Robust Extraction of Planar and Quadric Surfaces from Range Images

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Abstract

We present a new range image segmentation algorithm based on the extraction of planar and general quadric surfaces, which is of great relevance in man-made environments, built largely of low-order surfaces. We describe how our robust estimator can effectively reject erroneous surface hypotheses, while identifying points describing a general quadric surface, by means of an improved approximation to the true geometric distance between each point and the surface. We present thorough experimental results with quantitative evaluation against ground truth.

1. Introduction

A reliable segmentation of range images is of fundamental importance in tasks such as object recognition, autonomous navigation, industrial inspection, and reverse engineering. However, it remains a vexing challenge because it depends on correct feature extraction from data describing multiple structures, and corrupted by noise and other sensor errors.

Recently, a number of papers have addressed the quantitative evaluation of segmentation algorithms using metrics calculated by analyzing machine results against ground truth [6, 7, 8, 9, 13]. The evaluation results show that for some applications, even polyhedral object segmentation needs improvement. The main difficulties are the correct identification of small surfaces and the preservation of edge location, important for modeling and CAD-model based object recognition [7].

To deal with multiple structures and image degradation (i.e., statistical outliers) many segmentation algorithms have employed robust estimators [2, 11] for geometric characterization of range data. We have previously proposed [6] planar surface extraction using an improved robust estimator based on MSAC / RANSAC [16]. Like RANSAC, the estimation process verifies a (large) number of surface hypotheses generated from randomly sampled, minimal subsets of points (e.g. 3 points for a plane, 9 for a general quadric). Using the robust estimator’s cost function, each hypothesis is evaluated based on the residual distances of all range points and a set of inliers is identified. The inliers of the lowest-cost hypothesis are used in a regular least squares estimate to yield the final surface parameters. A genetic algorithm (GA) [10] can accelerate the optimization process by combining minimal subsets (chromosomes) of hypotheses (individuals) presenting low cost [3, 15]. An initial, rough edge map may also be used to partition the range data, and the resulting disjoint regions processed separately, reducing the number of erroneous hypotheses and the associated computational expense. In [6] we extended the MSAC / RANSAC estimator cost function to include local orientation to better identify outliers/inliers and reject incorrect surface hypotheses. We also presented a new GA to accelerate the search for the best surface hypothesis. Here, our approach is enhanced to extract planar and also general quadric surfaces.

2. Segmentation Algorithm

Our approach to range image segmentation comprises two main stages: preprocessing and surface extraction.

2.1. Preprocessing

The preprocessing stage accomplishes three objectives: (a) It estimates the local orientation at each pixel, (b) it provides an initial, rough partition of the data with step and roof edge detection [1], and (c) it provides an approximate classification of each pixel as flat, curved, or undetermined. To estimate the local orientation we use principal components analysis (PCA) in $11 \times 11$ neighborhoods; these neighborhoods are not allowed to straddle detected step edges. The orientation is taken to be the eigenvector of the local covariance matrix associated with the smallest eigenvalue. Once the orientation is available, roof edges can be detected [1] and an initial (often incomplete) edge map is built to partition the data for efficiency.

For each pixel $p$, we analyze the change in normal direction [5], in moving from $p$ to a neighboring pixel $q$.

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\[ \kappa(p, q) = \frac{||\vec{n}_p - \vec{n}_q||}{||p - q||} \] (1)
and we define a simple planarity measure as:
\[ m_p = \text{median}_q \{ \kappa(p, q) \} \] (2)

Using a lower \((m_L)\) and a higher \((m_H)\) threshold, each pixel is then labeled as flat, curved, or undetermined. The resulting planarity map improves the efficiency of our method by suggesting the surface model (planar or quadric) more likely to explain the greater number of points in a region.

### 2.2. Surface Extraction

This stage is iterative. At iteration \(i\), only the largest connected region \(R_i\) (as delimited by the edge map) is processed. \(R_i\) may include surfaces with smooth joins that produce no edges. At the end of the iteration, the connected subregion \(R_i^c\) corresponding to the largest (planar or quadric) surface within \(R_i\) is extracted, using our robust estimator, and its boundaries are added to the edge map. Points in \(R_i^c\) are unavailable to subsequent iterations.

A new iteration is then started, taking the largest connected region remaining as \(R_{i+1}\). Iteration ends when the size of \(R_n\), \(n > i\), falls below a threshold \(t_R\). Very small regions that are not extracted are considered to be unreliable (and to convey little information) and are left unlabeled in the output (segmented) image.

To extract region \(R_i^c\), we first select the more promising surface model and then use our estimator for a robust surface fit. This means finding a set of inliers in \(R_i\) for the best model hypothesis and then performing a least squares fit to those inliers only. Our algorithm uses two surface models, defined by the implicit functions of a plane and a general quadric, respectively:

\[ f_1(x, y, z) = ax + by + cz + d = 0 \] (3)
\[ f_2(x, y, z) = a_0x^2 + a_1y^2 + a_2z^2 + a_3xy + a_4xz + a_5yz + a_6x + a_7y + a_8z + a_9 = 0 \] (4)

Using the planarity map, if \(R_i\) has more points labeled flat than curved, we find the planar model and extract the largest such surface in \(R_i^c\). Curved points may then become the majority in a connected region \(R_j\), \(j > i\), of \(R_i - R_i^c\), in which case the quadric model would be used in a subsequent iteration (or vice-versa).

Once the model is chosen, the GA [6] finds the hypothesis (minimal subset) minimizing the robust estimator cost function. The GA chromosomes are the coordinates of either three (planar) or nine (quadric) points; its fitness function is based on the residual distance and orientation of each point in the region with respect to the model parameters suggested by the points in the chromosome. Of course, one could use pure random sampling if the number of subsets is large enough to guarantee, with probability near one, that at least one subset will have only “good” points from the same surface [14]. This number is very large for the 9-point quadric hypotheses and the GA’s ability to combine good ones through the crossover operation clearly reduces the total number of cost function evaluations.

If the selected surface model is the plane, minimal subsets are permitted to include only pixels labeled as flat or undetermined, but not curved. Likewise, when the quadric model is used, hypotheses cannot be generated using flat pixels. This avoids generating many clearly erroneous hypotheses, letting the GA focus on better ones by drawing only from “similar” (flat, or not) points.

Each hypothesized surface \(h\) (the exact fit to the points in the minimal subset) is assigned a cost based on the residual distance \(r_{p,h}\) and local orientation \(\vec{n}_p\) of each point \(p\):

\[ \text{cost}(h) = \sum_{p \in R_i} \rho(r_{p,h}, \vec{n}_p) \] (5)

based on our improved robust error term [6]:

\[ \rho(r_{p,h}, \vec{n}_p) = \begin{cases} r_{p,h} & \text{if } r_{p,h} < \sigma, \vec{n}_p \cdot \vec{n}_{h,p} > \cos(\theta) \\ \sigma & \text{otherwise \ (outlier)} \end{cases} \] (6)

where \(\sigma\) and \(\theta\) are, respectively, the inlier bound and angular difference thresholds. Therefore, point \(p\) is classified as an inlier if it is very close to the hypothesized surface and the local normal \(\vec{n}_p\) is consistent with the surface orientation at that point, \(\vec{n}_{h,p}\). Minimizing this cost function means minimizing the number of outliers, while also minimizing the sum of inlier residuals.

After the final surface parameters have been obtained by a least squares fit to the inliers of the best hypothesis, a region growing process begins from those inliers as seed points. The \(\sigma\) and \(\theta\) thresholds, initially set to small values to improve the robustly fitted model parameters, are multiplied by a small constant \(C\) (in our experiments, \(C = 3\)) and new “inliers” are added to the growing region and its largest connected component is taken as \(R_i^c\). The planarity map is not used in this step. The region is also allowed to grow beyond the limits of \(R_i\), but a merged point outside \(R_i\) remains available to subsequent iterations. At the end, points extracted multiple times are assigned to that surface with which they have the smallest residual. So when \(R_i^c\) is extracted (i.e. its boundaries are added to the edge map), the residuals of its points are also saved.

Using the general quadric model alone to extract both planar and curved surfaces results in poor planar fits and greater over-segmentation. This is because the general quadric can describe a planar region reasonably well but, beyond that region’s limits, will also curve...
to collect more inliers from neighboring surfaces, decreasing its cost. Using the two surface models and the planarity map, we avoid this problem.

2.3. Residual Distance to a Quadric Surface

Since there is no closed formula for the true geometric distance between a point \( p \) and a general quadric surface – and the computational expense of using iterative processes is prohibitive for our approach owing to the large number of cost function evaluations we must do – an approximation to this distance must serve as the residual value of \( p \).

Calculating the geometric distance can be formulated as first finding the closest point \( p_0 \equiv (x_0, y_0, z_0) \) on the surface, with \( f_2(x_0, y_0, z_0) = 0 \). \( p_0 \) can be found by intersecting the surface with a line parallel to the normal \( \vec{n}_0 \) at \( p_0 \) and passing through \( p \equiv (x, y, z) \). That is, \( p - p_0 = \alpha \vec{n}_0 \), for some scalar \( \alpha \). Since \( \vec{n}_0 \) is also unknown, the problem requires solving an equation of degree six, which is also not feasible for our approach.

Some segmentation algorithms [3] intersect the quadric surface with three lines, one parallel to each of the \( X \), \( Y \), and \( Z \) axes (Fig. 1(left)). That is, the vectors \((1, 0, 0)\), \((0, 1, 0)\), and \((0, 0, 1)\) are taken as approximations for \( \vec{n}_0 \). The problem is then to solve, for each approximation, one quadratic equation with one unknown: \( f_2(x, y, z) = 0 \), \( f_2(x, y, z) = 0 \), and \( f_2(x, y, z) = 0 \). This yields at most six solutions, and six distances \((d_1, d_2, \ldots)\); the smallest is taken as the residual value.

We improve on this idea as follows. Using our robust error term, it is possible first to verify whether the hypothesized quadric surface has orientation consistent with the local normal at \( p \) (i.e. \( \vec{n}_p \approx \vec{n}_{b,p} \)). If the orientation is inconsistent, we need not calculate the residual because \( p \) is an outlier. Otherwise, \( \vec{n}_0 = \vec{n}_p \equiv (a_p, b_p, c_p) \) is taken and the surface is intersected with a line passing through \( p \) and parallel to \( \vec{n}_p \), yielding a better approximation for the closest point \( p_0 \) (Fig. 1(right)). We need to solve only \( f_2(x, y, z) = 0 \), subject to

\[
\begin{align*}
\frac{x_p - x}{a_p} &= \frac{y_p - y}{b_p} &= \frac{z_p - z}{c_p} \\
(7)
\end{align*}
\]

expressing \( y \) and \( z \) in terms of \( x \). This has at most two solutions; the smaller distance is taken as the residual.

3. Experimental Results

We applied the algorithm to the 256 × 256 images from the Cyberware range image database, containing both polyhedra and curved-surface objects. Ground truth segmentations are also available for comparison, as described in [12].

The algorithm thresholds were set by visually inspecting the segmentation results for a training subset of the Cyberware database. For preprocessing, we set \( m_L = 0.007 \) and \( m_H = 0.015 \). For surface extraction, the minimal region size was set to \( t_R = 100 \) pixels. The robust error term thresholds were: \( \theta = 10 \) degrees, \( \sigma = 0.7 \) millimeters, and \( C = 3 \) for region growing. Fig. 2 shows the segmentation results for two Cyberware range images (not in the training set) alongside the corresponding ground truth. The small boxes (on the left result) indicate the pixels yielding the best hypothesis for each extracted surface.

Visually, the results present regions similar to those found in the ground truth if the region size is not close to \( t_R \). For very small regions, and in “tight corners” smaller
than the PCA neighborhood, over-segmentation may occur because of inaccurate local orientation estimation.

Surfaces in the range images not obeying the models are decomposed in quadric and planar patches of decreasing size. Also, pixels corresponding to large surfaces with very low curvature may be labeled as flat, since this is done locally, and the surface may be over-segmented into a few planar patches, 2 or 3 in our experiments.

Fig. 3 shows a quantitative evaluation of our segmentation results against the ground truth for the 40 Cyberware images. The average occurrences of correct detection, over- and under-segmentation, missed and noise regions are plotted according to the region overlap tolerance. For comparison, Fig. 3 also shows the correct detection curve obtained with the popular UB algorithm [8], whose thresholds were automatically tuned using evaluation feedback [12].

Our algorithm took an average of 30 seconds to segment each image, running on a Pentium IV 1.7GHz PC.

4. Final Comments

We presented a novel approach to extract planar and quadric surfaces from range images. An improved, GA-based robust estimator using local orientation and efficient residual distance approximations iteratively identifies the predominant region supporting the surface model suggested by a simple planarity test. Experimental results demonstrate the effectiveness of this approach in man-made environments with low-order surfaces. The approach is not appropriate, in its current form, for free-form objects. As future work, our planarity test will be developed to better detect low-curvature surfaces in the presence of statistical outliers. Finally, if an application relies on precise edge locations, the simple minimum distance criterion, used to merge points claimed by different surfaces, can be replaced by the more elaborate decision plane criterion [4].

References