Adaptive Learning and Information Diffusion*

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Abstract

This paper introduces a two-stage interactive cobweb model with information diffusion. The additional stage of information diffusion leads to a possibility of up to five equilibria. Stability properties under learning remain similar to those under a model with one-way information diffusion, but this modified assumption improves forecast efficiency for all firms involved.

Keywords: Adaptive Learning; Expectational Stability; Information Diffusion; Cobweb Model; Heterogeneous Expectations

JEL classification: C62, D84, E37

1 Introduction

Previous studies in the adaptive learning literature focus on an independent learning mechanism with either homogeneous or heterogeneous expectations.¹ Granato, Guse, and Wong (2008) (hereafter, GGW) present a modified “interactive” cobweb model to allow for both information heterogeneity and information diffusion. They assume that "Type-L" firms make initial forecasts based on exogenous information while "Type-F" firms form forecasts by observing Type-L’s expectations with possible interpretation (measurement) errors. This modification of the cobweb model leads to a possibility of up to three equilibria and expands the parameter space of potential learnable equilibria. They also discover the "boomerang effect" where Type-F firms’ inability to correctly observe Type-L firms’ expectations leads to a reduction in Type-L forecasting efficiency.

GGW argue that the boomerang effect exists because Type-L firms do not realize that interpretation errors create excess volatility. The purpose of this paper is to explore the equilibrium properties and firm forecast efficiency if a second-stage of information diffusion is introduced in the GGW model. After Type-F firms receive the initial expectations from Type-L firms to form expectations, we assume that Type-L firms can improve their expectations by observing Type-F firms’ forecasts with possible interpretation errors. This new setup leads to a possibility of up to five equilibria in the model, and for some parameter values the space of at least one learnable solution is expanded.

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¹See Evans and Honkapohja (2001).
Finally, the introduction of the second-stage information diffusion improves the forecast efficiency for all firms, and the boomerang effect will disappear if leading firms can perfectly observe following firms’ forecasts in the second stage.

2 The Setup of the Model

Consider a cobweb model of the following form: \( y_t = \beta E_{t-1} y_t + \gamma x_{t-1} + \eta_t \), where \( y_t \) is the price level at time \( t \), \( E_{t-1} y_t \) is the average expectation (not necessarily rational) of \( y_t \) formed at the end of time \( t-1 \), and \( \eta_t \sim iid (0, \sigma_\eta^2) \). \( x_{t-1} \) is an exogenous observable following a stationary AR\( (p) \) process driven by a white noise shock. We assume \( E x_t = 0 \) and \( E x_t^2 = \sigma_x^2 \). Under a cobweb model with a single good, it must be that \( \beta < 0 \); however, there exists variants of the cobweb model such that \( \beta \in (-\infty, \infty) \) (See Honkapohja and Mitra (2003) for an example).

GGW assume two different types of firms where Type-L firms have \( x_{t-1} \) in their information set while Type-F firms lack this information. Due to the information heterogeneity, firms in each group may believe that the other group’s expectations of \( y_t \) are misinterpreted. Thus, Type-L’s perceived law of motion (PLM) of \( y_t \) is:

\[
\hat{y}_{t;L} = \hat{y}_{t;L}^1 + \hat{y}_{t;L}^2 + \varepsilon_t,
\]

where \( \hat{y}_{t;L}^1 = y_{t;L}^1 + u_{t-1} \) is Type-L firms’ understanding of Type-L firms’ expectations of \( y_t, \) \( v_{t-1} \sim iid (0, \sigma_v^2) \) represents unobservable interpretation errors, and \( y_{t;L}^1 \) is given below. We present Type-F firms’ expectations of \( y_t \) as:

\[
y_{t;F}^1 = c (y_{t;L}^1 + v_{t-1}).
\]

GGW shows that if Type-L firms use the PLM: \( y_t = hx_{t-1} + \varepsilon_t \), then their mean squared error (MSE) is larger than what it would be if all firms are Type-L. If Type-L firms had the information of \( y_{t;F}^1 \), they would discover that their forecast error is highly correlated with this variable. Therefore, Type-L firms could improve their forecasts if they had access to the Type-F firms’ expectations.

We assume that Type-L firms will interact with Type-F firms to obtain information on \( y_{t;F}^1 \). However, the existence of interpretation errors makes Type-L firms less likely to perfectly observe \( y_{t;F}^1 \). As a result, Type-L’s second-stage PLM is:

\[
y_t = bx_{t-1} + d y_{t;F}^1 + \varepsilon_t,
\]

where \( y_{t;F}^1 = y_{t;F}^1 + u_{t-1} \) and \( \varepsilon_t \) is Type-F firms’ understanding of Type-L firms’ expectations of \( y_t \). Since Type-L firms have access to the Type-F firms’ expectations before obtaining \( y_{t;F}^1 \), Type-L firms can estimate the forecasting model presented in GGW (i.e., \( y_t = hx_{t-1} + \varepsilon_t \)). In this case, \( y_{t;L}^1 = hx_{t-1} \) is Type-L firms’ best attempt to form expectations of \( y_t \) without the knowledge of \( y_{t;F}^1 \).

Suppose the proportion of Type-F firms is \( \mu \) and that of Type-L firms is \( 1 - \mu \). The actual law of motion (ALM) is obtained by substituting average expectations of the market price into the structural equation:

\[
y_t = F (\phi) x_{t-1} + F_v (\phi) v_{t-1} + F_u (\phi) u_{t-1} + \eta_t,
\]

where \( \phi = (b, c, d, h) \).

Following Evans and Honkapohja (2001), we obtain a projected ALM associated with each particular PLM. The projections give a T-mapping from the three PLMs to their associated projected ALMs: \( T (\phi) = (T_b (\phi), T_c (\phi), T_d (\phi), T_h (\phi))^T \).

In the T-mapping, there are two parameters of interest: \( a_1 = \sigma_x^2 / \sigma_v^2 \) and \( a_2 = \sigma_c^2 / \sigma_u^2 \). The parameter \( a_1 \) is the ratio of variability of important forecasting information \( (\sigma_v^2) \) to misinterpretation variability \( (\sigma_u^2) \) presented in GGW. As \( a_1 \) increases, the first stage of communication becomes more useful. Since Type-L firms wish to extract \( v_{t-1} \) in the second stage of interaction, we introduce a new parameter \( a_2 \) which represents the ratio of the variability of "useful" forecasting information \( (\sigma_c^2) \) to misinterpretation variability \( (\sigma_u^2) \). As \( a_2 \) increases, the second stage of interaction becomes more useful. Under the setup of GGW, \( a_2 \) is implicitly set to zero.

\(^2\) Explicit representations of all equations are available upon request.
For a model with heterogeneous forecasting rules, Guse (2005) refers to a resulting stochastic equilibrium defined by the ALM and a fixed point of the T-map as a "mixed expectations equilibrium" (MEE). In the model presented above, a MEE is a stochastic process following the ALM where \( \phi = \tilde{\phi} \) and \( \tilde{\phi} = T(\tilde{\phi}) \).

The equilibrium function \( \tilde{\phi} = (b, c, d, h)' \) in the T-mapping is non-linear (a quintic), so there may exist multiple equilibria. Because there is no general technique to solve for solutions of a quintic function, we take a numerical approach to discuss number of equilibria, learnability of such stochastic equilibria, and the MSE for each forecasting model.

3 Equilibria and E-stability

Next, we examine the E-stability properties of the possible equilibria in \((a_1, \beta)\) space. To examine E-stability, we consider the following ordinary differential equation (ODE): \( \frac{d\phi}{d\tau} = T(\phi) - \phi \), where \( T \) is the mapping from the PLM, \( \phi \) to the implied ALM, \( T(\phi) \) and \( \tau \) denotes "notional" time. Evans and Honkapohja (2001) define an equilibrium to be E-stable (locally stable under least squares learning) if the ODE is stable when evaluated at the equilibrium values. We illustrate the conditions of uniqueness, multiplicity, and E-stability in Figure 1.

(Figure 1 about here)

Under a standard model of homogeneous expectations, the unique rational expectations equilibrium is (globally) E-stable when \( \beta < 1 \) and never E-stable when \( \beta > 1 \). In this modified model, we also find that at least one MEE is always E-stable when \( \beta < 1 \). In the first quadrant of Figure 1, which is consistent with GGW where \( a_2 = 0 \), we find that two of the three possible MEE are locally stable under adaptive learning in Region E. More importantly, there is a small region such that a MEE is E-stable when \( \beta > 1 \). The other quadrants show how the results change when information in the second stage of diffusion becomes more useful (i.e., \( a_2 > 0 \)). The second stage creates a possibility of up to five equilibria as shown in Region D. However, the E-stability results do not change much with the introduction of the second stage of information diffusion. When \( \beta < 1 \), at least one MEE continues to be E-stable, and as \( a_2 \) increases, the region of two E-stable MEE decreases. Although there is a region with a possible five solutions when \( \beta > 1 \), only one MEE will be E-stable. Finally, as \( a_2 \) increases, the E-stable region where \( \beta > 1 \) decreases. In fact, as \( a_2 \) approaches infinity, the E-stability condition collapses to \( \beta < 1 \) as in the homogeneous expectations case.

4 Information Diffusion and the MSE

Finally, we consider how the second stage of information diffusion affects forecasting efficiency of each firm type. At the MEE, the MSE for the Type-L and Type-F firms are the following respectively: \( MSE_L = E\left(y_t - \tilde{y}_{t,L}^{(1)}\right)^2 \) and \( MSE_F = E\left(y_t - (\bar{x}_{t-1} + \tilde{y}_{t,L})\right)^2 \). We introduce a measure called an "adjusted" MSE (AMSE) for each firm type. The AMSE for the Type-L and Type-F firms are: \( AMSE_L = (MSE_L - \sigma_r^2) / \sigma_r^2 \) and \( AMSE_F = (MSE_F - \sigma_r^2) / \sigma_r^2 \). The AMSE is the bias produced by a forecasting model relative to the variability of the noise produced in the first stage of information diffusion.

\[ ^3 \text{If parameter values are not given in the figures, we assume the following: } \beta = 0.75, \gamma = 2, \text{ and } \mu = 0.9. \]
GGW present the boomerang effect on the MSE from information diffusion where the Type-L MSE is increased due to imperfect information diffusion (i.e., $\sigma^2 > 0$). If $\beta$ and $\mu$ are sufficiently close to 1, then $\nu_{t-1}$ becomes more important information for forecasting $y_t$. In this case, Type-F firms do a better job at forecasting $y_t$ than Type-L so that $MSE_F < MSE_L$. We examine how the second stage of information diffusion affects the boomerang effect and how AMSE’s change with more accurate information in the second stage.

(Figure 2 about here)

Figure 2 shows that increases in $a_2$ will always lead to an efficiency gains for both forecasting models. For our parameter choices, $AMSE_L > AMSE_F$ for small values of $a_2$. As the information in the second stage becomes more useful (a larger $a_2$), Type-L firms have larger efficiency gains and therefore, the Type-L forecasting model becomes more likely to produce a smaller MSE than the Type-F forecasting model. These results are robust for different values of $\gamma$, $\mu$, and $\beta$. The GGW boomerang effect on the MSE exists when $AMSE_L > 0$. Each quadrant shows that the second stage information diffusion reduces forecasting bias for each type’s forecasting model. Further, we find that if there is perfect information diffusion in the second stage, then the boomerang effect disappears (i.e., $\lim_{a_2 \to \infty} AMSE_L = 0$).

5 Conclusion

In this paper, we modify an interactive cobweb model suggested by GGW where Type-L firms can improve their forecasts by imperfectly observing Type-F firms’ expectations. We find that there is a possibility of up to five equilibria in the model and the properties of E-stable equilibria are similar to GGW. The second stage of information diffusion improves the forecast efficiency for all firms, and the boomerang effect disappears if Type-L firms can perfectly observe Type-F forecasts in the second stage.

References


Figure 1: Equilibria and E-stability Properties

Figure 2: $AMSE_L$ and $AMSE_F$