A Semi-Blind Algorithm for Most Significant Tap Detection in Channel Estimation of OFDM Systems

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Abstract— In this paper, a semi-blind algorithm is proposed for the detection of most significant tap (MST) in the sparse channel estimation of OFDM systems. Based on an analysis of the second-order statistics of the signal received through a noise-free sparse channel, a direct relationship between the positions of the most significant taps (MST) of the sparse channel and the lags of the nonzero correlation functions is revealed, leading to an efficient semi-blind MST detection algorithm. By using the acquired MST position, a sparse least square channel estimate is then obtained. A number of computer simulation-based experiments are carried out to confirm the effectiveness of the proposed semi-blind MST detection algorithm and the associated sparse LS channel estimation method.

I. INTRODUCTION

A wireless channel can often be modelled as a sparse channel, in which the delay spread could be very large but the number of paths is normally very small [1]–[5]. Broadly speaking, there are two kinds of approaches for the sparse channel estimation. The first one estimates the complex amplitude and the delay of each path based on a non-sampling spaced parametrical channel modelling [1]. The second kind is based on the sparsity assumption of the equivalent discrete-time channel [2]–[5], in which only a few taps in the long tapped delay line are considered most significant. By exploiting the sparse structure of the channel, some improved channel estimation algorithms have been developed for OFDM systems [2], [5] and CDMA systems [3]. In this paper, we develop a very efficient sparse channel estimation approach for OFDM systems that requires only a small number of OFDM symbols. As the non-sampling spaced sparse channel estimation requires a large number of OFDM symbols for the estimation of the delay-subspace, we focus only on the sampling-spaced approach.

It should be mentioned that all sampling-spaced sparse channel estimation methods follow two steps: (1) detect the position of the most significant taps (MSTs) based on initial LS estimation, or by utilizing some techniques such as the generalized Akaike information criterion, the on-off-keying detection and matching pursuit method [2]–[4]; and (2) obtain an improved LS channel estimate by exploiting the position of the MSTs [2], [4]. The common problem of these sparse channel estimation methods is that a large number of pilots is needed in order to render an accurate MST detection. To increase the spectral efficiency, one cyclic-prefix (CP) based blind method has been proposed for the MST detection of OFDM systems [5]. However, this detection scheme needs a large number of OFDM symbols as well as a large CP length in order to obtain precise MST positions. In this paper, based on an analysis of the second-order statistics of the received signal passing through a sparse channel, we will develop an efficient semi-blind MST detection algorithm for the channel estimation of OFDM systems.

Throughout the paper, we adopt the following notations:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>†</td>
<td>Pseudo-inverse,</td>
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<tr>
<td>δ()</td>
<td>Delta function,</td>
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<tr>
<td>T</td>
<td>Transpose,</td>
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<tr>
<td>⊙</td>
<td>Circular convolution,</td>
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<td></td>
<td></td>
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<tr>
<td>diag()</td>
<td>A stacking of the elements of the involved vector into the diagonal elements of a diagonal matrix.</td>
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II. SPARSE LEAST SQUARE ESTIMATION OF OFDM CHANNELS

For an OFDM system, assume that the channel is constant during a number of consecutive OFDM symbols, the channel can be described by \( h(l) \in \mathcal{C}, (l = 0, 1, \cdots, L - 1) \). If the length of the cyclic prefix is not less than the channel length \( L \), the time-domain signal model for the frequency-selective fading channel is given by

\[
y(m, n) = h(n) \otimes x(m, n) + v(m, n), m \in \{0, \cdots, g - 1\}
\]

where \( g \) is the number of OFDM symbols within which the channel remains unchanged, and \( v(m, n) \in \mathcal{C} \) is a spatio-temporally uncorrelated noise with zero-mean and variance \( \sigma_n^2 \). In this paper, the channel is assumed to contain many zero taps in the uniform delay line as in [2]–[5]. In particular, the channel with respect to the \( d \)-th \( (d = 0, 1, \cdots, D - 1) \) most significant tap can be expressed as

\[
z(d) = h(l_d)
\]

where \( l_d \in \mathcal{L} = \{l_0, l_1, \cdots, D - 1\} \) are integers with \( 0 = l_0 < l_1 < \cdots < l_{D-1} \).

Prior to the development of the sparse LS algorithm for the estimation of the effective channel, we first briefly review the training-based LS channel estimation algorithm for OFDM systems. Assume that the \( K_p \) sub-carriers, say from \( \mathcal{I}_{\text{pilot}} \) to...
of each OFDM symbol carry the pilot signal. The transmitted and the received pilot vectors can be defined as
\[
X_pilot (m) \triangleq [X (m, i_{\text{pilot}1}), \ldots, X (m, i_{\text{pilot}K_p})]^T,
\]
\[
Y_pilot (m) \triangleq [Y (m, i_{\text{pilot}1}), \ldots, Y (m, i_{\text{pilot}K_p})]^T.
\]

It should be noted that the pilot signal might not be located at the same position in each OFDM symbol. Let \( F_1 \) be a \( K \times L \) matrix formed by the first \( L \) columns of a \( K \times K \) DFT matrix \( F_0 \). For the \( m \)-th OFDM symbol, one can form a \( K_p \times L \) matrix, say \( F (m) \), by taking only the rows of \( F_1 \) associated with the \( K_p \) pilot sub-carriers. It was shown in [6] that
\[
Y_pilot (m) = X_pilot - diag (m) F (m) h + \xi_pilot (m)
\]
(3)

where
\[
X_pilot - diag (m) \triangleq \text{diag} (X_pilot (m)),
\]
and \( \xi_pilot (m) \) represents the frequency-domain noise corresponding to \( v_i \) in \( (m, n) \) in \( (1) \). From \( (3) \), the received frequency-domain pilot signal with respect to \( g \) OFDM symbols can be obtained as
\[
Y_pilot = Ah + \xi_pilot
\]
(4)

where
\[
Y_pilot \triangleq [Y_pilot^T (0), \ldots, Y_pilot^T (g - 1)]^T,
\]
\[
A = \begin{bmatrix}
X_pilot - diag (0) F (0) \\
\vdots \\
X_pilot - diag (g - 1) F (g - 1)
\end{bmatrix},
\]
\[
\xi_{i_{\text{pilot}}, \text{pilot}} \triangleq [\xi_{i_{\text{pilot}}, \text{pilot}}^T (0), \ldots, \xi_{i_{\text{pilot}}, \text{pilot}}^T (g - 1)]^T.
\]

From \( (4) \), one can obtain an LS criterion
\[
\|Y_pilot - Ah\|^2
\]
(5)
The solution to this optimization problem is given by
\[
\hat{h} = A^\dagger Y_pilot
\]
(6)

In the above LS channel estimation method as well as some existing OFDM channel estimation methods, the sparse case of the wireless channel has not been taken into consideration. Thus, the channel estimation solution obtained is not efficient for the sparse channel considering that all the taps have to be estimated which in general leads to a high algorithmic complexity and a poor estimation accuracy. In the following, we would like to propose an sparse LS solution for OFDM channel estimation.

Assuming that the MSTs are correctly estimated and using \( (2), (3) \) can be rewritten as
\[
Y_pilot (m) = X_pilot - diag (m) \tilde{F} (m) z + \xi_pilot (m)
\]
(7)

where
\[
z = [z (0), \ldots, z (D - 1)]^T,
\]
and \( \tilde{F} (m) \) is a \( K_p \times D \) matrix, whose \( d \)-th column is the \( l_d \)-th column of \( F (m) \), \( d = 0, 1, \ldots, D - 1 \). Accordingly, \( (4) \) can be reexpressed as
\[
Y_pilot = \tilde{A}z + \tilde{\xi}_pilot
\]
(8)

where
\[
\tilde{A} = \begin{bmatrix}
X_pilot - diag (0) \tilde{F} (0) \\
\vdots \\
X_pilot - diag (g - 1) \tilde{F} (g - 1)
\end{bmatrix}.
\]
(9)

From \( (8) \), an LS estimate of the sparse channel with respect to the MSTs \( l_d \) \( (d = 0, 1, \ldots, D - 1) \) can be obtained as
\[
\hat{z} = (\tilde{A})^\dagger Y_pilot
\]
(10)

Obviously, the key step in the structured least square estimation of the effective channel lies in the detection of the MSTs. In the following, we first analyze the second-order statistics of the signal received through the sparse channel, and then propose a novel semi-blind most significant tap detection algorithm for OFDM channel estimation.

### III. Proposed Semi-blind MST Detection

#### A. Second-Order Statistics of the Received Signal through Sparse Channel

It is well known that the correlation function of the received signal vector \( y (m, n) \) plays a crucial role in blind or semi-blind channel estimation [7], which can be, in general, defined as
\[
r (l) \triangleq E \{ y (m, n) y^* (m, n - l) \}, \quad (l = 0, 1, \ldots, P).
\]
(11)

In this subsection, we would like to express \( r (l) \) in terms of the effective sparse channel \( z (d) \), \( d = 0, 1, \ldots, D - 1 \), in the absence of noise, and show that \( r (l) \) has only a few most significant lags (MSLs), i.e., most of \( r (l) \) with \( l \in [0, P] \) are zero values, due to the sparse feature of the channel.

Using \( (1) \) and \( (2) \) in \( (11) \), we obtain
\[
r (l) = z_A R_{x,D} (l) z_A^H
\]
(12)

where
\[
z_A \triangleq [z (0) \quad z (1) \quad \cdots \quad z (D - 1)]
\]
(13)

\[
R_{x,D} (l) \triangleq E \{ x_D (n) x_D^H (n - l) \}
\]
(14)

with
\[
x_D (n) \triangleq x (n), x (n - l_1), \ldots, x (n - l_{D-1}).
\]

Clearly, the value of \( r (l) \) mainly depends on \( R_{x,D} (l) \). Assuming that the transmitted signal is uncorrelated, namely,
\[
E \{ x (n - i) x^* (n - j) \} = \sigma_x^2 \delta (i - j),
\]
we can rewrite \( (14) \) as
\[
R_{x,D} (l) = \sigma_x^2 A_D (l)
\]
(15)
\( \Lambda_D (l) = \begin{bmatrix}
\delta (l) & \delta (l + l_1) & \cdots & \delta (l + l_D - 1) \\
\delta (l - l_1) & \delta (l) & \cdots & \delta (l + l_D - 1 - l_1) \\
\vdots & \vdots & \ddots & \vdots \\
\delta (l - l_D - 1) & \delta (l - l_D - 1 + l_1) & \cdots & \delta (l)
\end{bmatrix}
\)

(16)

It is obvious from (16) that the nonzero elements of \( \Lambda_D (l) \) occur only when \( l = l_i - l_j, (i, j = 0, 1, \cdots, D - 1; i \geq j) \).

Since the channel is sparse, i.e., \( D \ll L \leq P \), there is only a small number of choices of \( l \) which makes \( \Lambda_D (l) \) a nonzero matrix. It is clear from (12) and (15) that as long as \( \Lambda_D (l) \) is a zero matrix, \( R_{x,D} (l) \) is a zero matrix and so is \( R (l) \). Note that the position of the nonzero elements of \( \Lambda_D (l) \), and in turn that of the corresponding \( R_{x,D} (l) \) depend on the values of \( l \) and \( l_d \).

Let us consider the simplest case when \( D = 2 \). In this case, there are only two nonzero channel \( z (0) \) and \( z (1) \). Without loss of generality, we assume a unit signal variance, i.e., \( \sigma^2_z = 1 \). From (12), (13) and (15), one can find that there are only two nonzero values of \( r (l) \), i.e.,

\[
\begin{align*}
\tilde{r} (0) &= z (0) z^* (0) + z (1) z^* (1), \\
\tilde{r} (1) &= z (1) z^* (0),
\end{align*}
\]

which means that the MSL positions of \( r (l) \) are \( l = 0, 1 \).

When \( D = 3 \), \( R_{x,3} (l) \) has different sparse structures depending on the relationship between \( l_1 \) and \( l_2 \), which leads \( r (l) \) to have different expressions. Using (12), (13) and (15), one can easily verify the positions of the nonzero lags of \( r (l) \) as described below.

**Case D3.1:** if \( l_2 = 2l_1 \), three nonzero lags

\[
l = 0, l_1, \text{ or } l_2;
\]

(17)

**Case D3.2:** if \( l_2 \neq 2l_1 \), four nonzero lags

\[
l = 0, l_1, l_2 - l_1, \text{ or } l_2.
\]

(18)

In the case of \( D = 4 \), one can easily have that

**Case D4.1:** if \( l_1 : l_2 : l_3 = 1 : 2 : 3 \), four nonzero lags

\[
l = 0, l_1, l_2, \text{ or } l_3.
\]

(19)

In similar manner, one can obtain the expressions for the cases of \( D \geq 4 \). Due to the space constraint, its discussion has been omitted.

### B. Detection of Most Significant Taps (MSTs)

By exploiting the above relationship, we now propose an efficient MST detection method. Its first step is to detect the MSLs of \( \tilde{r} (l) \) by comparing its absolute value with a threshold

\[
\eta = \frac{K_e}{P_L + 1} \sum_{l=0}^{P_L} |\tilde{r} (l)|
\]

where \( P_L \) is a predetermined length and the coefficient \( K_e \) is used to adjust the average value of \( r (l) \). The second step of the new MST detection algorithm can be described as follows.

**Assume a total of \( W \) MSLs of \( \tilde{R} (l) \) have been correctly detected, whose positions are denoted as \( m_i \) with \( m_0 < m_1 < \cdots < m_{W-1} \).** We now utilize these positions to determine the MSTs. It is clear that the positions of the first and the last MSTs are simply given by

\[
l_0 = m_0 = 0, \\
l_{D-1} = m_{W-1}.
\]

(20)

(21)

Obviously, (20) and (21) give the only two nonzero taps if \( W = 2 \).

If \( W = 3 \), we have only the case D3.1 according to (17). Then, there is only one additional tap \( l_d \) to be determined, which is readily given by

\[
l_1 = m_1 = \frac{m_2}{2}.
\]

If \( W = 4 \), we should have case D3.2 from (18) or D4.1 from (19). Then, one can have the following three possible solutions,

**Case W4.1:** when \( D = 3 \) with \( l_2 > 2l_1 \), we have \( m_1 = l_1 \) and \( m_2 = l_2 - l_1 \), which gives \( l_1 = m_1 \) and \( l_2 = m_1 + m_2 = m_3 \);

**Case W4.2:** when \( D = 3 \) with \( l_2 < 2l_1 \), we obtain \( m_1 = l_2 - l_1 \) and \( m_2 = l_1 \), which yields \( l_1 = m_2 \) and \( l_2 = m_1 + m_2 = m_3 \);

**Case W4.3:** when \( D = 4 \) with \( l_1 : l_2 : l_3 = 1 : 2 : 3 \), we readily have \( l_1 = m_1 \) and \( l_2 = m_2 \).

In order to make a decision among the three choices, one can
obtain an LS criterion similar to the sparse LS channel estimation algorithm proposed in Section II. It can be easily found that the cost $J$ calculated by this criterion reaches its minimum when the estimated MSTs $l'_d \ (d = 0, 1, \cdots, D' - 1)$ gives true MSTs. In the case of $W = 4$, our MST detection only needs to calculate a few costs with respect to Cases W4.1, W4.2 and W4.3, denoted as $J_{4.1}, J_{4.2}$ and $J_{4.3}$. The complete scheme is shown in Fig.1, which gives three possible MST detection results, namely,

Case W4.1: $l_1 = m_1$; Case W4.2: $l_1 = m_2$ and Case W4.3: $l_1 = m_1, l_2 = m_2$.

In a similar manner, the idea of the MST detection can be extended to the case of $W \geq 5$.

IV. SIMULATION RESULTS

In our simulation, the number of subcarriers is set to 1024. A sparse Rayleigh channel modelled by a 3-nonzero tap FIR filter is assumed, in which each tap is i.i.d. complex normal distributed. The MSTs are $l_0 = 0, l_1 = 4$ and $l_2 = 11$. The constant $K_2$ for the calculation of the threshold $\eta$ is set to 0.8. The estimation performance is evaluated in terms of the MSE as in [8].

Experiment 1: SISO System

Here, the MSE performance versus the SNR is investigated. The simulation involves 2000 Monte Carlo runs of the transmission of 10 OFDM symbols with 30 pilot subcarriers. Fig. 2 shows the MSE plots of the proposed sparse and the original LS methods. It is seen that the sparse LS method significantly outperforms the original LS method. In particular, the gain of the sparse LS method over the original LS method is about 5.5 dB when SNR is 10 dB.

Experiment 2: MEsson versus pilot length

In this experiment, we investigate the channel estimation performance versus the number of pilot subcarriers. Fig. 3 shows the MSE plots from 2000 Monte Carlo iterations for 5 OFDM symbols with pilot subcarriers varying from 15 to 35 at an SNR of 10 dB. One can find that the sparse LS method can achieve a gain of $6 \sim 6.2$ dB over the LS method.

V. Conclusions

A semi-blind detection algorithm has been proposed for the most significant tap (MST) of sparse OFDM channels. The relation between the MSTs of the sparse channel and the most significant lags (MSL) of the correlation functions of the received noise-free signal has been disclosed. This relation has then been exploited to develop a highly efficient MST detection algorithm. By employing the acquired MST information, a sparse LS algorithm has been obtained for the estimation of the effective channel matrix. Simulation results have shown that, the performance of the sparse LS channel estimation method with the proposed MST algorithm is significant superior to that of the original LS method.

REFERENCES