Vibration Absorption in a Rotor-Bearing System Using a Cantilever Beam Absorber

A. Cabrera-Amado, M. Arias-Montiel, and G. Silva-Navarro
Centro de Investigación y de Estudios Avanzados del I.P.N.
Departamento de Ingeniería Eléctrica - Sección de Mecatrónica
Apdo. Postal 14-740, C.P. 07360 México, D.F. MÉXICO
alvaroca1@hotmail.com, mam7915@yahoo.com.mx and gsilva@cinvestav.mx

Abstract — This work treats the problem of vibration control in a rotor-bearing system using a passive absorber with controllable stiffness. The primary system consists of a rotor system mounted on two supports at its ends, one of them is a classical journal bearing and the other one is a bearing on a slider which can be displaced in order to change the distance between supports. The passive absorber is a concentrated mass mounted at the end of a cantilever beam which is an extension of the flexible shaft of the rotor system. The vibration attenuation in the rotor system is achieved modifying the system dynamics by the displacement of the slider bearing which causes changes in the absorber and primary system stiffness and as a consequence in their natural frequencies. The rotor-bearing system with the passive absorber is modeled using Finite Element Methods (FEM). To control the slider bearing position a PD control scheme is used, which is parameterized in terms of the spin speed of rotor. The vibration control scheme proposed is validated by numerical results, obtaining reductions up to 84% in the unbalance response of the primary system in relation with the open loop operation.

Keywords: Rotor-bearing system, passive absorber, semi-active vibration control.

I. Introduction

Vibration control in structures and machines has been a field of interest in varied branches of engineering, technology and industry for many years. The three most commonly used classification of vibration control systems are: passive, active and semi-active. Active and semi-active controllers require external power sources while the passive ones do not require external power for their operation. Active and semi-active control schemes in rotordynamics have been widely studied [11], [4]. In contrast, a vibration absorber is a mass-spring system with very little or no damping, which can absorb vibration excitations through energy transfer into it, thereby reducing the vibrations of the primary system [3]. Passive control of vibration is relatively simple and straightforward, robust, reliable and economical, but it has its limitations, namely, it is not possible to adjust the control forces once the device is designed and implemented. Therefore, this kind of vibration control systems are designed and performed for some specific frequencies.

Recently, vibration absorbers have been modified in order to improve their performance in a more wide frequency range with applications in structures. Chen and Wang [2] propose to use of mass-damper systems along a simply supported Timoshenko beam to reduce the vibrations of the first mode shapes, they present only numerical results for this method.

Blanco et al. [1] propose a Jeffcott-like rotor supported on two bearings, one of which can be horizontally moved, by means of some servomechanism in order to compensate the system response. The asymmetric motion of the disk, because of the motion of one of the supporting bearings, leads to a nonlinear and coupled model. The proposed methodology for dynamic stiffness control of the transient run-up of the rotor consists of two controllers: the first one, is a controller for the trajectory planning for the speed trajectory, and the second one, is a controller for the smooth switching on the position of the movable bearing. This technique allows to reduce and stabilize the unbalance response while passing through its first critical speed.

Jallili and Knowles [8] present an active resonator absorber for structural vibration control showing its effectiveness and stability by numerical simulations and proposing an inertial piezoelectric actuator for future applications. Sun et al. [9] shows a new kind of adaptive active resonator absorber which combines an adaptive passive vibration absorber with an active resonator absorber getting a reduction in the control effort observed by numerical results.

The works above mentioned apply vibration absorbers in structures. In this paper we propose an absorber to be used in a rotor-bearing system in order to attenuate its unbalance response. For this, the primary system and the absorber dynamics are modified by the displacement of one of the rotor supports, which is mounted on a slider. This motion is parameterized in terms of the spin speed of the rotor such that the slider tracks certain trajectory to avoid the resonant peaks and in this way reduce the unbalance response during the running up. Some numerical results show a good closed loop performance in comparison with the open loop system, obtaining reductions in vibration amplitudes up to 84%.
II. ROTOR-BEARING SYSTEM WITH PASSIVE ABSORBER

The primary system consists in a rotor system mounted on two supports at its ends, one of them is a classical journal bearing and the other one is a journal bearing on two sliders. A plane disk with mass $m_r$ is mounted on the flexible shaft of the rotor, it is supported in the left by a traditional journal bearing which is rigid enough with large stiffness $k_{iy}$ and the right journal bearing is mounted on two sliders to change the distance between supports into some small interval, which provides the appropriated longitudinal force to reduce vibrations of the primary system. The rotor has a cantilever beam absorber, which is an extension of the flexible shaft of the rotor-bearing system. It has a concentrated mass $m_a$ mounted on a ball bearing at the shaft end (this mass does not rotate). For regulation of the angular speed $\omega$ is considered a CD motor, which is controlled by a speed driver with an internal PID control (see Fig. 1).

![Fig. 1: Rotor-bearing system with a cantilever beam absorber.](image)

III. ROTOR-BEARING MODEL

The rotor is modeled using finite element methods, where three finite elements and four nodes are considered to construct the reduced model of the rotor-bearing system. The schematic diagram of the discretized model is shown in Fig. 2. Both journal bearings have different dynamics, with mass, stiffness and viscous damping. The finite elements are considered to construct the reduced model of the rotor-bearing system.

The mathematical model is analyzed in one plane of motion for the vertical $y$ dynamics. The FEM model assumes that each journal bearing has concentrated mass and stiffness in the corresponding nodes on the beam. The viscous damping in the rotor is considered proportional to the mass and stiffness. The finite elements are considered as Euler type beam and each element has 2 DOF per node as illustrated in Fig. 2.

![Fig. 2: Schematic diagram of the discretized rotor-bearing system.](image)

Here the vertical displacement of the disk is $y_r$, the unbalance mass in the disk $m_a$ and the disk eccentricity $e$. The left journal bearing has radial displacement $y_1$, a mass $m_1$ and the stiffness is approximated by $k_{iy}$. The sliding journal bearing has radial displacement $y_2$, the linear displacement $z$, its mass $m_d$ and its stiffness is approximated by a large value $k_{dy}$. The passive absorber is a cantilever like-beam that has a concentrated mass $m_a$ mounted on a ball bearing at the shaft end (this mass does not rotate).

The angular deflections of the beam are represented by coordinates, $\beta_y$ in the left journal bearing , $\beta_{sy}$ in the disk, $\beta_{dy}$ to the right journal bearing and $\beta_{ay}$ at the shaft end (see Fig. 2).

For the beam absorber is considered 2 DOF per each node, such that the overall model has 8 DOF, and an additional degree corresponding to the linear displacement of the sliding journal bearing. The beam model is endogenously perturbed by the unbalance force $f_y$ due to the unbalance mass $m_r$ in the disk.

The mass, stiffness and damping elemental matrices for the finite element model of a rotor-bearing systems are based in Genta [5].

The stiffness matrix $K_y$ of the rotor-bearing is given by

$$K_y = \gamma_0 \begin{bmatrix} 711 & 712 & 713 & 714 & 715 & 716 & 717 & 718 \\ 722 & 723 & 724 & 725 & 726 & 727 & 728 \\ 733 & 734 & 735 & 736 & 737 & 738 \\ 744 & 745 & 746 & 747 & 748 \\ 755 & 756 & 757 & 758 \\ sym & 766 & 767 & 768 \\ 777 & 778 \\ 788 \end{bmatrix}$$

where

$$\gamma_0 = EI; \gamma_{11} = \frac{12}{l_1^2} + \frac{k_{iy}}{EI}; \gamma_{12} = \frac{6}{l_1^2}; \gamma_{13} = \frac{-12}{l_1^2}; \gamma_{14} = \frac{6}{l_1^2}; \gamma_{15} = 0; \gamma_{16} = 0; \gamma_{17} = 0; \gamma_{18} = \frac{4}{l_1}; \gamma_{23} = \frac{6}{l_1^2}; \gamma_{24} = \frac{2}{l_1}; \gamma_{25} = 0; \gamma_{26} = 0; \gamma_{27} = 0; \gamma_{28} = 0; \gamma_{33} = \frac{12}{l_1^2} + \frac{12}{l_2^2}; \gamma_{34} = \frac{6}{l_2^2}; \gamma_{35} = -\frac{12}{l_2^2}; \gamma_{36} = \frac{6}{l_2^2}; \gamma_{37} = 0; \gamma_{38} = 0; \gamma_{44} = \frac{4}{l_1^2}; \gamma_{45} = -\frac{6}{l_2^2}; \gamma_{46} = \frac{2}{l_2^2}; \gamma_{47} = 0; \gamma_{48} = 0; \gamma_{55} = \frac{12}{l_1^2} + \frac{12}{l_3^2} + \frac{k_{dy}}{EI}; \gamma_{56} = \frac{6}{l_3^2} - \frac{6}{l_2^2}; \gamma_{57} = -\frac{12}{l_3^2} - \frac{788}{l_3^2}; \gamma_{58} = \frac{6}{l_3^2}; \gamma_{66} = \frac{4}{l_2^2}; \gamma_{76} = -\frac{6}{l_3^2}; \gamma_{77} = \frac{4}{l_3^2}; \gamma_{78} = \frac{4}{l_3^2};$$
The mass matrix $M_y$ is

$$M_y = a_0 \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} \\ a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} & a_{39} \\ a_{44} & a_{45} & a_{46} & a_{47} & a_{48} & a_{49} & a_{50} \\ a_{55} & a_{56} & a_{57} & a_{58} & a_{59} & a_{60} & a_{61} \\ a_{66} & a_{67} & a_{68} & a_{69} & a_{70} & a_{71} & a_{72} \\ a_{77} & a_{78} & a_{79} & a_{80} & a_{81} & a_{82} & a_{83} \\ a_{88} \end{bmatrix}$$

where

$$a_0 = \frac{ρA}{420}; a_{11} = 156 l_1 + m_r \frac{420}{ρA}; a_{12} = 22 l_1^2;$$

$$a_{13} = 54 l_1; a_{14} = -13 l_1^2; a_{15} = 0; a_{16} = 0; a_{17} = 0;$$

$$a_{18} = 0; a_{22} = 4 l_1^2; a_{23} = 13 l_1^2; a_{24} = -3 l_1^3; a_{25} = 0;$$

$$a_{26} = 0; a_{27} = 0; a_{28} = 0;$$

$$a_{33} = 156 l_1 + 156 l_2 + m_r \frac{420}{ρA}; a_{34} = 22 l_1^2 - 22 l_2^2;$$

$$a_{35} = 54 l_2; a_{36} = -13 l_2^2; a_{37} = 0; a_{38} = 0;$$

$$a_{44} = 4 l_1^3 + 4 l_2^3; a_{45} = 13 l_1^3; a_{46} = -3 l_2^3;$$

$$a_{47} = 0; a_{48} = 0; a_{55} = 156 l_2 + 156 l_3 + m_d \frac{420}{ρA};$$

$$a_{56} = 22 l_2^3 - 22 l_2^3; a_{57} = 54 l_1; a_{58} = -13 l_3^2;$$

$$a_{66} = 4 l_1^3 + 4 l_2^3; a_{77} = 156 l_3 + m_r \frac{420}{ρA}; a_{67} = 13 l_3^2;$$

$$a_{68} = -3 l_3^3; a_{78} = -22 l_2^2; a_{88} = 4 l_3^3.$$

The proportional damping matrix is a linear combination

$$C_y = 19.1 M_y + 18 \times 10^{-6} K_y$$

The dynamics of the sliding journal bearing is assumed as follows

$$m_d \ddot{z} + c_d \dot{z} = u$$

where $u$ denotes the control force.

The global system vertical dynamics is described as

$$M \ddot{q} + C \dot{q} + K \dot{q} = B u + F(t)$$

where $M$, $C$, $K \in R^{9 \times 9}$ and

$$M = \begin{bmatrix} M_y & 0 & 0 \\ 0 & m_d & 0 \end{bmatrix}; C = \begin{bmatrix} C_y & 0 & 0 \\ 0 & c_d & 0 \end{bmatrix}; K = \begin{bmatrix} K_y & 0 \\ 0 & 0 \end{bmatrix}$$

$$q = [ y_i \, \beta_{iy} \, y_r \, \beta_{ry} \, y_d \, \beta_{dy} \, y_a \, \beta_{ay} \, z]^T$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$F(t) = \begin{bmatrix} 0 & 0 & f_y(t) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$f_y(t) = m_r e \omega^2 \sin \omega t - m_r e \omega \cos \omega t$$

The physical parameters for the overall 9 DOF rotor-bearing system are given in Table I.

<table>
<thead>
<tr>
<th>Table I. Rotor-bearing system parameters</th>
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<tbody>
<tr>
<td>Left journal bearing</td>
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<tr>
<td>$m_r$ = 0.4kg</td>
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<tr>
<td>$k_{dy}$ = 4 x $10^6$ N/m</td>
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Other parameters of the system are: Young’s modulus $E = 200$ GPa, total length of the shaft $L = 0.512$m, first finite element length $l_1 = 0.19$m, second element length $l_2 = 0.19 + z$, third element length $l_3 = 0.1322 - z$, eccentricity $e = 43$µm, shaft diameter $D = 0.01$m, material density $ρ = 7807$ kg/m$^3$, inertia moment $I = \pi D^4/4$, transversal section $A = \pi D^2/4$, and the coordinate $z$ can be moved into the range $[-0.06, ..., 0.06]$m.

IV. Natural frequencies of the rotor-bearing system

The natural frequencies are obtained from the undamped rotor-bearing system (2) in free vibrations as

$$M \ddot{q} + Kq = 0, \quad q \in R^9$$

(3)

The natural frequencies associated with the vibration modes of the system are the eigenvalues of $M^{-1}K$ and two main natural frequencies are shown in Fig. 3, when the sliding journal bearing is moved into the range $[-0.05, ..., 0.05]$m. The first natural frequency $\omega_{n1}$ is associated to the disk $m_r$ and the second $\omega_{n2}$ to the mass $m_a$ at the shaft end.

![Fig. 3: The natural frequencies change their values, when the sliding journal bearing is moved into the range $[-0.05, ..., 0.05]$m.](image)

The main poles and zero change their values, when the sliding journal bearing is moved into the range $[-0.13, ..., 0.12]$m (see Fig. 4). They are obtained of the transfer function $G(s) = \frac{y_r(s)}{f_y(s)}$ and their numerical values are represented in Fig. 4.

V. Open Loop Simulation results

Consider the overall rotor-bearing system (2), whose parameters are given in Table I. The initial conditions in the rotor-bearing system are $y_i(0) = y_r(0) = y_d(0) = y_a(0) = 0$ and the initial position of the sliding support $z(0) = -0.04$m.
For the sliding support position regulation is implemented a PD control scheme as follows:

\[ u = -\alpha_1 (z - z^*) - \alpha_2 (\dot{z} - \dot{z}^*) \]  

where

\[ z^*(\omega) = p_1 \omega^4 + p_2 \omega^3 + p_3 \omega^2 + p_4 \omega + p_5 \]  

and \( z^* \) denotes the desired position of the sliding journal bearing, which is parameterized in terms of the antiresonance frequencies. These frequencies are associated to zeros of the transfer function \( G(s) = \frac{y_s(s)}{f_{\Omega}(s)} \) in the rotor-bearing system. The vibrations of the overall system are synchronous, meaning that the vibration frequency is identical to the motor angular speed \( \omega \) (see Figs. 5 and 4). The desired position \( z^* \) is saturated between \(-0.06\) and \(0.06m\). The design parameters of the PD control to regulate the sliding support position are selected as \( \alpha_1 = 1000, \alpha_2 = 990 \).
The sliding journal bearing is moved according to \( z^* \), which is based on the antiresonance frequencies \( \omega \) of the system and the optimal trajectory to follow is shown in Fig. 5. To apply the PD control scheme the measurements of the angular speed \( \omega \) and the linear displacement of the sliding journal bearing \( z \) are necessary. The vibration absorption in the primary system is shown in Fig. 9.

In Fig. 10 is shown the controllable position of the sliding support and the linear force to move it, which is contained between \(-0.4 \) and \(0.4N\).

**VII. Some experimental results**

Some experimental responses for the open-loop behavior were obtained on the experimental platform known as rotor Kit by Bently Nevada (shown in Fig. 11), which has a steel shaft with diameter \(10mm\), total length of \(512.2mm\) and a disk with unbalance of diameter \(75mm\), width \(25mm\) and its mass is \(0.808kg\). Some of the parameters are given in Table 1.

In Fig. 12 is shown the experimental vertical displacements of the disk \(m_r\), when the sliding journal bearing is positioned at different positions into the range \([-0.06, \ldots, 0.05]m\).

In Fig. 13 is shown the experimental vertical displacements of the mass \(m_a\) at the shaft end.

**Fig. 8:** Schematic diagram of the semiactive unbalance control scheme.

**Fig. 9:** Simulation results of the closed loop response of the disk \(m_r\) and the displacement at the shaft end \(m_a\).

**Fig. 10:** Controllable position of the sliding journal bearing.

**Fig. 11:** Rotor Kit experimental setup.

**Fig. 12:** Experimental responses for the open loop behavior \(y_r\), when the sliding journal bearing is positioned at \(z \in [-0.06, 0.05]m\).
VIII. Conclusions

In this work a 9 DOF model for a rotor-bearing system with a cantilever beam absorber is addressed. The passive absorber allows to reduce the vibrations due the unbalance mass of the disk by controlling its dynamic stiffness. To reduce the vibrations of the primary system is only needed to move the sliding journal bearing in terms of the angular speed into a specific range. The natural frequency of the cantilever beam absorber changes when the sliding journal bearing changes of position on two sliders. This simple technique allows to absorb vibrations of the overall rotor-bearing system, when passing through the first critical speed or natural frequency associated to the disk. The unbalance response of the disk is reduced up to 84% with respect to the open-loop response for a Jeffcott-like rotor.

References


