IMPACT OF FEEDBACK DELAY ON RATE ADAPTATION FOR MULTIPLE ANTENNA SYSTEMS

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I. INTRODUCTION

The design of spectrally efficient and robust schemes is vital to overcome the harsh wireless environment when delivering high data rate services. In the present paper, the performance of a constant power and discrete rate adaptation scheme is evaluated in a multiple antenna scenario. Such a technique has low complexity, and needs a low rate feedback channel to adapt the transmission rate. However, the feedback can become outdated when the channel is rapidly changing. In the light of the feedback delay, the authors propose to analyze the effect of outdated channel information.

Closed form expressions are obtained for the Bit-Error-Rate (BER) performance for MIMO diversity, and Spatial Multiplexing (SM) schemes. Through simulation results, we analyze the impact of feedback delay on the BER. The feedback delay proves to have different impact on each of the MIMO scheme when coupled to a rate adaptive system. We demonstrate that antenna selection suffers more from feedback delay than other diversity schemes.

The results can be used to assess the performance of different MIMO techniques, and may be applied to a switching mechanism, where, at each instant, the channel behavior is analyzed and the best fit MIMO mode is selected.

II. SYSTEM MODEL

The radio spectrum available for wireless communications is becoming increasingly more occupied as the demand for services increases more and more. Since the radio spectrum is a scarce resource, the design of spectral efficient schemes is vital for future wireless data communications systems.

Schemes that adapt the transmission parameters to the varying channel conditions considerably increase the spectral efficiency [bit/s/Hz] of the channel, and have received special attention from research in recent years [1]. Rate adaptive systems in particular provide substantial gains in performance with relatively low complexity, and reduced feedback overhead [2].

Multiple antenna schemes have been proposed to overcome the wireless channel impairments by providing diversity; thus, obtaining enhanced reliability. Traditionally, spatial diversity is exploited at the transmitter or at the receiver end. Space-Time Coding (STC) is an open-loop transmit diversity scheme where diversity is achieved without channel knowledge at the transmitter, e.g. Space-Time Block Coding (STBC) [3]. To improve performance, close-loop schemes that benefit from transmit diversity with partial channel knowledge has been recommended. Antenna selection is one such techniques, requiring very low over-head in the feedback channel. Along with diversity techniques, the use of the Multiple-Input-Multiple-Output (MIMO) channel for Spatial Multiplexing (SM) has attracted interest from research, since it dramatically increases a system’s data rate.

In [4], an adaptive multi-level Quadrature Amplitude Modulation (M-QAM) based on feedback was studied over Nakagami fading channels. In addition, other works also focused on similar adaptive system, as in [5], that combined an adaptive M-QAM with SM with Zero-Forcing (ZF) detection, while in [6] an STBC scheme is analyzed. Both these works study the impact of the feedback delay on the scheme considered, but do not compare different MIMO schemes.

A comparison between different multiple antenna techniques is called for in order to understand its advantages under a rate adaptive system. This scenario can be found in heterogeneous networks, where terminals with different MIMO capabilities coexist in the same air interface. Moreover, if the receiver is able to employ different MIMO techniques, a switching mechanism must exist to select the best MIMO mode at each instant, e.g. spatial diversity or spatial multiplexing. We intend to provide a tool for the switching process, so that the MIMO transmission delivering the best average BER performance is chosen at each moment, according to the channel.

In the present paper, we derive closed-form expressions for the performance of adaptive M-QAM modulation combined with different multiple antenna techniques over a Rayleigh fading channel. The schemes addressed are STBC, Maximum Ratio Combining (MRC), antenna selection, and SM with ZF detection. Furthermore, we analyze the impact of the feedback delay on the performance of the system. The study of the impact of outdated feedback shows how each MIMO scheme behaves when in presence of the channel time variability. As for the antenna selection scheme, this work provides novel performance results when coupled with an adaptive rate transmission.

The remainder of this paper is organized in the following manner. Section II outlines the system model and the channel statistics. The performance of the adaptive system is presented in Section III. The results are shown in Section IV, and conclusions drawn in Section V.

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The remainder of this paper is organized in the following manner. Section II outlines the system model and the channel statistics. The performance of the adaptive system is presented in Section III. The results are shown in Section IV, and conclusions drawn in Section V.
The feedback channel is used to communicate this information to the transmitter. Note that the overhead related to this scheme is considerably lower than transmitting the full channel knowledge, i.e., the channel gains, or the SNR value. Indeed, the feedback consists of \( \log_2(N) \) bits indicating the modulation size, when \( N \) constellations are available.

Once the information is sent to the transmitter, the correct modulation order is configured in the demodulator. The same modulation order is kept for the duration of a frame. At the receiver, a Maximum Likelihood (ML) detector is employed using the estimated channel gain.

A different number of antennas at transmitter and receiver ends is assumed. In this work, the authors consider a set of techniques available for multiple antenna links. These techniques have different goals i.e., to improve performance in terms of diversity, array gain, or multiplexing. Multiple antennas at the transmitter side can extract diversity by employing STBC or antenna selection, while multiple antennas at the receiver use MRC to obtain diversity and array gain. On the other hand, a MIMO configuration can also be applied to create parallel streams of data by using SM with ZF detection.

A. MIMO Channel Statistics

The wireless channel is a flat fading channel varying in time according to Jakes model. We assume the channel to be slow time-varying. Thus it may be regarded as constant for two consecutive symbols. The multipath fading environment is characterized by the Rayleigh fading channel. We assume that the channel at each branch is i.i.d., with PDF, characterized by the Rayleigh fading channel. We assume that the channel amplitude, \( \alpha \), is Rayleigh distributed. Moreover, for a Single-Input-Single-Output (SISO) link, the Signal-to-Noise Ratio (SNR) is defined by 

\[
\gamma = \frac{E_b}{N_0} |\alpha|^2,
\]

where \( E_b \) is the bit power, while \( N_0 \) is the noise power. Then, the SNR has a Probability Density Function (PDF) given by 

\[
p(\gamma) = \frac{1}{\bar{\gamma}} \exp \left(-\frac{\gamma}{\bar{\gamma}}\right),
\]

where \( \bar{\gamma} \) is the average SNR.

When a MIMO link is used to extract diversity from the system, both an STBC and MRC diversity schemes are employed. The \( N_r \) antennas at the transmitter encode the signal using STBC, without channel knowledge. The available power at the transmitter is divided among the transmission antennas when implementing STBC. On the other hand, the \( N_r \) receivers are applied for MRC, using channel knowledge at the receiver.

In this situation, the SNR of the signal is chi-square-distributed with \( 2N_rN_t \) degrees of freedom. The resulting \( \gamma \) and its PDF are then given by

\[
\gamma = \frac{E_b}{N_0} \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} |\alpha_{i,j}|^2
\]

\[
p(\gamma) = \frac{N_r N_t}{\gamma} \gamma^{N_r N_t - 1} \frac{1}{(N_r N_t - 1)!} \exp \left(-\frac{N_r \gamma}{\bar{\gamma}}\right).
\]

Again, within a MIMO link developed to obtain parallel streams, we consider an SM-ZF without channel knowledge at the transmitter, i.e., the available power is divided by the transmit antennas. The statistics of the SNR of such a system are described as a chi-square distribution with \( 2(N_r - N_t) + 1 \) degrees of freedom, [7]. In this paper, we impose the same number of antennas at transmitter and receiver ends, \( N \), for an SM system; thus, the SNR for each stream, and its PDF are given by [7]

\[
\gamma_k = \frac{E_b}{N_0 N_r} \frac{1}{[H^H H]^{-1}}_{k,k},
\]

\[
p(\gamma) = \frac{N_r}{\gamma} \exp \left(-\frac{N_r \gamma}{\bar{\gamma}}\right),
\]

where \([\cdot]_{k,k}\) is the diagonal value \( k \) of a matrix, while \( (\cdot)^H \) represents the hermitian of the matrix.

To improve performance of transmitter diversity, a small load feedback channel was selected to implement Antenna Selection, based on channel estimation at the receiver. Besides the rate, the feedback indicates also the index of the antenna with the highest SNR. In perfect channel estimation and no delay conditions, the PDF of the resulting SNR is obtained from second order statistics. Consider a system where \( L \) branches at the transmitter are available for selection. Assuming that the channel at each branch is i.i.d., with PDF, \( p(\gamma) \), and Cumulative Distribution Function (CDF), \( P(\gamma) \), the resulting SNR is

\[
\gamma = \max(\gamma_1, ..., \gamma_L)
\]

and its PDF is given by

\[
p(\gamma) = Lp(\gamma) (P(\gamma))^{L-1}.
\]
BER constraint, $BER_t$, such that $BER \leq BER_t$; thus, obtaining

$$\gamma_i = \frac{M_t - 1}{-1.6} \ln \left( \frac{BER}{0.2} \right).$$  \hspace{1cm} (5)

In Sections A. and B., closed form expressions of different performance measures are deduced. These results follow an analytical study of the channel statistics pursued for MIMO schemes with SNR statistics dictated by a chi-square distribution. Thus, it is not applicable to antenna selection case.

### A. Spectral Efficiency and Outage

The average spectral efficiency consists of the sum of the data rate average on the basis of the probability that the SNR lies within the boundaries of each fading region. It is then obtained through the following expression

$$\eta = \sum_{i=0}^{N} \log_2(M_i) \int_{\gamma_i}^{\gamma_{i+1}} p(\gamma) d\gamma.$$  \hspace{1cm} (6)

The probability of $n_{\text{th}}$ fading region can be expressed as follows [4]

$$\int_{\gamma_i}^{\gamma_{i+1}} p(\gamma) d\gamma = \frac{\Gamma(N_t, N_r, \frac{N_t}{\gamma_i}) - \Gamma(N_t, N_r, \frac{N_t}{\gamma_{i+1}})}{\Gamma(N_t, N_r)},$$  \hspace{1cm} (7)

where $\Gamma(.,.)$ is the complete gamma function, and $\Gamma(.,.,.)$ is the incomplete gamma function.

Outage occurs when the link experiences an SNR below the threshold for the first available modulation, $\gamma_1$. In such situation, no data is sent, and the outage probability is obtained by

$$P_{\text{out}} = \int_{0}^{\gamma_1} p(\gamma) d\gamma = 1 - \frac{\Gamma(N_t, N_r, \frac{N_t}{\gamma_1})}{\Gamma(N_t, N_r)}.$$  \hspace{1cm} (8)

### B. Average Bit Error Rate

In a discrete adaptive M-QAM system with an instantaneous BER constraint, the average BER experienced is smaller than the target BER, $BER_t$, [4]. The average BER can be computed using the following closed form expression

$$BER = \frac{\sum_{i=0}^{N} \log_2(M_i) \int_{\gamma_i}^{\gamma_{i+1}} BER(M, \gamma) p(\gamma) d\gamma}{\sum_{i=0}^{N} \log_2(M_i) \int_{\gamma_i}^{\gamma_{i+1}} p(\gamma) d\gamma},$$  \hspace{1cm} (9)

where the average BER within a single fading region can be expressed, for MIMO diversity, as

$$\int_{\gamma_i}^{\gamma_{i+1}} BER(M, \gamma) p(\gamma) d\gamma = 0.2 \left( \frac{N_t}{\gamma} \right)^{N_t N_r} \frac{N_t}{b_i} \frac{\Gamma(N_t, N_r, b_i \gamma_i) - \Gamma(N_t, N_r, b_i \gamma_{i+1})}{\Gamma(N_t, N_r)},$$  \hspace{1cm} (10)

where $b_i = \frac{N_t}{\gamma_i} + \frac{1.6}{M_t - 1}$.

### C. Channel Statistics with Feedback Delay

Until this stage, we have considered no delay in the feedback used by the adaptive system. However, in this Section we will introduce a delay $\tau$ between the time the SNR is measured and the time it is employed at transmitter. If this delay has a significant impact on the BER, then the adaptive technique will not produce the expected improvement, i.e. will not work appropriately. However, if a certain delay has a small impact on the BER performance, then it is valuable information. This fact can be used to minimize the rate of updating the constellation size, while meeting the requirement of the system.

It should be noted that the time delay, $\tau$, of the feedback in a time-varying channel affects the BER performance, but does not alter the spectral efficiency. The decision regarding the modulation size is performed at time $t$, based on channel $\gamma_t$, while the experienced channel when the signal is transmitted is $\gamma_{t+\tau}$.

In order to analyze the impact of the time delay, we need the second order statistics of the channel variation. Assuming Jakes model for time variation, the correlation between second order statistics of the channel variation. Assuming $J_0(2\pi f_D \tau)$. $f_D$ is the maximum Doppler spread, and $J_0(\cdot)$ is the zero-order Bessel function of the first kind. We consider $\gamma$ and $\gamma_t$, as the SNR at a time $t$ and $t + \tau$, respectively. Then, for a MIMO diversity scheme, the conditional PDF of $\gamma$ when $\gamma_t$ is known, is given by

$$p(\gamma | \gamma_t) = \frac{N_t}{(1 - \rho) \gamma} \exp \left( -\frac{N_t(\rho \gamma_t + \gamma)}{(1 - \rho) \gamma} \right)$$

$$\times I_{N_t, N_t-1} \left( \frac{2N_t \sqrt{\rho} \sqrt{\gamma}}{(1 - \rho) \gamma} \right).$$  \hspace{1cm} (11)

For a MIMO link used to perform SM, as described above, the conditional PDF is given by

$$p(\gamma | \gamma_t) = \frac{N_t}{(1 - \rho) \gamma} \exp \left( -\frac{N_t(\rho \gamma_t + \gamma)}{(1 - \rho) \gamma} \right) I_0 \left( \frac{2N_t \sqrt{\rho} \sqrt{\gamma}}{(1 - \rho) \gamma} \right).$$  \hspace{1cm} (12)

Once the conditional PDFs of each antenna transmission scheme is known, the average BER performance can be computed using a closed form expression. For the system considered in previous sections, the inclusion of a time delay results in the following expression for the average BER

$$BER = \frac{\sum_{i=0}^{N} \log_2(M_i) \int_{\gamma_i}^{\gamma_{i+1}} BER(\gamma) p(\gamma) d\gamma}{\sum_{i=0}^{N} \log_2(M_i) \int_{\gamma_i}^{\gamma_{i+1}} p(\gamma) d\gamma},$$

$$BER(\gamma) = \int_{0}^{\infty} BER(M, \gamma) p(\gamma) d\gamma.$$  \hspace{1cm} (13)

Following the procedure used in Section B., the closed form expression for the equation above is

$$\int_{\gamma_i}^{\gamma_{i+1}} BER(\gamma) p(\gamma) d\gamma = 0.2 \left( \frac{N_t}{\gamma} \right)^{N_t N_r} \frac{N_t}{b_i} \frac{\Gamma(N_t, N_r, b_i \gamma_i) - \Gamma(N_t, N_r, b_i \gamma_{i+1})}{\Gamma(N_t, N_r)},$$

where $b_i = \frac{N_t}{\gamma_i} + \frac{1.6N_t \rho}{1.6(1 - \rho) + N_t (M_t - 1)}.$  \hspace{1cm} (14)
The performance of the schemes described in Section A., coupled with a QPSK transmission over a Rayleigh fading channel is shown in Fig. 2. In this curve, the different advantages in the MIMO schemes in terms of diversity order and array gain, can be depicted.

The system considered in this paper is a discrete adaptive M-QAM, as described in Section III.. The M-QAM constellation sizes analyzed are: 4, 16, 64, 256 and with $BER_t = 10^{-3}$. The receiver measures the average SNR along a frame with a normalized length of $f_D T_{frame} = 0.05$. The M-QAM order is computed at the receiver, based on the threshold values expressed in (5). And it is then sent through a feedback channel to the transmitter. During one frame, the transmitter employs the constellation size obtained by feedback.

For the MIMO schemes mentioned in Section A., the simulation results for average spectral efficiency match the analytical expressions, and are shown in Fig. 3. As may be seen, SM has a significant advantage at higher SNR, providing higher throughput, while at lower SNR the performance is poorer when compared to the other MIMO techniques.

On the basis of the expressions found in Section III., we plot the analytical curves for the system described in this Section, applied to SISO, STBC, MRC and SM-ZF. The outage probability given in (7) is presented in Fig. 4. The advantage of employing diversity is significant in terms of achieving a satisfactory probability of outage. Finally, the average BER performance expressed in (9) is displayed in Fig. 5. The average BER is always below $BER_t$, due to the instantaneous constraint $BER \leq BER_t$.

We shall now show the simulation results for the current system in the presence of a performance degrading feedback delay. In Fig. 6, the average BER performance obtained through simulations for normalized feedback delay $f_D \times \tau = 0.1$, is given. In the case of a higher normalized time delay, performance degradation is significant, and is particularly evident in the case of SISO and SM schemes. In Fig. 6, one may also note that, for lower SNR values, STBC scheme performs better in terms of BER than MRC and antenna selection. This may be explained by the lower variance of the instantaneous SNR experienced with STBC, which is due to the channel hardening effect.

Finally, Fig. 7 illustrates the average BER performance degradation when the normalized time delay is increased for an average SNR of 14 dB. SM and SISO are the schemes that experience a higher degradation in performance caused by the feedback delay. Furthermore, one may observe that MRC and STBC experience a similar performance degradation, while antenna selection has a higher increase in BER. This behavior can be explained by the fact that the selection of the best antenna is also affected by the feedback delay. In fact, the choice of the antenna with highest SNR at time $t$, may not hold true at time $t + \tau$, on account of channel variation.

### RESULTS AND DISCUSSION

In this paper, a comparison of the performance of a rate adaptive system coupled with different MIMO schemes has been achieved. Closed form expressions of the average BER were derived so as to characterize a M-QAM system for MIMO diversity and SM over flat Rayleigh fading channels. The advantages of each scheme in terms of spectral efficiency, outage probability and average BER have been analyzed and quantified. Moreover, the analytical study was complemented with simulation results.

The feedback delay was shown to have different impact on each of the MIMO scheme when coupled to a rate adaptive system. Specifically, antenna selection proved to suffer higher degradation, due to the feedback delay than other diversity schemes (STBC and MRC).

As an area of further research, these results may be used in a MIMO switching mechanism, i.e. choosing the best scheme to employ multiple antenna, according to the channel conditions and feedback delay. One such technique can be applied in communications where the receiver is able to employ different MIMO schemes, but requires to be coordinated with the transmitter.

### CONCLUSIONS

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### REFERENCES


Figure 2: Non adaptive system (QPSK) for different MIMO modes

Figure 3: Discrete M-QAM adaptive system for different MIMO modes

Figure 4: Outage Probability

Figure 5: Adaptive M-QAM (M=0,4,16,64,256) and BER

Figure 6: Impact of feedback delay on average BER for different MIMO modes ($f_D \times \tau = 0.1$)

Figure 7: Impact of feedback delay on average BER for different MIMO modes ($\gamma = 14dB$)