Abstract—We develop a general framework for design of receivers for the wireless relay channel. We derive the optimum detectors for various degrees of channel state information (CSI) at the destination. We consider both the case when the destination has access to full knowledge of the CSI and the case when it only knows the statistics of the channel. High-SNR and low-SNR approximations of the detectors are presented as well.

I. INTRODUCTION

Relays in the form of analogue repeaters have been an essential building block of communication systems for a long time. The relays traditionally have been used to mitigate the effect of path loss for obtaining robust cross-continental and cross-oceanic communication. Satellite communication and optical communication are two examples of systems that use relaying. In radio communications, path loss and fading are two major sources of performance degradation. Therefore, cooperative communication in which two or more users in a network pool their resources to form a virtual antenna array has recently drawn much attention [1]–[7]. For example, one user (source) can benefit from an idle user (relay) in its proximity, by letting the idle user relay the source’s data to the destination, possibly after some processing. Since the information is transmitted through two independent wireless links, a diversity gain can be achieved. This new type of diversity is called user cooperation diversity. At the relay, two fundamental modes of operation are studied in the literature, where the relay either amplifies and retransmits (“amplify-and-forward”) [1], or decodes and retransmits (“decode-and-forward”) [2]–[6] the received signal. In the former case the relay is just a simple analogue repeater, while in the latter case the relay is regenerative and acts as a digital repeater [8]. In this paper we confine our study to the regenerative relays, with focus on uncoded transmission. Note that hard mapping at the relay strictly speaking is not optimal. However, the obtained gain by employing a nonlinear mapping at the relay was found to be quite negligible for an uncoded system in [9].

Reference [7] develops a framework for maximum-likelihood detection in cooperative diversity systems. For coherent BFSK modulation [7] derives the optimum receiver under the assumption that accurate channel state information (CSI) for all links are available at the destination. However, when the channel varies fast with time, the relays and the destination may not be able to obtain an accurate estimate of the channel. In this paper, we therefore derive (among other things) the optimum detector when the destination only has access to the statistics of the channel. The combination of both optimal receivers covers slow and fast fading scenarios. To differentiate among different system setups, we use the following definitions:

- **Single-Relay Channel**: a system where there is one relay in the network and no direct link between the source and the destination. (See Fig. 1.)
- **Parallel Relay Channel**: when there is more than one relay in the network and there is neither a direct link between the source and the destination, nor a link between the relays. (See Fig. 2.)
- **Cooperative Relay Channel**: when there is a direct link between the source and the destination and at least one relay in the network. (See Fig. 3.)

To put our contribution in context, we next present a summary of the results in this paper:

- We show that whenever the relay mapper is a sign preserving function, the optimum detector when the destination only has access to the statistics of the channel. The combination of both optimal receivers covers slow and fast fading scenarios.
- We derive the likelihood function for the two cases when the destination has access either to (i) full knowledge of the CSI or to (ii) the statistics of the channel. We also present low and high signal-to-noise ratio (SNR)
By means of simulation, we illustrate that the performance of the optimum detector which uses knowledge of the average source-relay link quality is quite close to the one with the instantaneous knowledge in the cooperative channel.

Maximum-ratio combining (MRC) is in general suboptimal but its performance reaches that of the optimum detector when the probability of a decision error at the relay approaches zero. We prove that the optimum detector for the parallel relay channel is a weighted version of MRC at low SNR.

Next we derive the optimum detector for the single-relay channel.

**Proposition 1.** If the error probability at the relay is less than 50%, the optimum detector for uncoded transmission over the single-relay channel is independent of the performance of the relay.

**Proof:** The optimum detector is given by

\[ \hat{s} = \arg \max_{\hat{a}} p(s | y, a_{rd}, a_{sr}) = \arg \max_{\hat{a}} p(y | s, a_{rd}, a_{sr}), \]

where the equality follows form the fact that the transmitted symbols are equiprobable. Thus, the destination uses the following decision rule

\[ p(y | s = 1, a_{rd}, a_{sr}) > p(y | s = -1, a_{rd}, a_{sr}). \]

The conditional distribution of the received signal at the destination given the channel coefficients and the transmitted symbol at the source is given by

\[ p(y | s, a_{rd}, a_{sr}) = \begin{cases} 1 - P_{e,r|a_{sr}} p(y | s_t = s, a_{rd}) + P_{e,r|a_{sr}} p(y | s_t = -s, a_{rd}), \end{cases} \]

where \( P_{e,r|a_{sr}} \) is the probability of a decision error at the relay and

\[ p(y | s_t = s, a_{rd}) = \frac{1}{\sqrt{2\pi \sigma_n^2}} e^{-\frac{(y - a_{sr}s)^2}{2\sigma_n^2}}. \]

Using (6), (5) simplifies to

\[ (1 - 2P_{e,r|a_{sr}}) p(y | s_t = 1, a_{rd}) > (1 - 2P_{e,r|a_{sr}}) p(y | s_t = -1, a_{rd}) \]

Assuming that \( P_{e,r|a_{sr}} < 1/2 \), (i.e., \( 1 - 2P_{e,r|a_{sr}} > 0 \)), the decision metric reduces to

\[ p(y | s_t = s, a_{rd}) > p(y | s_t = -s, a_{rd}), \]

which does not depend on \( P_{e,r|a_{sr}} \).

Observe that the Proposition 1 suggests that the optimum detector simply is

\[ \hat{s} = \text{sign}(y). \]

As we show in the next theorem, the detector (9) is optimum for a broad class of relay functionalities.

**Theorem 1.** The optimum detector for uncoded transmission over the single-relay channel with a sign persevering relay

\[ y = a_{rd}s_r + n_d, \]

where \( a_{rd} \) is the channel coefficient, \( s_r \in \{ \pm 1 \} \) is the transmitted symbol with \( p(s = 1) = p(s = -1) = \frac{1}{2} \) and \( n_d \) is zero-mean additive white Gaussian noise with variance \( \sigma_n^2 \). The received signal at the destination \( (y) \) is given by

\[ p_{a_{rd}}(x) = \frac{x}{\gamma^2} e^{-\frac{x^2}{2\gamma^2}}. \]
functionality (i.e., \( \text{sign}(f(y_t)) = \text{sign}(y_t) \) where \( f(\cdot) \) is the mapper at the relay) is
\[ s = \text{sign}(y). \]  
(10)

**Proof:** The destination makes a decision based on
\[ p(y|s = 1, a_{rd}, a_{sr}) \uparrow_{+1} p(y|s = -1, a_{rd}, a_{sr}) \]  
(11)
We have
\[ p(y|s, a_{rd}, a_{sr}) = \int_{-\infty}^{+\infty} p(y|s, y_r, a_{rd})p(y_r|s, a_{sr}) \, dy_r \]
\[ = \int_{-\infty}^{+\infty} p(y|y_r, a_{rd})p(y_r|s, a_{sr}) \, dy_r. \]

Let
\[ G(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \]  
(12)

Thus,
\[ p(y|s = 1, a_{rd}, a_{sr}) = \int_{-\infty}^{+\infty} G(y_r, a_{rd}f(y_t), \sigma_n^2) G(y_t, a_{sr}, \sigma_n^2) \, dy_t = \int_{-\infty}^{0} G(y_r, a_{rd}f(y_t), \sigma_n^2) G(y_t, -a_{sr}, \sigma_n^2) \, dy_t + \int_{0}^{+\infty} G(y_r, a_{rd}f(y_t), \sigma_n^2) G(y_t, a_{sr}, \sigma_n^2) \, dy_t \]  
(13)

By change of variable, we obtain
\[ p(y|s = -1, a_{rd}, a_{sr}) = \int_{-\infty}^{0} G(y_r, a_{rd}f(-y_t), \sigma_n^2) G(y_t, a_{sr}, \sigma_n^2) \, dy_t + \int_{0}^{+\infty} G(y_r, a_{rd}f(y_t), \sigma_n^2) G(y_t, -a_{sr}, \sigma_n^2) \, dy_t \]  
(14)

Using (13) and (14), the optimum decision metric reduces to
\[ \int_{0}^{+\infty} A(y_t) \cdot B(y_r, y) \, dy_t \uparrow_{+1} 0, \]  
(15)

where
\[ A(y_t) = \exp\left(-\frac{(y_t - a_{sr})^2}{2\sigma_n^2}\right) - \exp\left(-\frac{(y_t + a_{sr})^2}{2\sigma_n^2}\right) \]
\[ B(y_r, y) = \exp\left(-\frac{(y - a_{rd}f(y_t))^2}{2\sigma_n^2}\right) - \exp\left(-\frac{(y - a_{rd}f(-y_t))^2}{2\sigma_n^2}\right). \]

Note that the integration interval in (15) is \( R^+ \). The factor \( A(y_t) \) in (15) is non-negative since \( y_t \) is non-negative. Since \( f(\cdot) \) is a sign preserving function and \( y_t \) is non-negative, \( B(y_r, y) \) is also non-negative when \( y \) is non-negative and vice versa. This makes (15) equivalent to \( y \geq 0 \).

So far our discussion has been concerned with hard decisions only, but to properly combine multiple received signals, the destination needs to calculate the likelihood ratio of each received signal. This requires the availability of source-relay link quality information at the destination. The likelihood ratio of the received signal at the destination can be written as
\[ \ell = \frac{p(y|s = +1, a_{rd}, a_{sr})}{p(y|s = -1, a_{rd}, a_{sr})} = \frac{(1 - P_{c,r|a_{sr}})e^{-\frac{(y - a_{sr})^2}{2\sigma_n^2}} + P_{c,r|a_{sr}}e^{-\frac{(y + a_{sr})^2}{2\sigma_n^2}}}{(1 - P_{c,r|a_{sr}})e^{-\frac{(y - a_{sr})^2}{2\sigma_n^2}} + P_{c,r|a_{sr}}e^{-\frac{(y + a_{sr})^2}{2\sigma_n^2}}} \]  
(16)

**B. Soft detector based on statistics of CSI**

Equation (16) requires knowledge of accurate instantaneous CSI of the source-relay and the relay-destination links at the destination. This information may not be known. As a remedy we derive the likelihood ratio which needs only know the statistics of the channel. The channel statistics are much easier to obtain and can be used as a priori information at the destination.

Observe that the term \( P_{c,r|a_{sr}} \) in (16) can be readily replaced by \( P_{c,r} = E_{a_{sr}}[P_{c,r|a_{sr}}] \), where \( E[\cdot] \) denotes the expectation. To average out the \( a_{rd} \), we introduce the following result.

**Lemma 1.** The conditional distribution of the received signal at the destination given \( s_t \) is
\[ p(y|s_t) = E_{a_{sr}}[p(y|s_t, a_{sr})] = \frac{\sigma_n e^{-\frac{y^2}{2\sigma_n^2}}}{\sqrt{2\pi\sigma_n^2 + \gamma^2}} \left[ 1 + \sqrt{2\pi\gamma} \frac{e^{-\frac{y^2}{2\sigma_n^2}}}{\sqrt{2\pi\sigma_n^2 + \gamma^2}} Q(-\gamma y s_t) \right], \]
where
\[ m = \frac{\gamma}{\sigma_n \sqrt{\sigma_n^2 + \gamma^2}}. \]  
(17)

**Proof:** The conditional distribution of the received signal at the destination given \( s_t \) and \( a_{rd} \) is
\[ p(y|s_t, a_{rd}) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(y - a_{rd}s_t)^2}{2\sigma_n^2}}. \]  
(18)

Thus,
\[ p(y|s_t = 1) = \int_{0}^{\infty} p(y|s_t = 1, a_{rd} = x)p_{a_{rd}}(x) \, dx \]
\[ = \int_{0}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2 + \gamma^2}} e^{-\frac{(y - a_{rd}s_t)^2}{2\sigma_n^2}} \cdot x \, dx \]
\[ = \frac{1}{\sqrt{2\pi\sigma_n^2 + \gamma^2}} \int_{0}^{\infty} xe^{\gamma x} \, dx. \]
Thus, we have

\[
B(x) = \frac{\sigma_n^2 + \gamma^2}{2\gamma^{2}\sigma_n^2} \left[ \left( x - \frac{\gamma^2}{\sigma_n^2 + \gamma^2} \right)^2 + \frac{\gamma^2\sigma_n^2}{\sigma_n^2 + \gamma^2} y^2 \right].
\]

(19)

Therefore, we have

\[
p(y|s_t = 1) = e^{-\frac{2(\sigma_n^2 + \gamma^2)}{\sqrt{2\pi}\sigma_n^2}} \times \int_{0}^{\infty} x \exp \left[ -\frac{2\gamma^2}{2\sigma_n^2 + \gamma^2} \left( x - \frac{\gamma^2}{\sigma_n^2 + \gamma^2} \right)^2 \right] dx.
\]

Now let \( r = x - \frac{\gamma^2}{\sigma_n^2 + \gamma^2} y \), so that

\[
p(y|s_t = 1) = e^{-\frac{2(\sigma_n^2 + \gamma^2)}{\sqrt{2\pi}\sigma_n^2}} \times \int_{-\infty}^{\infty} \left( r + \frac{\gamma^2}{\sigma_n^2 + \gamma^2} \right) \exp \left( -\frac{r^2}{2\sigma_n^2 + \gamma^2} \right) dr = \frac{\sigma_n e^{-\frac{\gamma^2}{\sigma_n^2 + \gamma^2}}}{\sqrt{2\pi}(\sigma_n^2 + \gamma^2)} \left[ 1 + \sqrt{\frac{\pi}{\gamma^2}} my \right].
\]

where \( m \) is given by (17) and \( Q(x) = \int_{-\infty}^{\infty} e^{-\frac{t^2}{2}} dt \). It is easy to verify that \( p(y|s_t = -1) = p(-y|s_t = 1) \). We thus obtain

\[
p(y|s_t = -1) = \frac{\sigma_n e^{-\frac{\gamma^2}{\sigma_n^2 + \gamma^2}}}{\sqrt{2\pi}(\sigma_n^2 + \gamma^2)} \left[ 1 + \sqrt{\frac{\pi}{\gamma^2}} y \right].
\]

Therefore,

\[
p(y|s_t) = \frac{\sigma_n e^{-\frac{\gamma^2}{\sigma_n^2 + \gamma^2}}}{\sqrt{2\pi}(\sigma_n^2 + \gamma^2)} \left[ 1 + \sqrt{\frac{\pi}{\gamma^2}} my \right].
\]

C. Low- and high-SNR approximations

In this section we present approximations of the likelihood function at low and high SNR.

**Proposition 2.** The likelihood function (16) at low SNR can be approximated by

\[
\ell \approx 1 + 2(1 - 2P_{e,r|a_x}) \frac{\sigma_n y}{\sigma_n^2}.
\]

(22)

**Proof:** The likelihood function (16) can be written as

\[
\ell = \frac{1 - P_{e,r|a_x} + P_{e,r|a_x} e^{-\frac{2\sigma_n y}{\sigma_n^2}}}{(1 - P_{e,r|a_x}) e^{-\frac{2\sigma_n y}{\sigma_n^2}} + P_{e,r|a_x}}.
\]

Using that \( e^x \approx 1 + x \) when \( |x| \ll 1 \), we obtain

\[
\ell \approx 1 - P_{e,r|a_x} \frac{2\sigma_n y}{\sigma_n^2} + P_{e,r|a_x} \frac{2\sigma_n y}{\sigma_n^2}.
\]

(24)

Since \( \frac{1}{1+x} \approx 1 - x \) when \( |x| \ll 1 \), we have

\[
\ell \approx \left( 1 - P_{e,r|a_x} \frac{2\sigma_n y}{\sigma_n^2} \right) \left( 1 + (1 - P_{e,r|a_x}) \frac{2\sigma_n y}{\sigma_n^2} \right)
\]

\[
\approx 1 + 2(1 - 2P_{e,r|a_x}) \frac{\sigma_n y}{\sigma_n^2}.
\]

**Proposition 3.** The likelihood function (16) at high SNR can be approximated by

\[
\ell \approx \frac{1 - P_{e,r|a_x}}{P_{e,r|a_x}} \text{sign}(y).
\]

(25)

**Proof:** Using (23), we obtain

\[
\ell \approx \left\{ \begin{array}{ll}
\frac{1 - P_{e,r|a_x}}{P_{e,r|a_x}} y > 0 \\
\frac{1 - P_{e,r|a_x}}{P_{e,r|a_x}} y < 0
\end{array} \right.
\]

**Proposition 4.** The likelihood function (20) at low SNR can be approximated by

\[
\ell \approx 1 + \sqrt{2\pi} (1 - 2P_{e,r}) y.
\]

(26)

**Proof:** Using \( Q(x) \approx \frac{1}{2} - \frac{x}{\sqrt{\pi}} \) and \( e^x \approx 1 + x \) when \( |x| \ll 1 \), (20) simplifies to

\[
\ell \approx \frac{1 + \sqrt{2\pi} (\frac{1}{2} - P_{e,r})y}{1 - \sqrt{2\pi} (\frac{1}{2} - P_{e,r})y}.
\]

(27)

Using \( \frac{1}{1+x} \approx 1 - x \) when \( |x| \ll 1 \), we have

\[
\ell \approx \left( 1 + \sqrt{2\pi} (\frac{1}{2} - P_{e,r})y \right) \left( 1 + \sqrt{2\pi} (\frac{1}{2} - P_{e,r})y \right)
\]

\[
\approx 1 + \sqrt{2\pi} (1 - 2P_{e,r}) y.
\]
**TABLE I**

<table>
<thead>
<tr>
<th></th>
<th>Statistical Likelihood (i.e., ( p(a_{rd}) ) and ( p(a_{sr}) ) are known.)</th>
<th>Instantaneous Likelihood (i.e., ( a_{rd} ) and ( a_{sr} ) are known.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact Expression</td>
<td>( \frac{1 + \sqrt{2\pi(1 - P_{e,r} - Q(m_y)\int y \exp\left(\frac{(my)^2}{2}\right))} - 1}{1 + \sqrt{2\pi(1 - P_{e,r} - Q(m_y)\int y \exp\left(\frac{(my)^2}{2}\right))}} )</td>
<td>( \frac{1 - P_{e,r,i</td>
</tr>
<tr>
<td>High-SNR Approximation</td>
<td>( \frac{1 - P_{e,r,i</td>
<td>a_{sr}}}{P_{e,r}} ) ( \frac{\text{sign}(y)}{} )</td>
</tr>
<tr>
<td>Low-SNR Approximation</td>
<td>( 1 + \sqrt{2\pi(1 - 2P_{e,r})m_y} )</td>
<td>( 1 + 2(1 - 2P_{e,r,i</td>
</tr>
</tbody>
</table>

**Proposition 5.** The likelihood function (20) at high SNR can be approximated by

\[
\ell \approx \left(1 - P_{e,r,i|a_{sr}} \right) \left(\frac{P_{e,r,i|a_{sr}}}{P_{e,r}} \right) \left(\frac{\text{sign}(y)}{} \right) .
\]  

**Proof:** Using

\[
\lim_{m \to \infty} Q(m_y) = \begin{cases} 
0 & y > 0 \\
1 & y < 0
\end{cases}
\]

we obtain

\[
\ell \approx \begin{cases} 
\frac{1 - P_{e,r,i|a_{sr}}}{P_{e,r,i|a_{sr}} P_{e,r}} & y > 0 \\
\frac{1 - P_{e,r,i|a_{sr}}}{P_{e,r}} & y < 0
\end{cases}
\]

Table I summarizes the results obtained so far. In Table I, \( m \) is given by (17).

**IV. PARALLEL RELAY CHANNEL**

We next generalize the results to the wireless parallel relay channel (see Fig 2). The transmission over the channel is divided into two phases. In the first phase the source transmits its data and the relays listen to the transmitted signal. In the second phase each relay transmits a regenerated signal to the destination over mutually orthogonal channels. We assume that all source-relay-destination paths are independent and obey the model for a single-relay channel in Section III.

Since all transmission paths are independent, the likelihood ratio of the received signals at the destination can be written as

\[
\ell = \frac{p(s = +1|y)}{p(s = -1|y)} = \frac{p(y|s = +1)}{p(y|s = -1)} = \prod_{i=1}^{K} \frac{p(y_i|s = +1)}{p(y_i|s = -1)}. 
\]  

(30)

where \( y = [y_1, y_2, ... y_K] \) and \( y_i \) is the received signal from relay \( i \). The results of Section III can be used to calculate \( p(y_i|s = \pm 1) \).

Unlike the single-relay channel, the optimum detector for the parallel relay channel in general depends on the performance of the relays. There are exceptions however, as we now illustrate.

**Proposition 6.** Consider a 2-relay parallel relay channel and suppose both relays have the same average channel gain from the destination. Then the optimum detector with the knowledge of the average relay performance for uncoded transmission is independent of the relays’ performances.

**Proof:** The optimum detector is given by

\[
\hat{s} = \arg \max_{s} p(y_1|s_1)p(y_2|s_2), 
\]  

(31)

By assumption, the average bit error rates (BER) at the relays are the same. Denoting the average BER at the relays by \( \bar{\epsilon} \), we have

\[
p(y_k|s) = (1 - \bar{\epsilon})p(y_k|s_k = s) + \bar{\epsilon}p(y_k|s_k = -s). 
\]  

(32)

The optimum detector can be written as

\[
p(y_1|s = 1)p(y_2|s = 1) \geq_{+1} p(y_1|s = -1)p(y_2|s = -1). 
\]  

(33)

Using (32), (33) simplifies to

\[
(1 - 2\bar{\epsilon})p(y_1|s_1 = 1)p(y_2|s_2 = 1) \geq_{+1} (1 - 2\bar{\epsilon})p(y_1|s_1 = -1)p(y_2|s_2 = -1), 
\]  

(34)

Assuming \( \bar{\epsilon} < 1/2 \) (i.e., \( 1 - 2\bar{\epsilon} > 0 \)), the decision metric reduces to

\[
p(y_1|s_1 = 1)p(y_2|s_2 = 1) \geq_{+1} p(y_1|s_1 = -1)p(y_2|s_2 = -1), 
\]  

which does not depend on \( \bar{\epsilon} \). \( \blacksquare \)

In the following two propositions we derive low-SNR approximations of the optimum detector. While in previous sections we worked with likelihood ratios, we work with log-likelihood here as this proves to be more convenient.

**Proposition 7.** The log-likelihood ratio of the received signal with instantaneous CSI over the parallel relay channel at low SNR is

\[
\ln(\ell) = \frac{2}{\sigma_n^2} \sum_{i=1}^{K} (1 - 2\epsilon_i)a_{i,rd}y_i, 
\]  

(35)

where \( \epsilon_i \) is the BER of the \( i \)th relay and \( a_{i,rd} \) is the channel coefficient between the \( i \)th relay and the destination.
Proof: Using the low-SNR approximation given in Table I, we have
\[
\ln(\ell_i) = \sum_{i=1}^{K} \ln \left( 1 + 2(1 - 2\varepsilon_i) a_r d y_i \sigma_n^2 \right)
\]
\[
= \frac{2}{\sigma_n^2} \sum_{i=1}^{K} (1 - 2\varepsilon_i) a_r d y_i,
\]
(36)
where we used the approximation \( \ln(1+x) \approx x \) when \(|x| \ll 1 \).

**Proposition 8.** The log-likelihood ratio of the received signal with average channel knowledge over the parallel relay channel at low SNR is
\[
\ln(\ell) = \sqrt{2\pi} \sum_{i=1}^{K} (1 - 2\varepsilon_i) m_i y_i,
\]
(37)
where \( \varepsilon_i \) is the average BER of the \( i \)th relay and \( m_i = \frac{\sigma_n^2}{\sigma_n^2 + \sigma_i^2} \).

Proof: Similar to the proof of Proposition 7.

Propositions 7 and 8 suggest that the optimum detector should compute a linear combination of the received signals at the destination. This can be seen as a variant of maximum-ratio combining (MRC). Observe that Proposition 8 suggests that the detector at the destination with the same average performance of the relays (e.g., co-located relays) converges to equal gain combining (EGC) as the SNR approaches zero.

V. NUMERICAL RESULTS

We present simulation results for frequency non-selective fading channels which vary independently from one symbol to the next. Denoting the distance between the nodes \( i \) and \( j \) by \( d_{ij} \), the fading parameter is \( \gamma_{ij}^\alpha = d_{ij}^{-\alpha} \) where \( \alpha \) is the path loss exponent. We set \( \alpha = 4 \) in all simulations. We normalize all distances by the source-destination distance. We define the SNR by \( \frac{2E(\sigma_x^2)}{\sigma_n^2} \).

Fig. 4 shows the BER for a cooperative channel (Fig. 3) with one relay.\(^1\) It is assumed that all distances between the nodes are equal. We see that the optimum detector with average knowledge of the source-relay link quality performs close the optimum detector with instantaneous channel knowledge. The statistical receiver (i.e., the optimum receiver when only the statistics of the channel are known) shows a 3 dB loss compared to the optimum detector at high SNR. Fig. 5 shows the BER for a parallel relay channel (Fig. 2) with two relays where the relay SNRs are 10 and 30 dB respectively.

VI. CONCLUSIONS

We derived the optimum detectors for the single and parallel relay channels with regenerative relays when the destination has access either to full CSI or to the statistics of the channel. Among others, we showed that the performance of the optimum detector with average knowledge of the source-relay link is close to that of the receiver with instantaneous knowledge.

\(^1\)The presented results in Table I, easily can be extended to source-destination link in which \( P_{e,r} | a_{sr} = 0 \).

**REFERENCES**


