sum to obtain a reducible representation of a cyclic group generated by

$$\mathbf{U} = \chi_1 \oplus \cdots \oplus \chi_d$$

$$= \left(\begin{array}{ccc} e^{2\pi j k_1/N} & \cdots & 0 \\ 0 & \ddots & \vdots \\ 0 & \cdots & e^{2\pi j k_d/N} \end{array}\right),$$

where $\oplus$ is the direct sum and $\chi_i = e^{2\pi j k_i/N}$ for some $1 \leq k_i \leq N-1$. Hence, each codeword in the Fourier codebook is given by $\mathbf{f}_l = \mathbf{U}^\dagger \phi$, $l = 0, \ldots, N-1$, where $\phi = 1/\sqrt{d}[1, 1, \ldots, 1]^T$.

Unfortunately, there is no known exact low complexity search algorithm to obtain the coefficients $k_1, \ldots, k_N$. Instead, an exhaustive search of exponential complexity or a random search is usually employed [13]. To reduce the complexity, we propose an approximate low complexity algorithm in the next section to obtain the coefficients.
IV. PROPOSED LOW COMPLEXITY CONSTRUCTION

In this section, we propose a low complexity construction of the Fourier codebook based on the Grassmannian criterion. A key problem in CoMP that does not appear in the single cell case is that codebooks must be constructed for variable numbers of BSs and codebook sizes, which changes the dimension of the codebook. Our codebook construction has complexity of order $O(Nd)$ in contrast to the standard exhaustive search approach, which has exponential complexity.

Our proposed construction is based on an iterative procedure, which generates one coefficient at a time. In order to develop our construction, we require a new criterion based on the Grassmannian criterion in order to generate the coefficient at each step.

The first step is to observe that the norm-squared inner product—the basis of the Grassmannian criterion—between two codewords for $d = 2$ codebook is

$$|f_k^i f_l^i|^2 = \frac{1}{2} \sum_{m=1}^{2} e^{-2\pi j k_m (l_1 - l_2)/N}$$

$$= \frac{1}{4} e^{2\pi j (l_1 - l_2) (k_1 - k_2)/N} + e^{2\pi j (l_1 - l_2) (k_2 - k_1)/N}$$

$$= \frac{1}{2} + \frac{1}{4} (\psi_1^\ast \psi_2 + \psi_1 \psi_2^\ast),$$

(9)

where $\psi_i = e^{2\pi j (l_1 - l_2) k_i/N}$. Denote $l_1, l_2$ as the pair of codewords that maximize (9), with $l_1 \neq l_2$.

Now, the norm-squared inner product for the case where $d = 3$ and codebook size $M$ is given by

$$|f_k^i f_l^i|^2 = \frac{1}{3} \sum_{m=1}^{3} e^{-2\pi j k_m (l_1 - l_2)/M}.$$  

(10)

The next step is to choose $M(l_1 - l_2) = N(l_1 - l_k)$ for some $l_1, l_k \in \{1, 2, \ldots, M\}$ (recall that $l_1, l_2$ are the pair of codewords that maximize (9)). This allows (10) to be written in terms of $\psi_1, \psi_2$ and hence the coefficients $k_1, k_2$ to be obtained from (9). In particular, we have

$$\max_{i \neq j} |f_k^i f_l^j|^2 = \frac{1}{3} \sum_{m=1}^{3} e^{-2\pi j k_m (l_1 - l_2)/N}$$

$$\geq \frac{1}{9} \left( \psi_1^\ast \psi_2 + \psi_1 \psi_2^\ast + e^{2\pi j (k_1 - k_2)(l_1 - l_2)/N} + e^{2\pi j (k_2 - k_3)(l_1 - l_2)/N} + e^{2\pi j (k_3 - k_1)(l_1 - l_2)/N} \right),$$

(11)

where $\psi_1^\ast \psi_2 + \psi_1 \psi_2^\ast$ is the same as in (9). The bound follows by noting that the inner product of codewords $l_1$ and $l_2$ must be less than or equal to the maximum pair.

More generally, the following bound holds as the dimension is increased further

$$\max_{k \neq l} |f_k^i f_l^i|^2 \geq \frac{1}{d^2} \left( d + \sum_{i=1}^{d} \sum_{j=1, j \neq i}^{d} \psi_j \psi_i^\ast \right),$$

(12)

where $\psi_1 = 1$. As the dimension of the codebook is increased, the new characters can be computed as

$$\psi_i = e^{2\pi j k_i (l_1 - l_2)/N}.$$  

(13)

We observe that the contribution in (12) of each new term corresponding to a unit increase in the codebook dimension is given by

$$\sum_{i=1}^{d+1} \psi_i \psi_{d+1}^i + \psi_i^\ast \psi_{d+1}.$$  

(14)

This sum provides a method for choosing $k_{d+1}$. In particular, we obtain $k_{d+1}$ by solving the optimization problem

$$\min_{k_{d+1} = 1, 2, \ldots, N} \sum_{i=1}^{d+1} \psi_i \psi_{d+1}^i + \psi_i^\ast \psi_{d+1}.$$  

(15)

We now explicitly state our proposed construction for the Fourier codebook.

1) Obtain the coefficients $k_1, k_2$ using the Grassmannian criterion and exhaustive search for the $d = 2$ codebook with $N$ codewords. The exhaustive search is feasible due to the low dimension of the codewords.

2) Set $d = d + 1$ and choose $l_k, l_l$ in (13) such that $M(l_k - l_2) = M(l_1 - l_2)$, where $M$ is the size of the new codebook of dimension $d$.

3) Compute the characters $\psi_l$ in (13) and solve (15) based on the bound in (12) to obtain $k_d$.

4) Repeat from Step 2) until the desired codebook dimension is reached.

In contrast with the standard exhaustive search, this approach only requires $d$ searches over $N$ integers instead of a single search over $N^d$ integers. As such, the complexity of obtaining the coefficients using (15) to construct the codebook is of order $O(Nd)$. We compare the performance of the two approaches in Section VI, where we show that the proposed low complexity construction performs comparably with the exhaustive search approach.

We also note that our low complexity codebook construction can be employed as an initial codebook in other codebook constructions. In particular, it is directly applicable in the schemes proposed in [6, 14].

V. EQUIDISTANT TIGHT FRAMES

In this section, we propose a method to construct a sequence of codebooks with different dimensions using only one set of coefficients, while preserving the optimality of each codebook. This is achieved by using Fourier codebooks that are also equidistant tight frames—codebooks that optimize the Grassmannian criterion. More precisely, an ETF is a codebook that satisfies $\max_{i \neq j} |f_k^i f_l^j|^2 = \frac{N}{d(N-1)}$. In fact, this is the minimum achievable value of $\max_{i \neq j} |f_k^i f_l^j|^2$ as it achieves the Welch bound [15].

Before describing our proposed method, we require a result from frame theory and combinatorics. In particular, we require the notion of a cyclic difference set, which is defined as follows.
We provide two examples to illustrate this idea: subset property to as the particular, there is a class of cyclic difference sets that contain but so far neglected, property of cyclic difference sets. In constructing variable dimension codebooks using only a subset property holds, order the largest cyclic difference set elements of the largest cyclic difference set. This is proved in Proposition 1.

Example 1:
1) \(d = 3, \ N = 7\): \(k = \{1, 2, 4\}\);
2) \(d = 7, \ N = 15\): \(k = \{1, 2, 4, 0, 5, 8, 10\}\).

Example 2:
1) \(d = 4, \ N = 13\): \(k = \{0, 1, 3, 9\}\);
2) \(d = 13, \ N = 40\):
\[
k = \{0, 1, 3, 9, 5, 15, 22, 25, 26, 27, 34, 35, 38\}.\]

Our proposed method proceeds as follows:
1) Choose the dimensions and the number of codewords that are required for each codebooks.
2) Construct the cyclic difference sets for each codebook—each cyclic difference set is equivalent to the set of coefficients \(k_1, \ldots, k_d\) for that dimension. If the subset property holds, order the largest cyclic difference set such that the first elements correspond to smaller cyclic difference sets. This is illustrated in both examples above.
3) When a total \(d\) transmit antennas are employed, choose the first \(d\) elements of the largest cyclic difference set.
4) Construct the codebook using (8).

In order to employ our method for the largest possible class of cyclic difference sets, the performance of the codebook should not be changed by the order of the cyclic difference sets within the largest cyclic difference set. This is proved in the following proposition.

**Proposition 1.** Let \(k\) be the coefficients for a Fourier codebook \(F\), and \(k'\) be a permutation of \(k\) that generates the codebook \(F\). Then,
\[
\max_{i \neq j} |f_i^* f_j| = \max_{k \neq l} (f_k^*)^* f_l^*,
\]
where \(f_i, f_j\) are in \(F\) and \(f_i', f_j'\) are in \(F'\).

**Proof:** We observe that the permutation operator is unitary. As such, the codewords in \(F'\) can be written as
\[
f_i' = U_\pi f_j,
\]
where \(f_j\) is in \(F\) and \(U_\pi\) represents the permutation. Since the unitary transformation is an isometry, the proposition follows.

A useful property of difference sets is that the complement is also a difference set. This means that new ETFs can be easily constructed from pre-existing ones. In [15], the following property was proven.

**Proposition 2.** Let \(k\) be the coefficients of an ETF that form a cyclic difference set over \(\mathbb{Z}_N\). Then, \(k' = \mathbb{Z}_N \setminus k\) is also a cyclic difference set corresponding to an ETF.

As a consequence, we have the following useful proposition that shows that sequences of ETFs that are generated using subsets of difference sets exist in pairs. This can be exploited to construct additional codebooks corresponding to new dimensions by taking the complement of each set of coefficients.

**Proposition 3.** Let \(k_1 \subset k_2 \subset \cdots \subset k_p\) be a sequence of sets of coefficients that generate ETFs of dimensions \(d_1, \ldots, d_p\) and codebook sizes \(N_1, \ldots, N_p\) and \(\pi(\cdot)\) be a permutation. Then, the sequence \(\mathbb{Z}_{N_{\pi(1)}} \setminus k_{\pi(1)} \subset \mathbb{Z}_{N_{\pi(2)}} \setminus k_{\pi(2)} \subset \cdots \subset \mathbb{Z}_{N_{\pi(p)}} \setminus k_{\pi(p)}\) are coefficients that generate ETFs of dimensions \(N_1 - d_1, \ldots, N_p - d_p\).

VI. SIMULATION RESULTS

In this section, we demonstrate via Monte Carlo simulations that our low complexity construction performs comparably with the standard construction. While our proposed codebook construction is applicable to BSs with multiple antennas, for the purposes of simulation comparisons we assume that each BS has a single antenna. We assume that each channel is Rayleigh fading and independent. The distance between BS \(i\) and the user is \(R_i\). As such, the distribution of channel SNR between BS \(i\) and the user is given by
\[
f_{\gamma_i}(\gamma) = \frac{1}{\tau_i} e^{-\gamma/\tau_i},
\]
where \(\tau_i = P_i R_i^{-\alpha}/N_0\), \(P_i = 1\) is the transmit power of BS \(i\), \(\alpha = 2\) is the path loss exponent and \(N_0\) is the noise power. The feedback link is assumed to be error-free and zero-latency.

In Fig. 1, the throughput is plotted against the transmit SNR for varying numbers of BSs. The distance from each BS to the user is \(R_i = 1\), \(i = 1, \ldots, d\). We observe that our construction performs comparably with the exhaustive search construction for all numbers of BSs. The largest gap in performance is less than 0.15 bits/s/Hz at 15 dB and \(d = 6\). We also note that the performance gain by exploiting additional BSs diminish as the number of BSs increases. This is expected as each BS is at the same distance from the user.
We proposed a low complexity codebook construction for CoMP with multiple antenna BSs and a single antenna user. Our construction was based on a new recursive lower bound on the maximum inner product between codewords. This allowed us to deal with the new problem in CoMP of constructing variable dimension codebooks. We also proposed a new method to reduce storage requirements without compromising performance by requiring only one set of coefficients for several codebook dimensions. We demonstrated the performance of our low complexity construction via simulations. We showed that our construction can perform comparably with the standard exhaustive search construction, while only requiring construction complexity linear in the total number of antennas and codebook size.

VII. CONCLUSION

In Fig. 2, the throughput is plotted against the transmit SNR for varying numbers of BSs. In this case, the distance to the user for BS $1$ is $R_1 = 100$ and $R_i = 1, 2, \ldots, d$. We observe that our construction performs comparably with the exhaustive search. In fact, the performance reduction using the proposed construction is negligible. In this figure, increasing the number of BSs results in an improved performance gain compared to the scenario in Fig. 1. This is due to the fact that one BS-user link is degraded due to the large transmission distance.

Fig. 1. Plot of throughput versus transmit SNR when the proposed codebook and exhaustive search construction are employed for varying numbers of transmitting BSs. The distances between each BS and the user are $R_i = 1, i = 1, \ldots, d$.

Fig. 2. Plot of throughput versus transmit SNR when the proposed codebook and exhaustive search construction are employed for varying numbers of transmitting BSs. The distances between each BS and the user are $R_i = 100$, $R_1 = 1, i = 2, \ldots, d$.

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