Brain Emotional Learning Based Fuzzy Inference System (BELFIS) for Solar Activity Forecasting

Mahboobeh Pasrapoor, Urban Bilstrup

School of Information Science, Computer and Electrical Engineering (IDE), Halmstad University, Sweden
mahpar11@student.hh.se, urban.bilstrup@hh.se

Abstract— This paper presents a new architecture based on a brain emotional learning model that can be used in a wide variety of AI applications such as prediction, identification and classification. The architecture is referred to as: Brain Emotional Learning Based Fuzzy Inference System (BELFIS) and it is developed from merging the idea of prior emotional models with fuzzy inference systems. The main aim of this model is presenting a desirable learning model for chaotic system prediction imitating the brain emotional network. In this research work, the model is used for predicting the solar activity, since it has been recognized as a threat to critical infrastructures in modern society. Specifically sunspot numbers are predicted by applying the proposed brain emotional learning model. The prediction results are compared with the outcomes of using other previous models like the locally linear model tree (LOLIMOT) and radial bias function (RBF) and adaptive neuro-fuzzy inference system (ANFIS).

Keywords-component: brain emotional learning; fuzzy inference system; multi-year ahead prediction; solar activity forecasting; solar cycle 23; sunspot chaotic time series.

I. INTRODUCTION

The main purpose of this paper is to describe recently conducted work on using brain emotional learning (BEL) for solar activity prediction. It has recently been pointed out as an important issue to provide early warning of solar storms on the sun [1] to decrease the harmful effects on critical infrastructures such as: satellites, telecommunication and power systems [2]-[6]. Bio-inspired models, such as neural networks, neuro-fuzzy inference systems, and evolutionary computing techniques have a long tradition of being used as data-driven approaches for complex system modeling. Using brain emotional learning based computational models (BELs) [6]-[12] is a fairly new area of bio-inspired models. Recent studies have indicated that BELs can enhance the accuracy and reduce complexity for the prediction of complex systems [8]-[12]. Previous results [8]-[12] applying BELs for prediction and controller applications have shown good results. In this paper, a new model of brain emotional learning is applied to predict solar activity. A common measure for predicting solar activity is the number of sunspots which cyclically changes from a solar minimum to solar maximum throughout a solar cycle [1]-[6]. For the purpose of forecasting solar activity, sunspot time series that have nonlinear and dynamic characteristics can be predicted by data driven approaches. So far, well-known “black box” [13] models: linear and nonlinear auto regression methods, neural networks and neuro-fuzzy have been applied for the prediction of sunspots time series. Although many of these methods have shown reasonable results for prediction of sunspot time series, it seems that they are not powerful enough to obtain high accuracy results for long term prediction. Specifically, these methods often require a large training set. For prediction of such time series, with limited training data, a black box model with a simple structure is desirable. Low time complexity is also desirable, training algorithms of most data driven approaches require a large number of iterations to achieve high accuracy [4], [6] and [11]-[15]. In contrast to black box models, physically based methods with nonlinear mapping capability can be a feasible and reliable model for solar activity forecasting. However, the performance of “white box” [13] model depends on the ability to derive a physical model with high prediction accuracy [4], [5] and [11]. Thus, it is interesting to apply a model that is not a fully data driven model or a pure analytical model for sunspot series prediction. In this paper, a model that is neither a black box nor a white box model [13] is examined to predict sunspot time series.

The organization of this paper is as follows: the brain emotional learning is briefly explained and the prior computational models are reviewed in Section II. In Section III, the suggested model BELFIS, is described in more detail. Three case studies of solar activity forecasting, are examined by applying BELFIS and the results are compared with the previous studies in Section IV. In Section V, we conclude this paper by reviewing the main results and giving some remarks about the performance of BELFIS and we also give some ideas of future extension of BELFIS.

II. REVIEW OF BRAIN EMOTIONAL LEARNING

A. Emotional learning in the brain

A group of regions in the brain including the hippocampus, the amygdala, the thalamus, the sensory cortex and the orbitofrontal cortex, is called the limbic system [16], [17]. The functions of these regions in the context of emotional learning can be summarized as follows:
1) Thalamus: This region, which is the first part of the limbic system to receive emotional stimuli, is responsible for the provision of high-level information about the stimulus [17]-[21]. 2) Sensory cortex: It is a part of the sensory area of the brain and is responsible for distributing the received signal between the amygdala and the orbitofrontal region [18], [19]. 3) Amygdala: This region is the central part of the limbic system. It has direct connections with the thalamus, orbitofrontal and sensory cortex [18]-[24]. The amygdala participates in storing the emotional experiences and the emotional responses [25], evaluating the positive and negative reinforcement and the emotional responses [26], learning and predicting the association between unconditioned and conditioned stimuli [26]. The amygdala encompasses three main parts: the basolateral complex, the lateral and the centromedial nucleus [21] and [28]. The basolateral region is the largest part of the amygdala and performs the role of mediating memory consolidation [17]. The lateral is the part of the amygdala to pass the stimuli to other regions and also forms the stimulus-response association [26]. The centromedial part that encompasses the medial nucleus and the central nucleus, has a role in mediating the expression of emotional responses [17] and [28]. 4) Orbitofrontal cortex: This region involves processing the stimulus, learning the stimulus–reinforcement association. It also evaluates the reinforcement signal, and prevents the amygdala from providing inappropriate responses [17]-[19].

B. Emotional Learning Models

Computational models of emotional learning have been proposed on the basis of the brain emotional network of mammal brains [18]-[19]. Some types of computational models can be mentioned as, the Amygdala-orbitofrontal subsystem model [18], [19] the Cathexis model [30], the amygdala hippocampus model [31], the hippocampus-neocortex model [32], a model of the limbic system [32], reversal emotional learning [33], model of mind [34]. The amygdala-orbitofrontal subsystem model which is the basis of our model has a simple structure (see Fig. 2). It is inherited from the parts of the limbic system that have a role in emotional learning, where the interaction between these parts is formulated as mathematical relationships by the computational model [18]-[19]. The amygdala-orbitofrontal subsystem consists of four parts which interact with each other to form the association between the conditioned and the unconditioned stimuli [12], [18]-[19]. The output function of the model, E, has been defined by subtracting the response of the orbitofrontal from the amygdala’s response. The reward signal, REW, has been defined to update the parameters of the amygdala and the orbitofrontal. The Brain Emotional Learning-Based Intelligent Controller (BELBIC) was introduced as an intelligent controller that adopted the amygdala-orbitofrontal subsystem [8]. BELBIC has been successfully applied in control applications: controlling the heating and air conditioning, aerospace launch vehicles, intelligent washing machines, and micro heat exchangers [8], [12] and [27]. The excellent results of BELBIC and the simple structure of the amygdala-orbitofrontal system have been a great motivation to develop more instances of BELs and apply them for prediction applications, in particular prediction of chaotic time series. However, early results, when a simple structure as the amygdala-orbitofrontal system was applied for prediction, indicated undesirable performance. Thus, emotional learning models with different structures and functions than the amygdala-orbitofrontal subsystem have been suggested.

III. BRAIN EMOTIONAL LEARNING BASED FUZZY INFERENCE SYSTEM

In the following, we describe how the brain emotional model based on the fuzzy inference system (BELFIS) is divided into several parts in order to mimic the structure of the limbic system regions and how the function of BELFIS is defined to provide an input-output mapping. Furthermore, we explain how the BELFIS updates the learning parameters to learn the input-output mappings from the training samples.

A. Structure

The structure of the brain emotional learning based fuzzy inference system (BELFIS) is based on the amygdala-orbitofrontal system; it consists of four main parts (See Fig. 2): TH, CX, AMYG and ORBI. Structurally, the connections between these parts are similar to the structure of those regions of the brain’s limbic system which have a role in emotional learning. Functionally, these parts mimic some functionality of those regions. Certainly, the limbic system regions are very complex and it is not our intention to mimic all their functions and all connections in detail. The structure of BELFIS is explained as following: 1) The TH has connection with the AMYG and the CX to send input vector of BELRFS, \( i_{c,j} \) to the AMYG and the CX. It is subdivided into two parts, Fig. 4; the MAX and the AGG. 2) The CX provides \( s_{c,j} \) and distributes it between the AMYG and the ORBI. 3) The AMYG receives two inputs, \( th_{c,j} \) and \( s_{c,j} \), which originated from the TH and the CX. There is a bidirectional connection between this part and the ORBI (see Fig. 2). The connections and the internal parts of the amygdala are imitated by defining the BL and the CM. The BL corresponds to the set of the lateral and the basal and the CM corresponds to the accessory basal and centromedial part.

It should be noted that this definition is done for BELFIS and here is no scientific evidence to such a definition for the nuclei of the amygdala.
The BL provides \( r_{a,j} \) as a partial response and sends it to the CM which is the main part of the AMYG. The CM receives the partial response and sends reinforcement signals, \( P_{a,j} \) and \( P_{a,j}^e \) to both the AMYG and the ORBI respectively. The ORBI receives \( S_{c,j} \) from the CX and \( P_{a,j}^e \) from the CM and sends a partial response, \( r_{a,j} \), to the CM.

**B. Function**

To imitate the function of the brain emotional learning, the BELFIS adapts the adaptive neuro-fuzzy inference system. Before explaining how the BELFIS makes an appropriate output for an unseen input, we explain a simple model of an adaptive neuro-fuzzy inference system.

**Adaptive neuro-fuzzy inference system (ANFIS)**

Adaptive neuro-fuzzy inference system (ANFIS) is a type of fuzzy inference system which is structurally defined by adaptive networks. The ANFIS defines the architecture of five layers where each layer consists of adaptive nodes. To represent both Sugeno and Tsukamoto fuzzy inference system, different types of ANFIS are defined [29]. Fig. 3 describes the third type of ANFIS with two rules, which have been defined using the Sugeno fuzzy inference system. The Sugeno fuzzy if-then rules are expressed using equation (1) and (2).

If \( x_1 \) is \( \mu_{11} \) and \( x_2 \) is \( \mu_{12} \) then \( f_i = q_{11} x_1 + q_{12} x_2 + q_{13} \)  \( (1) \)

If \( x_1 \) is \( \mu_{21} \) and \( x_2 \) is \( \mu_{22} \) then \( f_i = q_{21} x_1 + q_{22} x_2 + q_{23} \)  \( (2) \)

The following steps explain the function of each layer of the third type of ANFIS [29], with a two-dimensional input vector \( X \).

**Layer 1:** Consists of square nodes with Gaussian or bell-shaped functions which are defined by (3) and (4) respectively. Where \( x_1 \) is \( j \) th dimension of input vector.

\[
\mu(x_j) = \exp(-\frac{1}{2} \frac{(x_j - c_j)^2}{\sigma_j^2})
\]

\( (3) \)

\[
\mu(x_j) = \frac{1}{1 + \frac{|x_j - c_j|^{2b_j}}{a_j}}
\]

\( (4) \)

**Layer 2:** It has circular nodes that are labeled with \( \prod \). The output of these nodes is multiplication of their input as (5).

\[
w_k = \prod_{j=1}^{n} \mu_{kj}(x_j)
\]

\( (5) \)

**Layer 3:** It consists of circular nodes that are labeled with \( \bar{\prod} \). The output of these nodes is defined as equations (6).

\[
b_k = \frac{w_j}{\sum_{k=1}^{m} w_k}
\]

\( (6) \)

**Layer 4:** It has several square nodes to provide the then part of the Sugeno fuzzy rules using (7).

\[
f_k = \sum_{j=1}^{n} q_{kj} x_j + q_{j(k+1)}
\]

\( (7) \)

**Layer 5:** Conducts summation (8) and provides the final output as (11) [29]. It also has two time delay units.

\[
F(X) = \sum_k w_k(X) f_k(X)
\]

\( (8) \)

To describe the function of BELFIS, we define a fuzzy function according to equation (8). The subscript \( k \) shows the number of fuzzy rules, \( X \) indicates an input vector and \( \bar{w}_k \) is calculated using (6).

Assuming \( i_{c,j} \) is input vector of \( I_c \) as (9).

\[
I_c = \{i_{c,1}, i_{c,2}, \ldots \}
\]

\( (9) \)

The functions of each part are as follows:

**TH:** The competitive function and the linear function describe the function of the MAX and AGG respectively. The output of MAX is calculated according to (10). The AGG is a linear neural network that passes \( i_{c,j} \) to the CX.

\[
\text{th}_{c,j} = [\text{Max}(i_{c,j})]
\]

\( (10) \)
thus, the nonlinear parameter would be; these sets are extracted from the

\( F \) is a fuzzy function according to (8).

\[ r_{a,j} = F(s_{c,j}, t_{c,j}) \]  \hspace{1cm} (11)

CM: Consists of an adaptive network that provides output of BELFIS as (12).

\[ r_j = F(t_{a,j}, r_{a,j}) \]  \hspace{1cm} (12)

ORBI: This part forms the input-reinforcement mappings using an adaptive network (see Fig. 2). The function of the ORBI is defined by (13).

\[ r_{a,j} = F(s_{c,j}) \]  \hspace{1cm} (13)

C. Learning Algorithms

For adjusting the linear and nonlinear learning parameters, the hybrid learning algorithm of ANFIS, is utilized. The hybrid learning algorithm consists of the steepest descent (SD) and the least-squares estimator (LSE). The SD updates the nonlinear parameter in a gradient related direction to minimize the loss functions, which are defined based on reinforcement signals \( p_{a,j}, p'_{a,j} \) and the outputs of the adaptive networks. The LSE is applied to update the linear parameters of the then part of the Sugeno fuzzy rules.

In BELFIS, the hybrid learning algorithm is independently applied to update the parameters of each adaptive network. Assuming \( t_{u,j} \) and \( r_{a,j} \) are an input and output vector from \( I_{u} = \{ i_{u,1}, i_{u,2}, \ldots \} \) and \( r_{u} = \{ r_{u,1}, r_{u,2}, \ldots \} \); these sets are extracted from the training samples. To train BELFIS using the SD, an overall loss function is defined according to (14).

\[ \text{lossfunc} = \frac{1}{1 + (\lambda_{a} r_{a,j} + \lambda_{w} r_{w,j} + \lambda_{r} r_{r,j} + \lambda_{u} r_{u,j})^2} \]  \hspace{1cm} (14)

The loss functions of adaptive networks are determined using the appropriate vector for \( \lambda \) as equation (15).

\[ \lambda = [\lambda_{a}, \lambda_{w}, \lambda_{r}, \lambda_{u}] \]  \hspace{1cm} (15)

For example, the weights of ORBI are updated using the loss function as equation (16).

\[ \text{lossfunc}_{\text{ori}} = \frac{1}{1 + (\lambda_{a} r_{a,j} + \lambda_{w} r_{w,j} + \lambda_{r} r_{r,j} + \lambda_{u} r_{u,j})^2} \]  \hspace{1cm} (16)

As we mentioned, the loss functions can be defined on the basis of the reinforcement signals. Equations (17) and (18) calculate the \( p'_{a,j} \) and its corresponding loss function.

\[ p'_{a,j} = \lambda_{a} r_{a,j} + \lambda_{w} r_{w,j} \]  \hspace{1cm} (17)

\[ \text{lossfunc}_{\text{ori}} = \frac{1}{1 + (\lambda_{a} r_{a,j} + p'_{a,j})^2} \]  \hspace{1cm} (18)

The loss functions are converged to minimum values using the SD that updates all nonlinear learning parameters. Defining the loss function as (17), BELFIS has the ability to update the nonlinear learning parameters in two learning phases. For the first learning phase, the value of \( \lambda_{a} \) should be greater than zero, \( \lambda_{a} > 0 \); thus, the nonlinear learning parameters are updated using the training samples. For the second learning phase, the value of \( \lambda_{a} \) should be equal to zero, \( \lambda_{a} = 0 \); thus, the nonlinear parameter would be incrementally updated using new data. The main advantage of defining two learning phases is to overcome the over fitting and under fitting problem. It should be noted that, the main parts of BELFIS are similar to components of the amygdala-orbitofrontal system, the output functions, the learning rules and reinforcement functions of these parts are significantly different from the previous models.

IV. FORECASTING SOLAR ACTIVITY USING BELFIS

To evaluate the performance of BELFIS, it is examined
for solar activity prediction and the obtained results are compared by the result of ANFIS and the results from other previous studies. Sunspot numbers can be used to estimate solar activity, the solar winds and the geomagnetic storms [2]-[6], [15], and [35]. Solar activity has a periodic behavior that is known as the solar cycle. Each solar cycle lasts approximately 11 years. In this paper, the sunspot numbers are gathered from the SIDC (World Data Center for the Sunspot Index). BELFIS is examined for long-term and short-term prediction of solar activities using the smoothed and non-smoothed monthly sunspots and smoothed yearly sunspots time series. To compare the accuracy of BELFIS with other studies the Normalized Mean Square Error (NMSE) and Mean Square Error (MSE) are considered as the error measures. It should be noted the max-min normalization is applied for all of the data.

A. Monthly Prediction

First, non-smoothed sunspot numbers from May 1987 to May 2012 (300 samples) are selected as test data for a multi-month horizon prediction. For this purpose, the first 2400 samples and the next 300 samples are considered as the training data set and the cross validation set, respectively. BELFIS and ANFIS are examined for 6, 12 and 18 months ahead prediction respectively. Table I presents the results have obtained from ANFIS and BELFIS using approximately an equal number of iterations. For long-term prediction, 12 and 18 months ahead, BELFIS provides a lower MSE than ANFIS. The noticeable fact is that BELFIS predicts more accurately than ANFIS using a small number of iterations. The needed number of iterations for BELFIS is 1600; we could not achieve lower MSE for ANFIS even if more than 4000 iterations is executed. In [35], the reported MSE for an 18 months ahead prediction for the first 300 samples of monthly sunspots is between 0.005 and 0.015. The MSE decreases to a value between 0.005 and 0.01 using a very large number of iterations, i.e. 10000 iterations. It should be noted that test data for BELFIS and FTLRNN are dissimilar. For the second experiment, non-smoothed monthly sunspots, a part of solar cycle 19, is predicted by BELFIS and ANFIS. The solar cycle 19 started in 1954 and ended in 1964; thus, the set contains the sunspots from 1950 to 1965 and includes the peak sunspot number of solar cycle 19, which occurred in 1957. The obtained results of applying BELFIS, ANFIS and other methods are listed in Table II. It can be seen that the NMSE of BELFIS is not lower than the NMSE of ANFIS; however, the number of iterations for BELFIS is less than the ANFIS. Notable is also that BELFIS predicts 240.093 (Fig. 5) for the peak of solar cycle 19 and is very close to the observed real value, 250. Table II also compares the results of BELFIS with ELFIS [2], and other types of BELs. In addition to the above experiments, the smoothed monthly sunspot time series from May 1996 to April 2007 are tested using recursive prediction by the BELFIS and the ANFIS. For this purpose, the monthly sunspots from 1700 until 1975 are considered as training data and the sunspots from January 1976 to April 1996 are predicted by these methods. Figure 6 shows the predicted values versus the observed values and indicates that for short-term prediction the performance of ANFIS and BELFIS are similar. For this prediction, the NMSE of BELFIS is 6.9779e-4, while the NMSE of ANFIS is 7.0352e-4. Although these error indices are close to each other, the number of required iterations for BELFIS is less than 100; in contrast, ANFIS requires more than 4000 iterations to converge. It can be concluded that BELFIS requires lower number of iterations to achieve accurate results in comparison with the ANFIS. Then, the predicted values without any correction to the observed values are added to the training data and the monthly sunspots from May 1996 to April 2007, a part of solar cycle 23 are predicted. The graph in Fig. 7 shows the recursively predicted values by ANFIS and BELFIS. It shows that both methods have excellent performance for recursive prediction. Thus, we can apply them for predicting solar cycles 24 and 25.

![Figure 5. Predicted values of monthly sunspots from 1950 to 1965 using the BELFIS and the ANFIS.](image)

**TABLE I.** THE MSE OF 6, 12 AND 18 MONTHS AHEAD PREDICTION USING DIFFERENT METHODS.

<table>
<thead>
<tr>
<th>Method</th>
<th>6 Month ahead</th>
<th>12 Month ahead</th>
<th>18 Month ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td>BELFIS</td>
<td>0.008</td>
<td>0.0083</td>
<td>0.007</td>
</tr>
<tr>
<td>ANFIS</td>
<td>0.008</td>
<td>0.0085</td>
<td>0.01</td>
</tr>
<tr>
<td>FTLRNN[35]</td>
<td>0.008-0.01</td>
<td>0.01-0.015</td>
<td>0.01-0.015</td>
</tr>
</tbody>
</table>

**TABLE II.** RESULTS OF DIFFERENT METHODS FOR ONE STEP AHEAD PREDICTION OF MONTHLY SUNSPOTS FOR SOLAR CYCLE 19.

<table>
<thead>
<tr>
<th>Method</th>
<th>Specifications</th>
<th>Predicted Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>BELFIS</td>
<td>0.1234</td>
<td>16rules(43)</td>
</tr>
<tr>
<td>ANFIS</td>
<td>0.1017</td>
<td>4rules (600)</td>
</tr>
<tr>
<td>ELFIS[2]</td>
<td>0.1386</td>
<td>3rules(--)</td>
</tr>
</tbody>
</table>
In order to compare with the other studies, the sunspots from January 1999 to January 2001 are also examined for recursive prediction. For this case, the training data and cross validation are similar to [4]. The test data includes the peak of solar cycle 23; it has 120.8 sunspots and occurred in April 2001. The predicted values by BELFIS and ANFIS are depicted in Fig. 8. It indicates that the predicted values by ANFIS and BELFIS are very close in the majority of points of test set; however, in peak value the performance of ANFIS is a little better than BELFIS. Table III compares the NMSE and the predicted values for the peak obtained using different data-driven methods. It states that the results of BELFIS and ANFIS are very close to each other. Both Fig. 8 and Table III verify that for short term prediction, one month ahead, the results of ANFIS are better than BELFIS. Of course, in comparison with the other methods, LoLiMoT and RBF-OLS, BELFIS has better performance.

B. Yearly Prediction

This section compares BELFIS to other studies that have been done for predicting the yearly sunspots. For this purpose, first, BELFIS and ANFIS are trained by the yearly sunspots from 1700 to 1799 and tested on the set that includes sunspots from 1800 to 2000. Table IV presents the comparison between these two methods and four other techniques: threshold autoregressive (TAR) method, WNET, LOGF-NN and a type of BELs [6]. It lists the resulting NMSEs of these methods for a one year ahead prediction of the test set. The result indicates that ANFIS and BELFIS predicts more accurately than the other listed methods.

The results show that the prediction error of BELFIS is equal to 16.97, the lowest error of all listed methods. Furthermore, the number of iterations for BELFIS is less than the iterations of ANFIS. In another experiment, BELFIS and ANFIS the training set that is defined from the 1700 to 1920 to test three sets. The sets are sunspot from 1921 to 1955, 1956 to 1979 and 1980 to 1994. The NMSEs of BELFIS, ANFIS, autoregressive threshold (AR) and a multilayer neural network (WNET) are presented in Table V. It indicates that the results of BELFIS have not changed for the different test sets, as happens for AR and WNET. Thus, as we have expected, BELFIS overcomes the over fitting issue. For the third test set, the predicted values using BELFIS and ANFIS are depicted in Fig. 9. It is noticeable that BELFIS has good ability to predict peak values. The results are summarized in table IV.

C. Solar Activity Forcasting for Solar Cycle 23

Solar cycle 23 lasted 12.6 years, it started in May 1996 and ended in December 2008. For solar cycle 23, different techniques, white box and black box, have been utilized to predict multi-year ahead. As white box methods, the precursor and the solar dynamo have been able to forecast cycle 23 more than five years in advance. Precursor methods predict the solar cycle using the statistical correlations between solar activity and the fluctuation of terrestrial magnetism [4], [15].

<table>
<thead>
<tr>
<th>Method</th>
<th>Specifications</th>
<th>Month</th>
<th>Magnitude</th>
<th>NMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BELFIS</td>
<td>April</td>
<td>122.2</td>
<td>0.032</td>
<td></td>
</tr>
<tr>
<td>ANFIS</td>
<td>April</td>
<td>122.0</td>
<td>0.030</td>
<td></td>
</tr>
<tr>
<td>LoLiMoT[4]</td>
<td>March</td>
<td>120.9</td>
<td>0.046</td>
<td></td>
</tr>
<tr>
<td>RBF-OLS[4]</td>
<td>June</td>
<td>120.3</td>
<td>0.032</td>
<td></td>
</tr>
</tbody>
</table>
Solar dynamo method or “dynamo-based solar activity prediction technique” [5] predicts the solar cycle using magnitude persistence. A good measure for magnitude persistence is the solar dynamo amplitude (SODA) index. Solar dynamo has shown reasonable accuracy for multi-year ahead prediction of solar cycles 21, 22 and 23 using the smoothed sunspot numbers [5] and [15]. As black box methods, previous studies [6], [15], [38] have stated that the LLNF and ANFIS are not able to predict the solar cycle for more than five years ahead. For prediction of less than five years ahead, the prediction accuracy of these methods decreases with an increase in prediction horizon [15]. Consequently, it is interesting to apply BELFIS to evaluate the prediction performance of BELFIS for long-term prediction of solar activity. For this purpose, the multi-year ahead of solar cycle is examined by BELFIS and ANFIS. Table VI lists the NMSEs and the predicted values of peak 120.8, for the methods. Table VI shows that the predicted error of BELFIS, the NMSE, for this five-year ahead prediction of solar cycle 23 is less than the ANFIS. Thus, the BELFIS can be considered as a reliable prediction technique for solar cycle predictions. Although, previous studies have stated that the neuro-fuzzy methods, LLNF and ANFIS, are not powerful for long-term prediction, BELFIS shows a fairly good performance predicting five and seven years ahead of solar cycle 23. For five year ahead prediction of solar cycle 23, using the BELFIS, the magnitude of solar maximum is predicted as 132.30. The BELFIS predicted value is approximately equal to the predicted values of two previous methods: ANFIS and LLNF.

V. Conclusion

In this paper, a new type of BELs named BELFIS is proposed and applied for solar activity prediction. BELFIS is a merge of previous proposed emotionally-inspired structures and neuro-fuzzy inference system. It utilizes three adaptive networks with slightly different training samples and the fact that the number and the type of membership functions of each adaptive network can be different from the other adaptive networks.. Because of these characteristics, BELFIS is a fairly accurate predictor model with high flexibility. In comparison with ANFIS, the model complexity of BELFIS is high; however, it converges faster than ANFIS. This characteristic has emerged from the emotionally-driven structure. The BELFIS suffers from the ‘curse of dimensionality’ problem; thus, it is not feasible for high dimension applications. The obtained results indicate that BELFIS is a reliable, nonlinear predictor model for solar activity forecasting, this is particularly shown by the prediction result of solar cycle 24. It can also be applied as a predictor model for the long-term and short-term prediction of chaotic and nonlinear systems. In future, the authors would consider adding an optimization method, e.g. genetic algorithm, to find the optimal values of the fiddle parameters of the BELFIS: the initial values of membership functions and the weights of loss functions. In addition, the BELFIS will be presented using other types of fuzzy inference system, e.g. LoLiMoT. We also intend to combine singular spectrum analysis (SSA) with the proposed model to increase the long term prediction accuracy of BELFIS. The next step would be to apply it to predict other indices of geomagnetic storms, disturbance storm time (Dst) index and global geomagnetic storm (Kp) index, identify complex systems and classify nonlinear data.
### TABLE VI. DIFFERENT METHODS FOR SOLAR CYCLE 23.

<table>
<thead>
<tr>
<th>Method</th>
<th>NMSE</th>
<th>Predicted value of peak</th>
<th>Prediction year, years ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td>BELFIS</td>
<td>0.1741</td>
<td>132.30</td>
<td>1995,5years</td>
</tr>
<tr>
<td>BELFIS</td>
<td>0.3858</td>
<td>145.51</td>
<td>1993,7years</td>
</tr>
<tr>
<td>ANFIS</td>
<td>0.3546</td>
<td>131.07</td>
<td>1995,5years</td>
</tr>
<tr>
<td>ANFIS</td>
<td>0.6085</td>
<td>172.96</td>
<td>1993,7years</td>
</tr>
<tr>
<td>Solar dynamo</td>
<td></td>
<td>170 ± 25</td>
<td>1993,7years</td>
</tr>
<tr>
<td>LLNF(PCA)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### REFERENCES


539