Sending Correlated Gaussian Sources over a Gaussian MAC: To Code, or not to Code

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Abstract—We consider 1-helper problem in which one source provides partial side information to the fusion center (FC) to help reconstruction of the main source signal. Both sources communicate information about their observations to the FC through an additive white Gaussian multiple access channel (MAC) without cooperating with each other. Two types of MAC are considered: orthogonal MAC and interfering (non-orthogonal) MAC. We characterize the tradeoff between the transmission cost, i.e., power, and the estimation distortion, \( D \), using Shannon’s separation source and channel coding theorem. We demonstrate that the separation-based coding strategy outperforms the uncoded transmission under an orthogonal MAC. However, in the symmetric case under an interfering MAC, below a certain signal-to-noise ratio (SNR) threshold, uncoded transmission outperforms the separation-based scheme. The threshold can be determined in terms of the correlation coefficient between the sources, \( \rho \), and in fact is an increasing function of \( \rho \). Finally, the optimal power scheduling to minimize the total power consumption in the network is derived.

Index Terms—source coding, partial side information, rate-distortion function, successive coding.

I. INTRODUCTION

We consider a special case of multiterminal source coding called \( m \)-helper problem in which multiple correlated sources transmit their information to a FC for further processing. One of these sources is the source of interest but other sources act as helpers by sending correlated information (which is called side information) to help reproduction of the first source signal [1]. This problem for the special case of two correlated memoryless Gaussian sources and squared distortion measures is investigated by Oohama [2]. Oohama derives an outer region for the rate-distortion region of the two-terminal source coding problem and demonstrates that the inner region obtained by Berger [3] and Tung [4] is partially tight [2]. He shows that his outer bound when combined with the inner bound of Berger and Tung determines the rate-distortion function of the 1-helper problem. Wagner et al. [5] completely characterize the rate-distortion function of the two-terminal source coding problem by showing that the inner bound of Berger and Tung in the sum-rate is tight, too [5].

In [1] the \( m \)-helper problem is considered where only one of the sources is reconstructed while other \( m \) sources work as helpers. For this case, the authors of [1] derive a lower bound on the rate-distortion function. Also, a partial solution for a special case is presented in a recent work by Oohama [6], where \( m \) source signals are independent noisy versions of the main source. The coding/decoding strategy of [1]-[6] is based on the joint decoding of all messages. As a less complex way, we suggested using the successive Wyner-Ziv coding [7] in the \( m \)-helper problem [8]. Although in general this simple strategy has a suboptimal performance, we have shown that for the Gaussian 1-helper problem, successively structured Wyner-Ziv codes can achieve the rate-distortion function of the problem.

We consider the problem of source-channel communication in the 1-helper problem where both correlated sources are transmitted through an additive white Gaussian multiple access channel to the FC. Since the final goal is to reconstruct the main source to within some prescribed distortion level at the smallest cost in the communication link, finding a suitable coding strategy to achieve this goal is critical. Our interest lies in the achievable power-distortion region, while the fidelity of estimation at the FC is measured by the mean squared error distortion. The only related work in this matter is the recent work of Lapidoth et al. [9] in which the authors have considered sending a memoryless Bi-variate Gaussian source over an interfering MAC. They have shown that in the symmetric case, where the source components are of the same variance and the transmitting signals are subjected to the same average power constraint, uncoded transmission is optimal below a threshold SNR.

Shannon shows that the separate designing of source and channel coding is an optimal strategy for the ergodic point-to-point communication (asymptotically as the delay becomes unconstrained) [10]. We first determine the power-distortion region achievable by Shannon’s separation coding approach. We also determine the optimal power scheduling strategy to minimize the total power consumption in the network. On the other hand, it is shown [11],[12] that for a point-to-point transmission of a single Gaussian source through an additive white Gaussian noise (AWGN) channel, a simple uncoded transmission achieves the optimal power-distortion tradeoff. Also, it is shown [12] that for a class of sensor networks, modeled by the Chief Executive Officer (CEO) problem, the uncoded transmission achieves a much larger power-distortion region than that achieved by the separate source and channel coding. Thus, we analyze the performance of the uncoded transmission approach in the 1-helper problem and obtain its
optimal power scheduling strategy. The results of orthogonal MAC and non-orthogonal MAC are different. The separate source and channel coding approach performs much superior to the uncoded transmission approach under an orthogonal MAC. However, in a symmetric case under an interfering MAC, the uncoded transmission can perform better than the separate coding strategy below a SNR threshold. This result is similar to the recent result of Lapidoth et al. [9].

The rest of this paper is organized as follows: In Section II, we present the system model and problem statement. In Section III, the power-distortion performance of the separation-based coding scheme in the 1-helper problem is evaluated. Section IV discusses the optimal power scheduling for the uncoded transmission approach. Comparison of coded and uncoded transmission schemes is also presented in this Section. Conclusions are given in Section V.

II. PROBLEM STATEMENT

We consider the source-channel communication in 1-helper problem. The model of the sensor network studied in this paper is shown in Fig. 1. Sensors cannot cooperate to exploit their correlation.

Let $X$ and $Y$ be correlated Gaussian random variables such that $\{(X(t), Y(t))\}_{t=0}^{\infty}$ are jointly stationary Gaussian memoryless sources. For each observation time $t = 1, 2, 3, \ldots$, the random pair $(X(t), Y(t))$ takes a value in real space $\mathcal{X} \times \mathcal{Y}$ and has a probability density function (pdf) $p_{X,Y}(x,y)$ of $\mathcal{N} \sim (0,\Lambda)$ where the covariance matrix $\Lambda$ is given by

$$
\Lambda = \begin{bmatrix}
\sigma_X^2 & \rho \sigma_X \sigma_Y \\
\rho \sigma_X \sigma_Y & \sigma_Y^2
\end{bmatrix}, -1 < \rho < 1
$$

and $\rho$ denotes the correlation coefficient between $X$ and $Y$. We represent $n$ independent instances of the Gaussian processes $\{X(t)\}_{t=1}^{\infty}$ and $\{Y(t)\}_{t=1}^{\infty}$ by data sequences $X^n = \{X(1), X(2), \ldots, X(n)\}$ and $Y^n = \{Y(1), Y(2), \ldots, Y(n)\}$, respectively. Data sequences $X^n$ and $Y^n$ are separately encoded to $\varphi_0(X^n)$ and $\varphi_1(Y^n)$ where the encoder functions are defined as

$$
\begin{align*}
\varphi_0 & : \mathcal{X}^n \rightarrow \mathcal{C}_0 = \{1, 2, \ldots, C_0\} \\
\varphi_1 & : \mathcal{Y}^n \rightarrow \mathcal{C}_1 = \{1, 2, \ldots, C_1\}
\end{align*}
$$

The sensors communicate the coded sequences to the FC through an orthogonal MAC under a transmission cost constraint. This constraint comes from the restrictions on the resources such as bandwidth or power that are available at sensor nodes. Here, the transmission cost constraint is in the form of

$$
\frac{1}{n} \sum_{t=1}^{n} E \left[ (U_i(t))^2 \right] \leq P_i + \delta \quad i = 0, 1
$$

where $U_i(t)$ is the transmitted signal to the FC and $\delta$ is an arbitrary prescribed positive number. This restriction is in fact the constraint on the transmission power for each sensor. The received signals at the FC are represented by $W_i(t)$ for $i = 0, 1$. The FC makes an estimate of the main source $X^n$ as $\hat{X}^n$ rather than estimation of both sources. The FC function is given by

$$
\psi_0 : \mathcal{W}_0 \times \mathcal{W}_1 \rightarrow \mathcal{X}^n
$$

where $\mathcal{W}_0$ and $\mathcal{W}_1$ are the domains of random variables $W_0(t)$ and $W_1(t)$, respectively. The FC produces the source estimate $\hat{X}^n$ to an acceptable degree of fidelity $D_0$. The measure of the fidelity is the average distortion criterion, i.e., $\Delta_0 = \frac{1}{2} E \left[ \sum_{i=1}^{n} d(x_i, \hat{x}_i) \right]$ where $d(x, \hat{x})$ is the mean-squared error (MSE) distortion measure.

Let $P = (P_0, P_1)$ and $\mathcal{F}_\delta^{(n)}(P_0, P_1)$ denote all 3-tuple encoder and decoder functions $(\varphi_0, \varphi_1, \psi_0)$ that satisfy (2)-(4). For a particular coding scheme $(\varphi_0, \varphi_1, \psi_0)$, the performance is determined by the required cost and the incurred distortion. For any target distortion $D_0 \geq 0$, the power-distortion region is defined in [13,14] as

$$
\mathcal{P}(D_0) = \{(P_0, P_1) \mid (P_0, D_0) \text{ is admissible} \}.
$$

A power-distortion pair $(P_0, D_0)$ is admissible if for any $\delta > 0$ and any $n \geq n_0(\delta)$ there exists a pair $(\varphi_0, \varphi_1, \psi_0) \in \mathcal{F}_\delta^{(n)}(P_0, P_1)$ such that $\Delta_0 \leq D_0 + \delta$. In other words, $(P_0, D_0)$ is admissible if there is a coding scheme that can achieve a distortion close to $D_0$ while satisfying the transmission cost constraints.

The objective is to determine the power-distortion region for the Gaussian 1-helper problem in an information-theoretic sense irrespective of delay and complexity. This includes determining the optimal power scheduling to minimize the total power consumption for any given distortion $D_0 \geq 0$.

III. SOURCE-CHANNEL COMMUNICATION BASED ON THE SHANNON’S SEPARATION THEOREM

One approach to characterize all achievable power-distortion pairs $(P, D)$ is based on the source and channel coding separation theorem which states that to obtain the optimal performance in the system, data compression and error correction can be optimized separately and performed sequentially [10]. As a result, all achievable pairs of $(P, D)$ can be obtained by combining the rate-distortion region and the orthogonal MAC capacity region. In other words, $(P, D)$ is admissible if the rate-distortion region $\mathcal{R}(D)$ and the capacity region $\mathcal{C}(P)$ intersects, i.e., $\mathcal{R}(D) \cap \mathcal{C}(P) \neq \emptyset$ [15]. Hence, there are two steps to characterize all achievable $(P_0, P_1, D)$: (1) the source coding part which is to characterize the rate-distortion region of the 1-helper problem and (2) the capacity of a MAC and then applying the Shannon’s separation theorem.
A. Rate-Distortion Region of the 1-helper Problem

To obtain the rate-distortion function, we assume that the channel between encoders and the FC are noiseless rate-constrained channels. The coded sequences are sent to the fusion center with the rate constraints

$$\frac{1}{n} \log_2 C_i \leq R_i + \delta_1 \quad i = 0, 1$$

(5)

where $\delta_1$ is an arbitrary prescribed positive number. Since we have considered noiseless channel between encoders and the decoder, the decoder observes $\varphi^2 = (\varphi_0(X^n), \varphi_1(Y^n))$, and makes an estimate of the main source. The decoder function is given by

$$\psi_0 : C_0 \times C_1 \rightarrow X^n.$$

(6)

By applying the successive coding/decoding strategy in the $m$-helper problem, the problem can be decomposed into $m$ successive noisy Wyner-Ziv stages [16]. Each encoder encodes its message while previously decoded messages that are available at the FC act as the decoder side information [17]. Those messages are used to improve the source estimate or to reduce the rate for a given distortion. At the FC, instead of joint decoding, messages from encoders are decoded sequentially in order to increase the fidelity of estimation at each decoding step.

**Proposition 1:** For the 1-helper coding system, the successive coding strategy can achieve the rate-distortion function which can be expressed as

$$R_0(D_0) = \frac{1}{2} \log \left[ \frac{\sigma^2_X}{D_0} \left(1 - \rho^2 + \rho^2 2^{-2R_1}\right) \right],$$

(7)

where $\log^+(x) = \max\{\log x, 0\}$.

**Proof:** Let $Z_2$ denote the output signal generated by the helper at the decoder. Then, $X_0$ can be encoded at the Wyner-Ziv rate

$$R_0(D_0) = \frac{1}{2} \log \left( \frac{\sigma^2_X | Z_2}{D_0} \right).$$

(8)

The helper forms the minimum mean squared error (MMSE) estimate of $X$ from $Y$, given by

$$X_2 = E[X | Y] = \rho \frac{\sigma_X}{\sigma_Y} Y.$$

(9)

Then, the helper encodes $X_2$ at rate $R_1$, thus the quantization error (distortion) can be expressed as

$$D_1 = \text{var}(X_2)^{2^{-2R_1}} = \rho^2 \sigma^2_X 2^{-2R_1}.$$

(10)

Finally, $Z_2$ can be written as $X_2 + E_2$, where $E_2$ is the quantization error with variance $D_1$ given above, and $E_2$ is independent of $X_2$ in the limit of large block length. The result is that the conditional variance of $X$ given $Z_2$ is equal to the estimation MMSE between $X$ and $X_2$ plus the quantization error variance $D_1$. We have

$$\sigma^2_{X | Z_2} = E[|X - X_2|^2] + D_1 = \sigma^2_X (1 - \rho^2) + \sigma^2_X \rho^2 2^{-2R_1}.$$

(11)

Replacing this into (8), we obtain the result of (7). This is the achievable rate-distortion region by the successive coding strategy. Comparing our result with the results of [2] and [5] shows that by applying the successive coding strategy, the rate-distortion function for the 1-helper coding system is achievable. This completes the proof.

B. Capacity Region of a MAC

The capacity-region of a MAC means the set of all achievable rate pairs $(R_0, R_1)$ when the channel inputs satisfy the power constraints. In this paper, we consider the source-channel communication in the 1-helper problem under two kinds of MACs: orthogonal MAC and interfering MAC.

1) Capacity Region of an Orthogonal MAC: By considering multiple-access schemes such as time/frequency/code division multiple access (TDMA/FDMA/CDMA), the channels between sensors and the FC are orthogonal [18]. In other words, the Gaussian MAC is reduced to an array of 2 independent single-user Gaussian channels. These channels can be modeled as AWGN channels with individual channel gains $\{\sqrt{g_i} : i = 0, 1\}$ [13]. The sensor network model for this case is shown in Fig. 1. Channel noises are independent, identically distributed (i.i.d.) over time with variances $N_0$ and $N_1$, respectively. Since sensor $i$ has a transmission power constraint of $P_i$, the capacity region can be represented as

$$C(P) = \left\{(R_0, R_1) \mid 0 \leq R_i \leq \frac{1}{2} \log \left(1 + \frac{g_i P_i}{N_i}\right)\right\}. $$

(12)

2) Capacity Region of an Interfering MAC: Consider the Gaussian sensor network model of Fig. 3 where $Z$ represents the AWGN of the MAC with the variance of $\sigma^2_Z$. The capacity-region of an interfering MAC is determined when the messages of different users are independent [19]. But in the $I$-helper problem, the messages are correlated. The capacity region of a Gaussian MAC with correlated data is not known. Instead, we use the upper bound on the sum-rate presented in [9].

In fact, the capacity region of a two-user Gaussian interfering MAC is located inside the following region:
C. Shannon’s Separation Theorem

Based on the separate source and channel coding, a power-distortion tradeoff \((P, D)\) is achievable only if the rate-distortion region \(R(D)\) and the capacity region \(C(P)\) intersects. Shannon proves that separate source and channel code design is an optimal strategy for the ergodic point-to-point communication scenario [10]. Combining the rate-distortion achieved by the successive coding strategy in (7) and the capacity region of the MAC in (12) and (13), we obtain the achievable power-distortion region of the 1-helper problem under orthogonal and non-orthogonal MAC.

1) Orthogonal MAC: The achievable power-distortion region for the 1-helper problem under an orthogonal MAC is:

\[
\mathcal{P}^o(D_0) = \{ (P_0, P_1) | D_0 \geq \frac{N_0 \sigma^2 (N_1 + \rho g_1)}{(N_0 + g_0 P_0)(N_1 + g_1 P_1)} \}.
\]

(14)

Now we want to minimize the total power consumption, i.e., \(P_{\text{total}} = P_0 + P_1\), in the network. In other words, we want to find \((P_0, P_1)\) for the following problem:

\[
\begin{align*}
\min & \quad P_0 + P_1 \\
\text{s.t.} & \quad (P_0, P_1) \in \mathcal{P}(D_0)
\end{align*}
\]

(15)

Claim 1: Optimal power scheduling in the 1-helper problem for any given distortion \(D_0 \geq 0\) can be expressed as

\[
\begin{align*}
P_0^{\text{opt}} &= \frac{N_0 \sigma^2}{g_0} \left( \frac{1}{D_0} - 1 \right) - \left( \frac{\sqrt{\frac{\sigma^2 N_0}{D_0}}}{D_0} \right)^2 \\
P_1^{\text{opt}} &= \sqrt{\frac{\sigma^2 N_0 N_1}{D_0 g_0 g_1}} - \frac{N_1}{g_1}
\end{align*}
\]

(16)

where \((x)^+\) equals 0 where \(x < 0\).

Proof: To minimize the total power consumption with the constraint \(D_0 = \frac{N_0 \sigma^2 (N_1 + \rho g_1)}{(N_0 + g_0 P_0)(N_1 + g_1 P_1)}\), we introduce a Lagrange multiplier \(\lambda\) and define

\[
J = P_0 + P_1 + \lambda \left( N_0 + g_0 P_0 - \frac{N_0 \sigma^2 (N_1 + \rho g_1)}{(N_0 + g_0 P_0)(N_1 + g_1 P_1)} \right).
\]

(17)

Differentiating with respect to \(P_0\) and \(P_1\) and setting the results to 0 leads to

\[
\left( 1 + \frac{g_1 P_1}{N_1} \right)^2 = \frac{\sigma^2 \rho^2 g_1 N_0}{D_0 g_0 g_1}.
\]

(18)

Therefore, the optimal value of \(P_1\) can be expressed as \(P_1^{\text{opt}} = \left( \sqrt{\frac{\sigma^2 \rho^2 N_0 N_1}{D_0 g_0 g_1}} - \frac{N_1}{g_1} \right)^+\). The optimal power of \(P_0\) will be obtained by substituting \(P_1^{\text{opt}}\) in the equation of the constraint.

From Claim 1, it is clear that the optimal power scheduling depends on the degree of fidelity (distortion) at which the underlying source can be estimated by the FC. In other words, if the acceptable distortion is high, we allocate all the power to the main source, i.e., if \(\frac{\sigma^2 \rho^2 g_1 N_0}{g_0 N_1} \leq D_0 \leq \sigma^2 \), then \(P_0 = \frac{N_0 \sigma^2}{g_0} (\frac{1}{D_0} - 1)\) while \(P_1 = 0\).

2) Interfering MAC:

Claim 2: A necessary condition for the achievability of \((P_0, P_1, D_0)\) using separation-based coding strategy in the 1-helper problem under an interfering MAC can be expressed as:

\[
\mathcal{P}^i(D_0) = \left\{ (S_0, S_1) | D_0 \geq \frac{\sigma^2}{(1+S_0+S_1)(1-\rho^2 + 2\rho \sqrt{S_0 S_1}(1-\rho^2))} \right\}
\]

(19)

where \(S_i = \frac{g_0 P_{i}}{\sigma^2}\). In other words, the distortion achieved by the separation-based scheme behaves at best like (19).

Proof: It is evident that by increasing \(R_0\) and \(R_1\) (or equivalently \(P_0\) and \(P_1\)) the final estimation distortion decreases. From the capacity region of an interfering MAC, which is illustrated in Fig. 4, we can see the optimum values of \(R_0\) and \(R_1\) which minimize the final estimation distortion \(D_0\) in the rate-distortion function of (7) are located on the line of sum-rate capacity, i.e., line BC. We optimize over the fraction of sum-rate allocated to each user: \(R_0 = \lambda \bar{R}\) and \(R_1 = (1-\lambda) \bar{R}\), given the total fixed communication rate \(R_0 + R_1 = \bar{R}\). Taking the derivative of \(D_0\) with respect to \(\lambda\) and set the result equal to zero reveals that the optimum point on line BC is point B, where \(R_0\) is maximum. By using the outer region of (13) for the capacity region of an interfering MAC and substituting \(R_0\) and \(R_1\), corresponds to point B, into the rate-distortion function of (7), the final result of (19) will be obtained.

Motivated by the optimality of the uncoded transmission of a Gaussian source across a point-to-point AWGN channel, and also its optimality for a class of sensor network, modeled by the CEO problem, we evaluate the power-distortion performance of the uncoded transmission in the 1-helper problem.

IV. UNCoded TRANSMISSION IN THE 1-Helper PROBLEM

We study the performance of single-letter uncoded transmission strategy applied to the 1-helper problem. In this
approach which is also called “analog forwarding” or “amplify-and-forward” [20] approach, each sensor transmits the scaled version of its observation, scaled to its power constraint, i.e.,

\[
\begin{cases}
U_0(t) = \alpha_0 X(t) & \text{where } \alpha_0 = \frac{P_0}{\sigma_X^2} \\
U_1(t) = \alpha_1 Y(t) & \text{where } \alpha_1 = \frac{P_1}{\sigma_Y^2}
\end{cases}
\]  
(20)

A. Orthogonal MAC

For this case, the system model is shown in Fig. 5. The received signals at the FC, \(W_0(t), W_1(t)\), can be expressed as

\[
W_0(t) = \frac{g_0 P_0}{\sigma_X^2} X(t) + V_0(t)
\]

\[
W_1(t) = \frac{g_1 P_1}{\sigma_Y^2} Y(t) + V_1(t)
\]

where \(V_i(t)\) for \(i = 0, 1\) are channel noises that are i.i.d. over time with variances \(N_0\) and \(N_1\), respectively. Since the encoding is memoryless, the optimum estimator is the minimum mean squared error (MMSE) estimator of \(X(t)\) from received signals \(\{W_0(t), W_1(t) : 1 \leq t \leq \infty\}\), which can be obtained by \(\hat{X}(t) = E[X(t) | W_0(t), W_1(t)]\). The average cost of MMSE estimator, which is the MSE distortion, \(D_a\), satisfies

\[
D_a = \frac{\sigma_X^2 N_0 (N_1 + g_1 P_1 (1 - \rho^2))}{(N_0 + g_0 P_0) (N_1 + g_1 P_1) - \rho^2 g_0 P_0 g_1 P_1}.
\]  
(22)

Hence, for any given \(D_a \geq 0\), the achieved power-distortion region by the uncoded transmission in the 1-helper problem under an orthogonal MAC can be expressed as:

\[
P_a^0 (D_a) = \left\{ (P_0, P_1) \mid D_a \geq \frac{\sigma_X^2 N_0 (N_1 + g_1 P_1 (1 - \rho^2))}{(N_0 + g_0 P_0) (N_1 + g_1 P_1) - \rho^2 g_0 P_0 g_1 P_1} \right\}.
\]  
(23)

Now, we use the Lagrange multiplier method to minimize the total power consumption while achieving a given average distortion \(D_a\). The proof involves simple calculations and is omitted.

For any given distortion \(D_a \geq 0\), the optimal power scheduling of the uncoded transmission strategy in the 1-helper problem under an orthogonal MAC is as follows:

\[
P_{0, opt} = \begin{cases} 
\frac{\sigma_X^2 N_0}{D_a g_0} + \frac{\rho^2 g_0 N_1}{g_1 g_0 (1 - \rho^2)} & \rho^2 \geq \frac{g_0 N_1}{g_1 N_0} \\
\frac{\rho^2 N_0}{g_0} & \rho^2 < \frac{g_0 N_1}{g_1 N_0}
\end{cases}
\]  
(24)

and

\[
P_{1, opt} = \begin{cases} 
\frac{\rho^2 g_1 N_1}{g_0 g_1 (1 - \rho^2)} & \rho^2 \geq \frac{g_0 N_1}{g_1 N_0} \\
0 & \rho^2 < \frac{g_0 N_1}{g_1 N_0}
\end{cases}
\]  
(25)

B. Interfering MAC

It is shown that in a class of sensor networks, modeled by the CEO problem, the uncoded transmission approach has much superior performance than conventional separate source and channel coding. In this topology of sensor network, \(L\) sensors communicate their information to a central FC through an interfering MAC. Motivated by this result, we want to evaluate the performance of uncoded transmission approach in the 1-helper problem under an interfering MAC.

The received signal at the FC, \(W(t)\), can be expressed as

\[
W(t) = \sqrt{\frac{g_0 P_0}{\sigma_X^2}} X(t) + \sqrt{\frac{g_1 P_1}{\sigma_Y^2}} Y(t) + Z(t).
\]  
(26)

It can be shown the achievable MSE distortion, \(D_a\), in estimating the source \(X(t)\) satisfies

\[
D_a = \frac{\sigma_X^2 (1 + S_1 (1 - \rho^2))}{1 + S_0 + S_1 + 2 \rho \sqrt{S_0 S_1}}.
\]  
(27)

Because of the nature of an interfering MAC, the helper may not be always useful. It means that depending on the required degree of fidelity, the no-helper system may perform better than the 1-helper scheme. The power-distortion region can be completely characterized by obtaining the intersection point of MSE achieved by both 1-helper and no-helper schemes.

Claim 3: For any given \(D_a \geq 0\), the achievable power-distortion region by the uncoded transmission in the 1-helper problem under an interfering MAC can be expressed as:

\[
P_a^1 (D_a) = \{ (P_0, P_1) \mid P_0 \geq f(D_a) \}
\]  
(28)

where \(f(D_a) = \frac{\sigma_X^2}{\rho^2 g_0} (\frac{\sigma_X^2}{D_a} - 1)\) for \(0 \leq D_a \leq D_a^*\) and \(f(D_a) = \left( \frac{\sigma_X^2 + g_1 P_1 (1 - \rho^2)}{g_0} (\frac{\sigma_X^2}{D_a} - 1) - \rho \sqrt{\frac{g_1 P_1}{g_0}} \right)^2 \) for \(D_a^* < D_a \leq D_1^*\), while

\[
D_a^* = \frac{\sigma_X^2 S_1 (1 - \rho^2)^2}{S_1 (1 - \rho^2) + 2 \rho^2 (S_1 + \sqrt{1 + S_1 (1 - \rho^2)})}.
\]  
(29)

Proof: By putting the MSE of 1-helper and no-helper equal to each other, we obtain:

\[
P_1 = \frac{4 \rho^2 \sigma_Y^2 g_0 P_0}{g_1 (g_0 P_0 (1 - \rho^2) - \rho^2 \sigma_Y^2)}.
\]  
(30)

Substituting the MSE of no-helper scheme in (30) and doing some manipulations will result in

\[
D_a = \frac{\sigma_X^2 S_1 (1 - \rho^2)^2}{S_1 (1 - \rho^2) + 2 \rho^2 (S_1 + \sqrt{1 + S_1 (1 - \rho^2)})}.
\]  
(31)

It means that for the required degree of fidelity less than or equal to (31), it is preferred to not use the helper. We call this value of the MSE, \(D_a^*\). For larger values of the required distortion, we use the MSE of the 1-helper scenario. By rewriting (27) for a fixed value of \(P_1\), a quadratic equation
with respect to $P_0$ will be obtained, i.e.,
\[
P_0(D,g_0) + 2\sqrt{P_0} \left( \frac{\rho \sqrt{g_0} g_1 P_1}{\sqrt{g_0} g_1} \right) + \frac{D_a g_1 P_1}{2} D_a \sigma^2_Z - \sigma^2_X \left( \frac{\sqrt{2}}{\sqrt{\rho g_1}} \right) = 0.
\]

Therefore, the optimal value of $P_0$ for a given $P_1$ can be obtained as
\[
P_0 = \left( \frac{\sqrt{2}}{\sqrt{\rho g_1}} \right) \left( \frac{\sigma^2_X + g_1 P_1 (1 - \rho^2)}{g_0} \right)^2.
\]

To have a positive value for $P_0$, the distortion should satisfy $D_a \leq \sigma^2_X \left( 1 + S_1 \right)$. This completes the proof.

We again apply the Lagrange multiplier method to minimize the total power consumption while achieving a given average distortion, $D_a$.

Claim 4: For any given distortion $D_a \geq 0$, the optimal power scheduling of the uncoded transmission in the 1-helper problem under interfering MAC can be expressed as follows:

For $0 \leq D_a \leq D_a^*$,
\[
\{ P_0^opt = \frac{K \sigma^2_X \left( \frac{a}{\sqrt{a + 2 + a}} \right)^2}{g_0 K + g_1 + 2\rho \sqrt{g_0} g_1 - \frac{\sigma^2_X}{\rho} g_1 (1 - \rho^2)}, \quad P_1^opt = g_1 \}
\]

and for $D_a^* < D_a \leq D_1^*$,
\[
\{ P_0^opt = \frac{\sigma^2_X}{\rho} \left( \frac{a}{\sqrt{a + 2 + a}} \right), \quad P_1^opt = 0 \}
\]

where the parameters are defined as $K = \left( \frac{a + \sqrt{a + 4}}{2} \right)^2$ and $a = \frac{1}{\sqrt{g_0} g_1} \left( g_0 - g_1 + \sigma^2_X \frac{D_a}{\sigma^2_Z} g_1 (1 - \rho^2) \right)$.

Proof: For small values of acceptable distortion, we have no-helper. Therefore, the optimal power scheduling is to allocate all the power to the main source. For $D_a > D_a^*$ consider a Lagrange multiplier $\lambda$ and define
\[
J = P_0 + P_1 + \lambda \{ D_a \left( g_0 P_0 + g_1 P_1 + 2\rho \sqrt{g_0} g_1 \right) + D_a \sigma^2_Z - \sigma^2_X \left( \frac{\sqrt{2}}{\sqrt{\rho g_1}} \right) \}.
\]

Differentiating with respect to $P_0$ and $P_1$ and setting the results to 0 leads to
\[
\frac{P_0}{P_1} - \frac{P_1}{P_0} = \frac{1}{\sqrt{g_0} g_1} \left( g_0 - g_1 + \frac{\sigma^2_X}{\sigma^2_Z} g_1 (1 - \rho^2) \right).
\]

Let $\sqrt{\frac{P_0}{P_1}} = x$ and $\frac{1}{\sqrt{g_0} g_1} \left( g_0 - g_1 + \frac{\sigma^2_X}{\sigma^2_Z} g_1 (1 - \rho^2) \right) = a$. Thus, (35) would be a quadratic equation with respect to $x$. The acceptable root of this equation is $\frac{a + \sqrt{a + 4}}{2}$. Putting this result in the constraint gives the optimal power scheduling of (32) and (33).

C. Comparison of the Uncoded transmission and the Separate Source and Channel Coding

1) Orthogonal MAC: By comparing the power-distortion region achieved by the separate source and channel coding approach (14) and power-distortion achieved by the uncoded transmission approach (23), we observe that the separate source and channel coding strictly outperforms the uncoded transmission approach. In other words,
\[
P_0^opt(D_0) \subset P^opt(D_0),
\]
i.e., the uncoded transmission approach is suboptimal for signal transmission in the 1-helper problem under orthogonal MAC.

As an example, consider two correlated sources $X$ and $Y$ with the correlation coefficient of $\rho = 0.8$. For a given distortion $D = 1$, the power-distortion regions achieved by the separate source and channel coding and the uncoded transmission approach in the 1-helper problem are shown in Fig. 6. It demonstrates that the separate source and channel coding approach strictly outperforms the uncoded transmission approach. In Fig. 7 the total power consumption versus the MSE distortion is illustrated. It shows that having a one helper correlated source can improve the total power-distortion performance of the system.

2) Interfering MAC: Comparing (19) and (28) and doing some manipulations reveals that if $g_1 P_1 = g_0 P_0 = g P$ and $0 < \text{SNR} = \frac{g P}{\sigma^2_Z} < f^*(\rho) = 2\rho$ for $0 < \rho < 1$, then, the uncoded transmission approach performs superior to the separate coding scheme. Therefore, in the symmetric 1-helper problem, where the transmitting signals through the MAC are subjected to the same average power constraint, below a threshold SNR, $f^*(\rho)$, uncoded transmission outperforms the separation-based coding scheme. In general, the region that uncoded transmission approach performs better than the separate coding scheme can be described as
\[
g(S_0, S_1) = \sqrt{S_0 S_1} < 2\rho \quad \forall S_0 \geq 0, S_1 \geq 0.
\]
channel coding results, where uncoded transmission performs superior to the coded transmission in low signal-to-noise ratios. From Fig. 8 we observe that as the correlation between the main source and the helper increases, the region becomes wider.

V. CONCLUSIONS

In this paper, based on the separate source and channel coding approach, the power-distortion regions of the 1-helper problem under orthogonal and non-orthogonal MAC were derived. We also determined the optimal power scheduling which minimizes the total power consumption for any given distortion $D$. We demonstrated that the separation-based coding scenario strictly outperforms the uncoded transmission approach under an orthogonal MAC. However, under an interfering MAC, below a certain SNR threshold, uncoded transmission has a performance much superior to that of the separation-based scheme.

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