Complementary Approaches to Accurately Evaluate the Performance in Optically Pre-Amplified DPSK Receivers with Direct Detection

Luis G. C. Cancela and João J. O. Pires
Instituto de Telecomunicações, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal.
lcancela@mail.telepac.pt, jpires@lx.it.pt

Abstract—A rigorous approach to evaluate the performance in optically pre-amplified differential phase-shift keying (DPSK) receivers with arbitrary optical and electrical filtering and using direct detection is presented. This approach is used to assess the degree of accuracy of the well known simplified approach that assumes a wide-band optical filter and an integrate-and-dump electrical filter. The performance results show that these two approaches can be used in a complementary way: the simplified approach is acceptable for time-bandwidth products larger or equal to 5, underestimating the receiver sensitivity by less than ~0.1 dB, while for time-bandwidth products smaller than 5 the rigorous approach should be used instead. It is shown that for a time-bandwidth product equal to 1 the simplified approach underestimate the receiver sensitivity by ~1.1 dB.

I. INTRODUCTION

The differential phase–shift keying (DPSK) modulation format resurfaced in the recent years usually linked with long haul optical transmission systems, mainly due to the higher receiver sensitivity and improved immunity to nonlinear transmission impairments in comparison with the traditional on–off keying (OOK) format [1]. More recently, some advantages of using this modulation format in the broader context of optical networks were also unveiled, namely its greater tolerance to in-band crosstalk when compared with the OOK format [2]. In this way, the performance modeling of optically pre-amplified DPSK receivers using direct detection has been an active research topic [1]-[6].

In general, the proposed approaches use an expansion of the signal and the amplified spontaneous emission (ASE) noise in terms of a set of orthogonal functions and rely on the moment generating function (MGF) to describe the statistics of the decision variable. However, they differ on the way how the orthogonal functions are obtained. In particular, in [2]-[4] the authors used, as a simplified assumption, the fact that for a combination of a wide-band ideal optical filter and an integrate-and-dump electrical filter the complex exponential functions are an appropriate set of orthogonal functions [7], while [1] and [6] treat the case of arbitrary optical and electrical filtering, and in this case the orthogonal functions are obtained as the eigenfunctions of an integral equation.

The approaches that rely on the resolution of an integral equation, albeit being rigorous, are complex, since they have to deal with a matrix eigenvalue problem [6]. In contrast, the simplified approach has the advantage of providing a closed-form expression for the MGF, without the necessity of solving any eigenvalue problem, and in this way, it seems appealing for engineering applications. Nevertheless, its degree of accuracy should be assessed and its range of applicability clearly defined. In the literature, to the best of the author’s knowledge, there are only some brief comments about its accuracy. In [3], it is qualitatively asserted that for time-bandwidth products equal to 1, the results were outside the range of validity of the model, while in [8] the simplified approach is briefly compared with a rigorous one, but it is considered that the modulation format is the OOK, and it is assumed that the signal is not affected by the optical filter.

Therefore, the main purpose of this paper is to find out the range of validity of the simplified approach, so that it can be used in a complementary way with a rigorous approach to accurately evaluate the performance of optically pre-amplified DPSK receivers. In this way a rigorous approach that can deal with arbitrary optical and electrical filtering is presented. This approach uses an eigenfunction expansion technique of the signal and noise at the optical filter output in the time domain.

II. RIGOROUS APPROACH

A. General case: arbitrary optical and electrical filtering

The DPSK receiver, as depicted in Fig. 1, consists of an optical pre-amplifier with gain $G$ and noise factor $F$, an optical filter, a delay interferometer with a differential delay equal to the bit period $T$, a balanced photodetector, and a post-detection electrical filter. The optical filter is assumed to have an arbitrary low-pass equivalent impulse response $h_o(t)$ and a
The electrical field at the interferometer input can be expressed as
\[ E(t) = E_s(t) + E_n(t), \]  
where the first term corresponds to the signal, and the second one to the ASE noise originated from the optical pre–amplifier. The electrical field of the signal can be written in complex baseband form as
\[ E_s(t) = \sqrt{2GP_s} u(t) \exp[j\theta_s(t)] + h_s(t), \]  
where \( P_s \) denotes the signal power at the amplifier input, \( u(t) \) a rectangular pulse of unitary amplitude within the interval \([0,T]\), and \( \theta_s(t) \) the signal phase. This phase is given by \( \theta_s(t) = \theta_s(t-T) + \pi(1-a_d)/2 \), where \( \theta_s(t-T) \) is the phase in the preceding time interval, with \( a_d = 1 \) for symbol “one” and \( a_d = -1 \) for the symbol “zero”. The ASE noise is considered to be a zero mean white stationary Gaussian noise with the same polarization as the signal and a single-sided power spectral density given by \( N_o = h\nu_s(G-1)F/2 \), where \( h\nu_s \) is the photon energy. The field \( E_n(t) \) can be expressed in terms of in–phase \( n_i(t) \) and quadrature \( n_q(t) \) components, giving for the case of a polarized receiver \( E_n(t) = [n_i(t) + jn_q(t)]*h_s(t) \).

The electrical fields at the interferometer outputs are \( E_c(t) = (1/2)[E(t) + E(t-T)] \) for the constructive port and \( E_d(t) = (1/2)[E(t) - E(t-T)] \) for the destructive port. These fields are detected using a pair of identical photodiodes with unitary responsivities, and the resulting currents are subtracted and electrically filtered. The resulting signal is sampled at \( t_d \), originating the decision variable
\[ v = v_s - v_n, \]  
with
\[ v_s = \frac{i}{2} \int \int A(\tau_1, \tau_2)[y_s(t_d-\tau_1) + n_s(t_d-\tau_1)]x, \]  
\[ v_n = \frac{i}{2} \int \int A(\tau_1, \tau_2)[y_n(t_d-\tau_1) + n_n(t_d-\tau_1)]x, \]

The definitions for the terms present in (4) are the following:

\[ A(\tau_1, \tau_2) = \int \int h_s(\zeta)h_s(\tau_1 - \zeta)h_s(\tau_2 - \zeta)d\zeta, \]  
\[ n_1(t) = (1/2)[n_s(t) + n_s(t-T)] , \]  
\[ n_2(t) = (1/2)[n_s(t) + n_s(t-T)] , \]  
\[ n_3(t) = (1/2)[n_s(t) - n_s(t-T)] , \]  
\[ n_4(t) = (1/2)[n_s(t) - n_s(t-T)] , \]  
\[ y_s(t) = \sqrt{2\alpha_s GP_s} u(t), \]  
with \( \alpha_s = (1+\alpha)/4 \).

The central idea of this approach is to expand the signal and the ASE noise at the optical filter input in terms of a set of orthogonal functions \( \{\phi_k\} \), so that the decision variable can be written as a series of uncorrelated random variables. Since these variables are Gaussian distributed they are also independent. In this way, the series expansion of the signal and the ASE noise can be written as
\[ y_s(t) = \sum_{k=0}^{\infty} y_k^s \phi_k(t), \]  
\[ n_i(t) = \sum_{l=0}^{\infty} n_{i,l} \phi_{i,l}(t), \text{ for } l = 1,\ldots,4, \]

where \( y_k^s \) and \( n_{i,l} \) are the respective series expansion coefficients. The functions \( \{\phi_k\} \) must obey to the following condition,
\[ \int \phi_l(t) \phi_k(t) dt = \delta_{kl}, \]
where \( \delta_{kl} \) is the Kronecker delta function. These functions are obtained as the eigenfunctions of the following integral equation,
\[ \int \Lambda(\tau_1, \tau_2) \phi_k(\tau_2) d\tau_2 = \lambda_k \phi_k(\tau_2), \]
where the values \( \{\lambda_k\} \) are the corresponding eigenvalues. Moreover, it can be demonstrated that the random variables \( \{n_{i,l}\} \) are mutually independent Gaussian variables with zero mean and variance given by \( N_o/2 \). Using the series expansions given in (8) and (9), the decision variable \( v \) reduces to
\[ v = \frac{i}{2} \sum_{k=0}^{\infty} \lambda_k(y_k^s + n_{i,k})^2 + \frac{1}{2} \sum_{l=0}^{4} \lambda_{i,l}[(y_{i,l}^s + n_{i,l})^2 + n_{i,l}^2] \]  
Bearing in mind that \( v \) is the difference between two independent random variables \( v_s \) and \( v_n \), the calculation of its MGF is straightforward [6], giving
\[ M_s(s) = M_{v_1}(s)M_{v_2}(-s), \]

where

\[ M_{v_1}(s) = \prod_{k=0}^{\infty} \exp \left( \sum_{k=0}^{\infty} \frac{s\lambda_k u_k^2 GP\alpha_k}{1-s\lambda_k N_0/2} \right), \]

with \( u_k = \int_{-\infty}^{\infty} u(t)\phi_k(t) dt \) and the quantity \( GP\sum_{k=0}^{\infty} u_k^2 \)

denoting the signal energy. A new parameter, \( \xi_k \), is defined as

\[ \xi_k = u_k^2 / T, \]

so according to the definition of \( u(t) \) it reduces to

\[ \xi_k = \left[ \int_{-\infty}^{\infty} \phi_k(t) dt \right]^2. \]

This parameter has a key role in this study, because the value of the summation \( \sum_{k=0}^{\infty} \xi_k \)
can be used as measure of how the optical filter affects the signal energy: as close this summation
approximates unit more negligible is the filtering effect on the signal.

B. Particular case: ideal optical filter and integrate-and-dump electrical filter

For an ideal optical filter with 3-dB bandwidth \( B_o \), the respective low-pass equivalent impulse response can be given by,

\[ h_o(t) = \frac{\sin \Omega t}{\pi t}, \]

where \( \Omega = \pi B_o \). The impulse response for the integrate-and-dump electrical filter is given by

\[ h_i(t) = 1, \quad t \in [0, T] \] and zero elsewhere. \hspace{1cm} (18)

In this way, the function \( \Lambda(\tau_1, \tau_2) \) can be written as

\[ \Lambda(\tau_1, \tau_2) = \int_{-\infty}^{\infty} \frac{\sin \Omega(\tau_1-u)\sin \Omega(\tau_2-u)}{\pi(\tau_1-u)\pi(\tau_2-u)} dt. \]

\hspace{1cm} (19)

It is interesting to note that when this particular case is solved with the approach proposed in [5] and [6], the kernel of the resultant integral equation is the noise autocorrelation function after the optical filter. With this approach the eigenfunctions obtained are the well known prolate spheroidal wave functions (PSWF), [10], [11]. In section IV the behaviour of the PSWF and the functions \( \{\phi_k\} \)
present in (11) is compared. The eigenvalues obtained in this situation are, however, the same as the ones obtained from (11), [9].

III. SIMPLIFIED APPROACH

The approach described in the previous section can be simplified by using the wide-band assumption, this is by choosing an ideal optical filter with a large time-bandwidth product \( (B_o T) \). For an integrate-and-dump electrical filter this simplification is obtained by making the following approximations: 1) \( \lambda_k = 1 \), for \( k < B_o T \) and zero elsewhere; 2) \( \sum_{k=0}^{B_o T-1} \xi_k = 1 \). The first approximation results from assuming that the filtered ASE noise samples have uniform noise power, while the second one results from assuming that the signal energy is not affected by the optical filter. Thus, the MGF of the decision variable is still given by (13), but with

\[ M_{v_1}(s) = \frac{1}{(1-s N_0/2)^{sT}} \exp \left( \frac{GSP\alpha_1}{1-s N_0/2} \right). \]

IV. DISCUSSION AND RESULTS

The rigorous approach developed in section II is used to find out the range of validity of the simplified approach presented in section III. The parameters used for evaluating the receiver performance are the following: a bit rate of 10 Gb/s, and an optical amplifier with \( G = 30 \text{ dB} \) and \( F = 5 \text{ dB} \). Before evaluating the receiver performance with these two approaches the behavior of the eigenfunctions and the eigenvalues of (11) is clarified. These quantities are obtained using the Gaussian quadratures rules integration technique and solving the resultant matrix eigenvalue problem [6]. In particular, with the purpose of making the problem numerically treatable, (11) uses an appropriate bounded integration limits rather than infinite limits, without affecting its accuracy. This integral is then evaluated with a Gaussian quadrature and the number of points used, as well as its integration limits will depend on the \( B_o T \) value, as will be discussed at the end of this section. Moreover, the series in (12) can be also truncated, without losing accuracy [8].

Fig. 2 (a) plots the eigenfunction \( \phi_o(t) \) for different values of \( B_o T \), while Fig. 2 (b) and Fig. 2 (c) do the same for the eigenfunctions \( \phi_1(t) \) and \( \phi_2(t) \), respectively. In Fig. 2 (a) the PSWF are also plotted for comparison purposes. It can be observed that as \( B_o T \) increases the energy of both \( \phi_o(t), \phi_1(t) \) and \( \phi_2(t) \) tend to become more concentrated in the interval \([-T/2, T/2]\) (note that the integration interval of (19) is shifted to \(-T/2 \) and \( T/2 \), for the sake of simplifying the calculations), so that for \( B_o T = 10 \) there is practically no energy outside this region. From these figures it can be observed that the even order functions have even symmetry around zero, whereas the odd order functions have odd symmetry. Moreover, all of these functions have an aperiodic behavior and its oscillation frequency increases with \( B_o T \).

These observations about the symmetry and aperiodic behavior of the eigenfunctions \( \{\phi_k\} \) have been also reported for the PSWF in [11]. Note that the eigenfunctions \( \{\phi_k\} \) obtained in this paper progressively provide a fairly accurate estimation of the PSWF as the product \( B_o T \) increases, as can be seen in Fig. 2 (a), where these functions are represented in the interval \([-T/2, T/2]\). It is shown in this figure that for \( B_o T = 1 \) there is
a visible difference between the two functions, whereas for $B_oT > 5$ this difference completely disappears.

simplified formulation, $\sum_{k=0}^{B_oT-1} \xi_k = 1$, does not hold for low values of $B_oT$.

Fig. 3 shows the parameter $\xi_k$, and the summation of all $\xi_k$, as a function of $B_oT$.

In Fig. 4 the eigenvalues $\lambda_k$ are represented for various values of $B_oT$. It is shown that the values of $\lambda_k$ for larger values of $B_oT$ are close to 1 for $k < B_oT$, and fall off rapidly to zero for $k > B_oT$. Nevertheless, for $B_oT = 1$ all the eigenvalues are smaller than 1 ($\lambda_0 = 0.783$, $\lambda_1 = 0.205$, $\lambda_2 = 0.011$ etc), indicating that the first requirement of the simplified formulation ($\lambda_k = 1$, for $k < B_oT$ and zero elsewhere) does not hold for low values of $B_oT$.

Fig. 4 Eigenvalues for various values of $B_oT$.

Fig. 5 compares the receiver sensitivity, defined as the received optical power required to achieve an error probability of $10^{-12}$, obtained with the rigorous and the simplified approaches (note that if $F = 3$ dB and the error probability is set to $10^{-9}$ the results reported in [3] are obtained). The error probability is evaluated using the MGF and applying a saddle-point approximation method [4]. As can be seen, the accuracy of the simplified approach, resulting from the wide-band assumption, improves with increasing values of the time-bandwidth product, but this approach is clearly inappropriate to provide rigorous performance estimates for products closer to 1. In particular, for $B_oT = 1$ the power penalty for using the wide-band approximation is around
1.1 dB, whereas for \( B_T = 10 \) this penalty is negligible. The reason for the inadequacy of the simplified approach resides, mainly, in the fact that it does not take into account the effect of optical filtering on the energy of the signal waveform, what, as seen (Fig. 3), is detrimental for low time-bandwidth products.

Besides the inadequacy of the simplified approach for low time-bandwidth products, Fig. 5 also shows that the two curves have a different behavior as \( B_T \) increases. The curve obtained with the simplified approach shows that the receiver sensitivity gets worse as \( B_T \) increases, whereas the rigorous curve shows that the receiver sensitivity starts improving and then from \( B_T = 1.5 \) it starts deteriorating. The existence of an optimum value for \( B_T \) in the rigorous approach is due to the fact that both the ASE noise and the action of the optical filter on the signal energy are affecting the system performance. In particular, the action of the filter on the signal energy decreases with increasing \( B_T \) products, whereas the influence of ASE noise increases with increasing \( B_T \) products. As for the simplified approach only the influence of the ASE noise is taken into account, so the receiver sensitivity deteriorates with increasing values of \( B_T \).

Also, from Fig. 5, a validity range for the simplified approach can be defined, by considering that this approach is adequate to evaluate the performance of a pre-amplified DPSK receiver when there is less than ~0.1 dB difference between the sensitivities of the simplified and the rigorous approaches. In this way, the simplified approach can be used for \( B_T \geq 5 \), whereas the rigorous one should be used for \( B_T < 5 \).

The rigorous approach, despite giving the exact results for all situations require a long processing time in comparison with the simplified approach (several hours versus few seconds). This processing time depends mainly on the integration limits of (11) and on the number of points used in the respective Gaussian quadrature, which ultimately depends on the \( B_T \) value. In particular, for \( B_T = 1 \) the eigenfunctions extend far beyond the interval \([-T/2, T/2]\) (see Fig. 2), therefore the integration limits of (11) were set to \(-10T\) and \(10T\), whereas, for \( B_T = 10 \), the eigenfunctions tend to have their energy concentrated inside the interval \([-T/2, T/2]\), so the integration limits of (11) were set to \(-5T\) and \(5T\). Despite this reduction on the integration interval, the number of points used in the Gaussian quadrature (1000 points) was preserved, since the eigenfunctions for \( B_T = 10 \) have a much more oscillatory behavior than the eigenfunctions for \( B_T = 1 \). In these situations the processing time required is about 10 hours, where ~95% of this time is due to matrix multiplications needed to describe the matrix eigenvalue problem, while the computation of the eigenvalues and eigenfunctions of (11) only requires the remaining 5% of the time. The computer used has an Intel Celeron D processor working at 2.8 GHz with a 1GB RAM and the software was developed in C programming language.

V. CONCLUSIONS

A rigorous approach to evaluate the performance of optically pre-amplified DPSK receivers with arbitrary optical and electrical filtering has been presented. This approach is used to find out the range of validity of the well known simplified approach that assumes an ideal wide-band optical filter and an integrate-and-dump electrical filter.

The results show that these two approaches can be used in a complementary way: the simplified approach can be used for time-bandwidth products larger or equal to 5, while for smaller values the use of the rigorous approach is required. It is observed that the processing time associated with this approach is several hours, whereas a few seconds are enough to use the simplified approach. Thus, despite the accuracy of the rigorous approach, its processing time can be a limiting factor, in particular, in applications where fastness is an important requirement, like physical-layer-aware network planning tools.

REFERENCES


Fig. 5 Receiver sensitivity as a function of \( B_T \).