Behavioural Analysis of Internal Mechanism of Nonlinear Distortion in OFDM Signal Systems

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Abstract—The IMP distortion occurring with the amplification of OFDM signals through a nonlinear RF power amplifier (PA) is analysed with a view to establishing internal relations and impairment significance of different types and combinations – composition categories – of IMPs. Through this, a novel estimation method was deduced to calculate power spectral densities (PSD) of nonlinear distortion. This method also helps expose some PA independent and operating point (OP) independent characteristics of PSD of nonlinear distortion. The simulation and analysis is based on the use of an IEEE 802.11a OFDM signal and a nonlinear PA model is derived from envelope characteristic measurements of an X-band and S-band GaN power amplifier.

Keywords Inter-modulation products, composition type, OFDM, power spectral density

I. INTRODUCTION

Orthogonal frequency division multiplex (OFDM) modulation, as a multi-carrier technology, is widely used in many modern wireless systems for reasons of their excellent usage of the limited RF spectral resource, and their robustness in multipath environments. A key weakness of these signals is their highly variable envelopes—high peak to average power ratio (PAPR). With any nonlinear signal processing this leads to significant inband and out-of-band impairment effects. The major source of this is nonlinear high power amplification process of the final RF high-powered amplification stage in RF transmitters, [1, 2]. The PAPR value is related to number of independent input subcarriers of OFDM signals, a number that varies widely from one communications system (and standard). In this OFDM signals tend to be more sensitive to the nonlinearity of amplification process when compared with classical single carrier signals where the options to shape envelopes towards constant envelope are greater.

With OFDM signals nonlinear distortion produces vast numbers of IMPs in and around the fundamental OFDM signal band and around the harmonics. The focus however can be on the former only – the fundamental band IMPs– as these will be the ones to impair the modulation fidelity of data subcarriers and cause out-of-band adjacent channels’ interference with them spurious emissions in these channels, [3, 4].

harmonic distortion will be well blocked through transmitter tuned circuits and filters.

Mostly nonlinear distortion is analysed as a macro level phenomenon especially in respect of behavioural analysis. Such investigations do nonetheless yield good models and results, e.g., [5, 6]. However this paper looks into what we may call internal mechanisms and micro-level phenomenon in IMP distortion effects. It is worth pointing out that such analysis in OFDM systems requires the handling of vast numbers of IMPs. This presents its own difficulties as will be apparent below.

Some behavioural models, such as complex power series model, do not lend themselves to complex IMP analysis especially when the number of (sub-)carriers is large; cf. also [7–9]. The Bessel-Fourier (BF) nonlinear PA behavioural model can be resolved into quite a manageable decomposed model which is what is required for IMP analysis down to the individual IMP level. Thus, in this paper, it is used. The model is derived from the measured envelope and the IMPs may be calculated individually by using Mixed Frequency and Time Domain (MFTD) methodology, [10–12].

Our analysis approach is to classify IMPs into various composition categories according to their individual average power over time. In this way the power spectral densities (PSDs) of dominant low orders components of nonlinear distortion are approximated by using the average power of single IMP and density distribution functions (DDF) of IMPs. This PSD estimation method is found to have little or no dependency on neither the specific PA nonlinearity characteristic nor the PA operating point. This result is verified by simulation results using a previously published statistical simulation approach, ‘Stat’ approach [11, 12], where the OFDM signal is characterized as a zero mean complex Gaussian process.

Measurements for the PA modelling are taken from a X-band and S-band GaN PA. The OFDM signal modelled is an un-coded IEEE 802.11a signal with 16-QAM applied. In section II, the PA measured envelope and BF model are introduced. In section III, the classification of IMPs, introduction of DDFs, and the approximation method to calculate PSD are presented. Validation, against ‘Stat’ simulation results, of PSD estimations of this technique, and analysis of PSD variation functions are presented in section IV. Section V presents the conclusions.
II. BESSEL-FOURIER MODEL AND MEASURED PA ENVELOPE

Letting the measured memoryless AM-AM and AM-PM envelop conversion functions be denoted as $g[A]$ and $\Phi[t]$, representing signals in their analytic form, the PA’s output in the fundamental band, $s_n(t)$, for input $s(t)$ may be written as, [13]:

$$s_n(t) = \frac{g[A(t)]}{A(t)} \cdot e^{j\Phi(A(t))} \cdot \Phi(t)$$

(1)

where $s_n(t) = A(t) \cdot e^{j\Phi(t)}$ (2)

where, in its most general form, all the PA’s input may be considered to be contained in $[A(t), \Phi(t)]$ combination. A BF behavioral model of the fundamental band output envelope, the nonlinear transfer function may be expressed as a linear sum of finite Bessel series terms of the first kind, that is, [10, 13],

$$g[A(t)] \cdot e^{j\Phi(A(t))} = \sum_{l,k} b_{l,k} \cdot J_{\alpha_l} (\alpha_l A(t))$$

(3)

which is a linear sum of finite Bessel series terms of the first kind, $J$, the argument of which is a function of the PA’s input envelope amplitude, $A(t)$, and $\alpha - a$ parameter related to the modelled dynamic range of BF model. The $b_{l,k}$ are the series complex model coefficients.

The measured envelope AM-AM modelled here is shown in figure 1. It can be clearly seen to be memoryless over the bandwidth range being considered. The input and output are expressed as the back off relative to the saturation power; the saturation power point used here is that defined in [14]. AM-PM conversion for this GaN is quite negligible, and having negligible impact on impairment results could in fact be disregarded, [14, 15].

III. CLASSIFICATION OF IMPS AND APPROXIMATION METHOD TO CALCULATE OUTPUT IMP PSD

MFTD signal representation combined with the BF behavioural model is an approach which facilitates the derivation of a decomposed representation of the PA output, decomposed that is into a sum of all the individual harmonics and IMPS generated, including the wanted amplified signal in the fundamental band. This then is suitable for the analysis of the nonlinear amplification of OFDM signals. The PA input OFDM signal may be written, [11, 12]:

$$s_n(t) = \sum_{l=1}^{M} \sum_{s=1}^{N} A_{l,s} \cdot e^{j\theta_{l,s}} (t - sT + \phi_{l,s})$$

(4)

where $A_{l,s}$ and $\phi_{l,s}$ are the amplitude and phase of the $\theta^{th}$ subcarrier of angular frequency $\omega_l$ in the $s^{th}$ symbol; notably $A_{l,s}$ may remain unchanged for some adjacent OFDM symbols. $M$ and $N$ denote the number of input frames and number of input subcarriers, respectively. The PA output may be shown to be, [10, 11]:

$$s_n(t) = \sum_{l=1}^{L} b_{l,s} \cdot \left[ \prod_{k=1}^{n} J_{\alpha_l} (\alpha_l A(t)) \right] \cdot e^{j\sum_{l=1}^{n} l \omega_l + \phi_{l,s}}$$

(5)

where $J_{\alpha_l}$ denotes the $n^{th}$ order term of a Bessel function of the first kind. Each IMP/harmonic maps to a unique realization of the parameter set $\{n_l\}$. The values of $\sum_{l=1}^{n} n_l$ and $\sum_{l=1}^{n} \omega_l$ denote the harmonic and order of IMP. The fundamental components may be selected by setting the condition $\sum_{l=1}^{n} n_l = 1$. Besides harmonics and IMP orders, it is possible to further classify IMPS by subcarrier composition defined by values of non-zero elements in the set $\{n_l\}$. The dominant $3^{rd}$ and $5^{th}$ order IMPS in the fundamental band may be classified into 2 and 6 composition types, respectively, as given in Table I, in which only non-zero $\{n_l\}$ values are presented.

<table>
<thead>
<tr>
<th>Type</th>
<th>${n}$</th>
<th>Type</th>
<th>${n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3A</td>
<td>(2, -1)</td>
<td>5C</td>
<td>(2, 1, -2)</td>
</tr>
<tr>
<td>3B</td>
<td>(1, 1, -1)</td>
<td>5D</td>
<td>(1, 1, 1, -2)</td>
</tr>
<tr>
<td>5A</td>
<td>(3, -2)</td>
<td>5E</td>
<td>(2, 1, -1, -1)</td>
</tr>
<tr>
<td>5B</td>
<td>(3, -1, -1)</td>
<td>5F</td>
<td>(1, 1, -1, -1)</td>
</tr>
</tbody>
</table>

From the MFTD approach, it may be readily shown that the power of a single IMP, which maps to a specific realization $\{n_l\}$, may be expressed as:

$$P_{n_k} = \frac{1}{2R} \sum_{l=1}^{L} b_{l,s} \prod_{k=1}^{n_l} \left| J_{\omega_l} (\alpha_l A(t)) \right|^2$$

(6)

where $R$ is the output load. Since the OFDM signals include lots of input frames, the average power over time of an IMP mapping to a specific $\{n_l\}$ combination may be approximated as follows:

$$\overline{P_{n_k}}(\sigma) = \frac{1}{2R} \sum_{l=1}^{L} b_{l,s} \prod_{k=1}^{n_l} \left| J_{\omega_l} (\alpha_l A(t)) \right|^2$$

(7)
where \( \bar{A} = \sqrt{\frac{\sigma^2}{2R \cdot N}} \)  

(8)

In equation (8), \( \sigma^2 \) denotes the input power, i.e., the PA operating point (OP). It may be seen from equation (7) that the average powers of two IMPs of one composition type are identical.

The estimated average powers of single IMP of all 8 types in Table I are presented in figure 2, and these average powers are calculated by using equations (7) and (8). The powers in figure 2 are normalized, and the reference powers used in normalisation at each OP is the average output power of wanted subcarriers at that OP. It may be seen from figure 2 that the average power over time of any single IMP of type 3B and 5F is greater than that of any single IMP of the other type of the same order, since IMPs of types 3B and 5F are composed of higher numbers contributing subcarriers –i.e., of higher complexity. This finding observed in figure 2 may be expressed by equations as:

\[
PIMP_{3B}(\sigma^2) > PIMP_{3A}(\sigma^2)
\]

and

\[
PIMP_{5F}(\sigma^2) > PIMP_{5A,5E}(\sigma^2)
\]

where \( PIMP_{\text{TYPE}} \) indicates the average power over time of single IMP of type ‘\( \text{TYPE} \)’.

The IMP Distribution Density Function (DDF) of IMPs of various composition types, when an IEEE 802.11a signal applied, are presented in figure 3. As would be expected from figure 3, IMPs of composition types with higher complexity have relatively higher-valued DDFs. That is, DDFs of IMPs of 5th order composition types generally are higher than those of 3rd order composition types, and DDFs of IMPs of type 3B and 5F are the highest ones among DDFs of IMPs of all other 3rd and 5th order composition types, respectively. DDF is an important role in approximation method to calculate PSDs of nonlinear distortion. DDF is determined by the OFDM signal standard used (or more specifically on the number of active subcarriers); it is naturally independent of either the PA characteristic or PA OP.

\[
PIMP_{3B}(\sigma^2) \cdot PIMP_{3B}(\sigma^2) \gg PIMP_{3A}(\sigma^2)
\]

and

\[
PIMP_{5F}(\sigma^2) \cdot PIMP_{5F}(\sigma^2) \gg PIMP_{5A,5E}(\sigma^2)
\]

\[
\text{PSD}_3(f, \sigma^2) = DDF_{3B}(f) \cdot PIMP_{3B}(\sigma^2)
\]

and

\[
\text{PSD}_5(f, \sigma^2) = DDF_{5F}(f) \cdot PIMP_{5F}(\sigma^2)
\]

where \( \text{PSD}_3(f, \sigma^2) \) and \( \text{PSD}_5(f, \sigma^2) \) denotes the total PSD of the 3rd and 5th order IMPs at frequency offset \( f \), and \( \sigma^2 \) denotes the input power of associated PA OP.

IV. VALIDATION OF NEW IMP PSD APPROXIMATION METHOD AND THE PSD VARIATION FUNCTION

In figure 4, the normalized PSD of the output ‘wanted signal’ components, and the 3rd and 5th order components of nonlinear distortion are presented. Here the PSDs look dependent on both PA characteristic and the PA OP. The reference power used in the normalization at each OP is the total output power at that OP. Figure 4 is obtained from model simulation results using Stat approach, [11, 12], with the same PA input signal configuration and the same GaN PA.
In figure 5, the simulated $PSD_3(0, \sigma^2)$ and $PSD_5(0, \sigma^2)$ are presented for comparison with predicted $PSD_3(0, \sigma^2)$ and $PSD_5(0, \sigma^2)$. ‘Simulated’ here refers to results obtained using the STAT approach and the ‘predicted’ to results obtained by using the PSD approximations as set out in equations (13) and (14). The reference power used in the normalisation is the average power of wanted subcarriers in output signal. It may be seen that the fitness between predicted graphs and those obtained by practical simulation is near perfect.

Executing a decibel based logarithmic transformation of equation (13), we may write:

$$PSD_i(f, \sigma^2)[dB] = \frac{1}{2} [\log [DDF_{i, \sigma^2}(f)] + \log [PIMP_{i, \sigma^2}]]$$

(15)

Thus:

$$\frac{\partial [PSD_i(f, \sigma^2)][dB]}{\partial f} = \frac{1}{2} [DDF_{i, \sigma^2}(f)]$$

(16)

Defining the variation function of $\Delta PSD_i(f, \sigma^2)[dB]$ as:

$$\Delta PSD_i(f, \sigma^2)[dB] = PSD_i(f, \sigma^2)[dB] - PSD_i(0, \sigma^2)[dB]$$

Thus, it may be easily obtained:

$$\frac{\partial [\Delta PSD_i(f, \sigma^2)][dB]}{\partial \sigma^2} = 0$$

(17)

Similarly it may be obtained:

$$\frac{\partial [PSD_i(f, \sigma^2)][dB]}{\partial f} = \log [DDF_{i, \sigma^2}(f)]$$

(18)

and

$$\frac{\partial [\Delta PSD_i(f, \sigma^2)][dB]}{\partial \sigma^2} = 0$$

(19)

That is, it is predictable that the variation functions of $PSD_i(f, \sigma^2)$ and $\Delta PSD_i(f, \sigma^2)$ and $\Delta PSD_i(f, \sigma^2)$ in equations (18) and (20), are determined by the DDF of OFDM signal used, and they are independent with specific PA characteristics and OP. This predicted result given in equations (18) and (20) are validated by practical simulation using Stat approach, [11]. This validation is presented in figure 6, where the simulated $\Delta PSD_3(f, \sigma^2)$ and $\Delta PSD_5(f, \sigma^2)$ are clearly seen to be independent with PA OP as predicted in equations (18) and (20).

V. CONCLUSION

In this paper, the internal mechanism of nonlinear distortion of OFDM signals is investigated. IMPs of dominated low orders are classified into various composition types, and the average powers of all IMPs, for the OFDM signal example chosen here, of any composition type tend to be very similar if not identical. The 3rd and 5th IMPs of types (1, 1, -1) and (1,1,1,-1,-1) –types 3B and 5F as labeled here– dominate the 3rd order and 5th order components of nonlinear distortion both through their powers, PIMPs, and DDFs. It is shown that the total PSDs of the 3rd and 5th order IMPs may be approximated using the PIMPs and DDFs of IMPs of these two types 3B and 5F respectively. It is also demonstrated here that the variation of the $PSD_i(f, \sigma^2)$ and $\Delta PSD_i(f, \sigma^2)$ functions are
determined by the DDF of OFDM signal used, independently of the specific PA characteristics and PA operating point.

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