An efficient finite volume model for shallow geothermal systems—Part II: Verification, validation and grid convergence

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ABSTRACT

This part of the series of two papers presents the computational capability of the finite volume model, described in Part I, to simulate three-dimensional heat transfer processes in multiple borehole heat exchangers embedded in a multi-layer soil mass. Geothermal problems which require very fine grids, of the order of millions of finite volumes, can be simulated using coarse grids, of the order of few to tens of thousands elements. Accordingly, significant reduction of CPU time is gained, rendering the model suitable for utilization in engineering practice. A verification example comparing the computational results with an analytical solution of a benchmark case is given. A validation example comparing computed results with measured results is presented. Furthermore, numerical examples are presented describing the possible utilization of the model for research works and design.

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1. Introduction

In Part I of this series of two papers, a computationally efficient three-dimensional finite volume model for heat and fluid flow in a shallow geothermal system, in particular a ground-source heat pump, is formulated. Two main aspects have contributed to the computational efficiency: the borehole heat exchanger model, and the discretization technique. In the first aspect, heat flow in a borehole heat exchanger is modeled using a pseudo three-dimensional line element, developed by Al-Khoury et al. (2005, 2006). This model includes thermal interactions between the BHE components in the governing partial differential equations, thus alleviating the need for geometrical discretization of the individual components. In the second aspect, solution of the energy equation of the soil mass, which is the most CPU time consuming, is conducted via a combination of a locally refined Cartesian grid and a hierarchical multigrid iterative solver. This combination offers computational efficiency and stability.

In this part, numerical accuracy and computational capability and capacity of the proposed model are studied in three aspects: accuracy, validity and grid convergence. The accuracy of the model is verified by comparing the computational result to a benchmark case. The van Genuchten and Alves (1982) analytical model for solute transport in 1D is utilized for this purpose. The structure of the paper and many parts of the text are similar to those published in Al-Khoury et al. (2010). The difference, however, is that, in the previous paper, the proposed model was based on the finite element method, but here, the model is based on the finite volume method.

2. Analytical verification

Verification of the model accuracy is illustrated by comparing its computational results with the solution of a one-dimensional convective-dispersive solute transport, developed by van Genuchten and Alves (1982). This model can be utilized for solving heat transfer in a single 1D pipe in contact with a constant temperature. The temperature distribution of a fluid moving in a heat pipe is given by

\[ T(z,t) = T_{\infty} - \frac{T_{s} - T_{\infty}}{2} \left( e^{\frac{z-u t}{\sqrt{\alpha t}}} \text{erfc} \left( \frac{z-u t}{\sqrt{2 \alpha t}} \right) + e^{\frac{z+u t}{\sqrt{\alpha t}}} \text{erfc} \left( \frac{z+u t}{\sqrt{2 \alpha t}} \right) \right) \]  \hspace{1cm} (1)

in which erfc is the complementary error function, \( z = \frac{r}{c} \) is the thermal diffusivity, \( u \) is the fluid velocity, \( T_{s} \) is the temperature in the surrounding medium, \( T_{\infty} \) represents the temperature of the fluid at the inlet of the pipe, and

\[ \nu = \frac{u^2}{1 + \frac{4n\kappa}{u^2}} \]  \hspace{1cm} (2)
with \( \eta = 2b_{sg}/(r_i \rho C) \). The geometry and material parameters are as the following:

- Pipe length: 1 m
- Pipe radius, \( r_i = 0.013 \) m
- Fluid \( \rho C = 4.1298 \times 10^6 \) J/m\(^3\) K
- Fluid \( \lambda = 0.38 \) W/m K
- Fluid velocity, \( u = 3.75 \times 10^4 \) m/s
- Pipe \( b_{sg} = 12 \) W/m\(^2\) K

The boundary conditions are: \( T_{s} = 10 \) °C, and \( T_{in} = 50 \) °C. Fully implicit time integration scheme with full upwinding was utilized. The time step was 1 s. The pipe was discretized using 2000 line cells. This example is taken from Diersch et al. (2011a,b), emphasizing on the model accuracy to simulate heat transfer in short times.

The numerical results of the temperature distribution along the pipe after 1728 s, compared to Eq. (1), are shown in Fig. 1. Apparently, the results are nearly identical. Fig. 2 shows time reconstruction of the fluid temperature histories at different depths. As expected the temperature is attenuated and the signal is propagating at a constant speed.

### 3. Experimental validation

Validation of the computational model is conducted by simulating an operational vertical ground-source heat pump. The experimental site consists of 7 double U-tube borehole heat exchangers inserted vertically in the ground to a depth of 100 m. The top view of the borehole distribution is shown in Fig. 3. At 1 m below the ground surface, the pipes run horizontally without insulation, and connected to a heat pump exchanger inside the building. The temperature was measured at the inlet of pipe-in

<table>
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<td>Measured data.</td>
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<td>Per pipe Q</td>
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<td>Fluid volume</td>
<td>2 m(^3)</td>
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| Pipe length At surface | 15 m in average each in and out!
and at the outlet of pipe-out, inside the building. Details of the measurements are shown in Table 1, where it shows the time interval of the measurements, the inlet temperature of pipe-in, the outlet temperature of pipe-out, the flow rate and some other operational information. The heating system was operated for 25 min, followed by 4 h of switching off mode. For the first 45 min, the measurements were done every 1–5 min. The measurements were limited to fluid temperature at the inlet of pipe-in and outlet of pipe-out, i.e. no measurements were made along the pipes. Also, the soil temperature was not measured.

The experimental site was simulated using the finite volume model, described in Part I. The soil mass was assumed to consist of three layers with 100 m × 100 m × 150 m in dimension. As the geothermal system operated for only 25 min, thermal interaction

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Table 2
Material and geometrical parameters.

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<td>Borehole diameter</td>
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<td>Volume flow rate/pipe</td>
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<td>Specific heat capacity</td>
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<td>Dynamic viscosity</td>
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Grout

| Thermal conductivity | 2.65 W/m K |
| Specific heat capacity | 2000 J/kg K |
| Mass density         | 1100 kg/m³ |

Soil

| Layer 1: Thermal conductivity | 2.6 W/m K |
| Layer 2: Thermal conductivity | 1.5 W/m K |
| Layer 3: Thermal conductivity | 1.0 W/m K |
| All layers                    |            |
| Mass solid density            | 2400 Kg/m³ |
| Porosity                      | 0.3        |

Heat pump

| Heat pump capacity | 35 kW |

---

Fig. 4. (a) Multi-level finite volume mesh, (b) 3D initial condition, and (c) temperature profile.

---

Table 3
Input of heat pump activity in the Energy option.

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<th>Time [min]</th>
<th>Measured power [W/m]</th>
<th>Pump activity [%]</th>
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Fig. 5. Calculated versus measured temperatures using $T_{in}$ input option.
between the boreholes is negligible, and hence, simulation of one borehole suffices. The vertical part of the borehole, 100 m in length, was simulated, and the horizontal part that runs to the building was not considered. The finite volume mesh is shown in Fig. 4(a). The mesh consists of 35,940 cells for the soil mass, and 100 cells for the BHE. The material and geometrical properties of the BHE and the soil are shown in Table 2.

Simulation of the heat process is conducted using two calculation methods: one based on using the measured inlet temperature as input, from which all other temperatures along pipe-in, pipe-out, grout and soil are calculated, and another based on the power (energy) as input, from which all temperatures, including the inlet of pipe-in, are calculated. The comparison between the experimental data and the calculation results can only be done at the inlet and the outlet of the system.

Fig. 6. Calculated versus measured temperatures using energy input option.

Fig. 7. Temperature distribution along a BHE: (a) 2 min, (b) 7 min, (c) 15 min, and (d) 25 min.
3.1. Calculation phases

Simulation of the initial conditions: As indicated earlier, the soil temperature was not measured, and therefore, it has to be calculated from data given in “Action” column of Table 1 and some common knowledge on the regional weather. As shown in Table 1, at time \( t=0 \), \( T_{\text{in}} \) and \( T_{\text{out}} \) were equal to 13 °C. This indicates that the soil temperature at the surface was also around 13 °C. From many field measurements, which have been conducted in many regions in Europe, it is known that at a depth of 10–15 m below the surface and down to approximately 150 m, the temperature is almost constant (or slightly increasing) of around 10–12 °C. The simulation of such an initial condition requires two phases:

Phase 1: steady state with a constant temperature \( T_{\text{bottom}}=T_{\text{top}}=10 \) °C.
Phase 2: transient, with \( T_{\text{bottom}}=10 \) °C and \( T_{\text{top}}=13 \) °C. The BHE’s are switched off. The calculation is conducted for 10 years with 1 year time step.

The calculated initial temperature distribution in the soil is shown in Fig. 4(b) and its profile is shown in Fig. 4(c). The temperature is 13 °C at the top and reaches almost 10 °C at a depth of around 40 m, and stays constant to the bottom of the geometry.

Simulation of the system operation: As shown in Table 1, the heating system was operated for 25 min after which it was switched off. However, the measurements continued for around 4 h.

3.1.1. Calculation based on fluid \( T_{\text{in}} \)

Using the fluid temperature at the inlet of pipe-in, \( T_{\text{in}} \) as input, two phases are necessary for simulating the switching on and off of the system.

Phase 3: This phase simulates the first 25 min of operation. \( T_{\text{in}} \) was used as input value and \( T_{\text{out}} \) was calculated. The input values for \( T_{\text{in}} \) and their corresponding time steps were taken from the fourth and second columns, respectively, of Table 1.
Phase 4: This phase simulates 68 min of switching off the system. For this, the heat pump consumption (energy) method of calculation, described in the next section, is necessary. In this method it is possible to define the activity of the system, wherein 100% indicates that the system is fully operational and consumes 35 kW, and 0% indicates that it is non-operational. This power is used as input for calculating the temperature distribution in the whole system, including those at the inlet of pipe-in, \( T_{\text{in}} \) and pipe-out, \( T_{\text{out}} \). However, an initial value of \( T_{\text{in}} \) is necessary and it was prescribed to 13 °C. 23 time steps representing 93 min were conducted.

3.1.2. Calculation results. The calculated temperatures at the inlet of pipe-in and the outlet of pipe-out together with those measured consumed by the heat pump is known a priori, as it is specified by the designer. The amount of power extracted from a BHE is calculated as

\[
P = \rho \cdot c_r \cdot Q (T_{\text{out}} - T_{\text{in}})
\]

in which \( P \) is the power obtained from a BHE, \( \rho \), is the mass density of the fluid, \( c_r \) is the specific heat of the fluid, \( Q \) is the flow rate, and \( T_{\text{out}} \) and \( T_{\text{in}} \) are the temperatures at the outlet and inlet of the BHE respectively. Using the data of Table 1 and the above listed material and geometrical parameters of the BHE, it can readily be calculated that the heat pump capacity of the system was 35 kW. This implicitly indicates that for a system consisting of 7 BHEs with 100 m length for each, the BHE capacity was 50 W/m.

As for the previous calculation method, the initial condition was initiated by Phase 1 and Phase 2 with the temperature distribution shown in Fig. 4.

Phase 3: Using Eq. (3), the measured power is calculated. Table 3 shows the measured power and the activity of the system, wherein 100% indicates that the system is fully operational and consumes 35 kW, and 0% indicates that it is nonoperational. This power is used as input for calculating the temperature distribution in the whole system, including those at the inlet of pipe-in, \( T_{\text{in}} \) and pipe-out, \( T_{\text{out}} \). However, an initial value of \( T_{\text{in}} \) is necessary and it was prescribed to 13 °C. 23 time steps representing 93 min were conducted.

3.1.2.1. Calculation results. The calculated temperatures at the inlet of pipe-out together with those measured

![Table 4](attachment:table4.png)

<table>
<thead>
<tr>
<th>BHE</th>
<th>Type</th>
<th>Double U-tube</th>
</tr>
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<tbody>
<tr>
<td>Borehole diameter</td>
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<td></td>
</tr>
<tr>
<td>Length</td>
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<td></td>
</tr>
<tr>
<td>Outer pipe diameter</td>
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<tr>
<td>Wall thickness</td>
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<tr>
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<tr>
<td>Volume flow rate/pipe</td>
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<tr>
<td>Fluid</td>
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<tr>
<td>Thermal conductivity</td>
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<td>Specific heat capacity</td>
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<td>Dynamic viscosity</td>
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<td>Prandtl number</td>
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<tr>
<td>Grout</td>
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<tr>
<td>Specific heat capacity</td>
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<tr>
<td>Mass density</td>
<td>1100 kg/m³</td>
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<td>Soil</td>
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<tr>
<td>Thermal conductivity</td>
<td>2.65 W/m K</td>
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</tr>
<tr>
<td>Mass solid density</td>
<td>2400 Kg/m³</td>
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<tr>
<td>Layer 2</td>
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<td>Mass solid density</td>
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<td>Layer 3</td>
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<tr>
<td>All layers</td>
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<td>BHE</td>
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<tr>
<td>Heat pump</td>
<td>Heat pump capacity</td>
<td>500 kW</td>
</tr>
</tbody>
</table>

![Fig. 5](attachment:fig5.png)

The first 25 min represent the operation mode of the system, followed by a switching off mode. Here it can be observed that the inlet temperatures in the first 25 min are identical to those input in the finite volume model, later on, it was calculated using the energy method (see below). The calculation results are in the order of ± 0.5 °C from those measured from the field.

3.1.2. Calculation based on power consumption (energy method)

In engineering practice, the temperature at the inlet of pipe-in is continuously changing. However, the amount of heat power
are shown in Fig. 6. The first 25 min represent the operational mode of the system, followed by a switching off mode. The calculation results are in the order of ±1 °C from those obtained from the field, constituting an error of less than 4% on average.

The calculation results for different time steps are shown in Fig. 7. The figure shows the temperatures in pipe-in, pipe-out, grout and soil in contact with the borehole. This figure shows the logical variation of the temperature with time. The first to be affected by the system operation is pipe-in, where it can be seen that after 2 min, its temperature distribution started to change, especially at the top (between 0 m and 20 m). These results, however, cannot be compared with experimental data since no measurements had been conducted in the fluid along the BHE. The CPU time for 23 time steps was less than 83 s in an AMD Phenom X6 3.2 GHz PC.

4. Grid convergence

Here, we study the grid convergence (grid sensitivity) of the proposed model. We examine the model computational efficiency in handling large cases that require, in standard models, millions of cells/elements with hours or days of CPU time.

Consider a soil mass of 150 m × 150 m × 150 m, embedded in which 100 double U-tube borehole heat exchangers, configured in 10 × 10 matrix, 5 m distances between neighboring boreholes. Two soil types were assumed: soil 1 constitutes the upper layer (from 0 m to −50 m) and the bottom layer (from −125 m to −150 m), and soil 2 constitutes the layer in the middle (from −50 m to −125 m). The geometrical and material parameters of the soil mass and the boreholes are presented in Table 4.

Four grid sizes were utilized: G1 = 4096, G2 = 16,472, G3 = 66,788 and G4 = 297,508 cells. Fig. 8 shows cross-sections at the middle of the geometry. It can be seen that to capture the high gradient in temperature field, local grid refinements are made near the borehole heat exchangers and along the ground surface.

4.1. Calculation phases

Phase 1: calculating soil temperature distribution. The initial condition was first assumed to be 13 °C for the whole profile.
Then the air temperature was set to 0 °C for 10 years. The computed temperature profile is shown in Fig. 9.

Phase 2: calculating temperature distribution of the geothermal system (soil-pipe interaction) for 6 months. A heat pump power of 500 kW was assumed. This implicitly indicates that for a system consisting of 100 double U-tube BHE with 100 m in length for each, the borehole heat capacity was 50 W/m, a typical heat capacity recommended by geothermal engineers. The pump was operated at 30% of its capacity. \( T_m \) is fixed to 0 °C, and the time step is 30 days. The thermal coefficients, \( b_{ig} \), \( b_{bg} \), and \( b_{bg} \) are calculated automatically using Eqs. 16 and 24 of Part I.

Fig. 10 shows a sample result representing temperature profiles along pipe-in, pipe-out and grout for a BHE at the center, for all grids. The figure shows that there is no significant difference in the results between the four grids, though the result of the coarse grid, G1, is less accurate. However, it exhibits no numerical oscillations. Nevertheless, it can be deduced that the model converged at G2 grid. The CPU time for 6 time steps were: 
\[ G1 = 20 \text{ s}, \quad G2 = 23 \text{ s}, \quad G3 = 32 \text{ s}, \quad \text{and} \quad G4 = 75 \text{ s} \]

in an AMD Phenom X6 3.2 GHz PC with OpenMP parallelization. Without parallelization, the CPU time becomes around 4.5 times that with parallelization. Using standard finite volume, finite difference or finite element methods to solve such a geothermal system with 100 double U-tube BHE might require millions of cells/elements with days of CPU time.

5. Numerical example

The aim of this numerical example is to study heat transfer processes in a shallow geothermal system for 7 days of system operation, in the absence and presence of groundwater flow. Two calculation phases were conducted. One describes the temperature distribution in the soil mass when subjected to monthly variation of air temperatures. And another describes temperature distribution along the geothermal BHE in 7 days of operation.

Similar to the previous example, the geothermal system is assumed to consist of 100 BHEs embedded in a three-layer soil mass. G3 grid, with 66,788 cells for the soil and 100 cells for each BHE, was utilized. The average cell size for the soil is 5.5 m, and for the BHE is 1 m.

5.1. Calculation phases

Phase 1: calculating soil temperature distribution in 12 months with varying air temperature. The initial condition was first assumed to be 13 °C for the whole profile. Then the air temperature was set to 0 °C for 10 years, just to get the temperature profile shown in the left-hand side of Fig. 12(a). 12 time steps, each with a 1 month period, were conducted. The air temperature was varied between 0 °C and 15 °C, following the time scheme shown in Fig. 11. The pipes were kept non-operational. The calculation results for January, July and December are shown in Fig. 12. The figure shows also the...
temperature profile along the depth. It can be seen that with varying air temperatures between summer and winter, the upper 12 m of the soil mass was affected, while at a higher depth, the temperature stayed almost constant.

**Phase 2:** calculating temperature distribution of the geothermal system (soil-pipe interaction) for 7 days in September. The soil temperature profile is shown in Fig. 13, where the temperature at the surface is 10.2 °C and at the bottom is 13 °C, calculated in Phase 1. The thermal coefficients, $b_{ps}$, $b_{og}$, and $b_{sg}$ are calculated automatically using Eqs. 16 and 24 of Part I.

As for the previous case, a heat pump power of 500 kW was assumed. It was also assumed that after every day of full operation, a day of non-operation was followed. The initial refrigerant temperature entering pipe-in, $T_{in}$, was 5 °C, and updated every 0.5 day. This means that 14 calculation time steps were necessary. An example of the calculation result is shown in Fig. 14. The figure shows temperature distributions along pipe-in, pipe-out and grout of one of the BHEs at the center for different times. It can be seen that the temperature in the pipe was gradually cooling down. However, during the non-operational period, some recovery took place. The CPU time was 50 s for 14 time steps in an AMD Phenom X6 3.2 GHz PC, i.e., less than 3.6 s for each time step.
5.2. Effect of groundwater flow

The effect of the groundwater flow (GWF) was also studied. The geometry and physical and thermal properties of the soil and the BHEs were as those of the previous example. The boundary conditions were as for December. Two analyses were conducted: one without groundwater flow and one with groundwater flow occurring in the soil layer (soil 2) at the middle of the geometry. The velocity of the groundwater flow was assumed $1 \times 10^{-6}$ m/s. All pipes were activated, and the analyses were conducted to simulate 10 years of 30% operational capacity.

Fig. 15 shows the soil temperature at $-100$ m, near to the bottom of a BHE at the center of the geometry, obtained from both analyses. The figure shows clearly the effect of groundwater flow on maintaining higher soil temperature. Fig. 16 shows three-dimensional temperature distribution in the soil mass and around the BHEs for both analyses.

6. Conclusion

In this part of the series of two papers, experimental validation, verification, grid convergence and a numerical example describing

![Fig. 14. Temperature distribution versus depth in a heat pipe at different times: (a) after 12 hours, (b) after 2 days, (c) after 3.5 days, (d) after 5 days, (e) after 6 days and (f) after 7 days.](image-url)
the computational capabilities of a finite volume model for heat flow in three-dimensional shallow geothermal systems, formulated in Part I, are presented. Experimental validation of the model showed that the model is capable of simulating measured data using relatively coarse grids. Analytical verification showed that the model gives very accurate results for even very short times. Grid convergence test showed that the model converges to the right solution at relatively coarse grid. The numerical examples showed that a large geothermal system consisting of 100 borehole heat exchangers, of 100 m in length for each, embedded in 3-layer soil mass could be modeled using coarse grids. As a result, the CPU time for conducting the calculations, using AMD Phenom X6 3.2 GHz PC, was in the order of 3.6 s per time step.

Obviously such computational requirements are minor as compared to typical 3D finite volume, finite difference or finite element requirements, where millions of cells/elements are needed to simulate the presented geothermal systems. The computational efficiency is attributed to the quasi-3D borehole heat exchanger model and the finite volume discretization and solution techniques. As a result, it is expected that the gained computational efficiency will boost considerably the possibilities for more insight into geothermal analysis, which might improve the procedures for BHE design.

![Fig. 15. Soil temperature versus time in the presence of GWF and without.](image)

![Fig. 16. Soil temperature distribution. Upper: general view, left: without GWF, right: with GWF. Lower: cross section around the BHEs, left: without GWF, right: with GWF.](image)
References


