JOINT FILTERING OF SAR INTERFEROMETRIC PHASE AND AMPLITUDE DATA IN URBAN AREAS BY TV MINIMIZATION

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1. INTRODUCTION

This paper investigates the use of a popular regularization model, the Total Variation minimization (TV), to filter SAR interferometric images (amplitude and phase data). This model has been extensively used for its property of edge preservation and is therefore well adapted for urban areas. Using a TV model adapted to multi-dimensional data, we propose to do a joint filtering of phase and amplitude images. Due to the many local minima, the minimization of such a model is hard to perform. A new fast approximate discrete algorithm is presented. Besides, the regularization strength is set automatically. The filtering can be applied as a preprocessing step for further image analysis steps such as 3D reconstruction. Results on real images are presented.

2. PROPOSED MODEL

It is assumed that an image $u$ is defined on a finite discrete lattice $S$ and takes values in a discrete integer set $\mathcal{L} = \{0, \ldots, L\}$. We denote by $u_s$ the value of the image $u$ at the site $s \in S$. We note by $(s, t)$ a clique of order two related to a chosen neighborhood system and by $N_s$ the local neighborhood of site $s$. A solution $\hat{u}$ regularizing $u$ is searched for. It can be shown that under the assumption of Markovianity of $\hat{u}$ and with some independence assumption on $u$ conditionally to $\hat{u}$, the MAP problem is an energy minimization problem:

$$\hat{u}^{(\text{MAP})} = \arg \min_{\hat{u}} E(\hat{u}|u) \quad \text{with} \quad E(\hat{u}|u) = \sum_s U(u_s|\hat{u}_s) + \beta \sum_{(s,t)} \psi(\hat{u}_s, \hat{u}_t)$$

$U(u_s|\hat{u}_s) = -\log p(u_s|\hat{u}_s)$ and $\psi$ is a function modeling the prior chosen for the regularized field. In the case of the minimization of the Total Variation, total variation (TV), $\psi(\hat{u}_s, \hat{u}_t) = |\hat{u}_s - \hat{u}_t|$.

2.1. Distributions of interferometric phase and amplitude

The synthesized radar image $z$ is complex-valued. Its amplitude $|z|$ is very noisy due the interferences that occur inside a resolution cell. Under the classical fully developed speckle model of Goodman, the amplitude $a_s$ of a pixel $s$ follows a Nakagami distribution depending on the square root of the reflectivity $\hat{a}_s$. This likelihood leads to the following (non convex) energetic term:

$$U(a_s|\hat{a}_s) = M \left[ \frac{a_s^2}{\hat{a}_s^2} + 2 \log \hat{a}_s \right]$$

In the case of SAR interferometric data, the interferometric product is obtained by complex averaging of the hermitian product $\gamma$ of the two SAR images. A good approximation of the distribution of the phase $\phi_s$ is a Gaussian which leads to a quadratic energy:

$$U(\phi_s|\hat{\phi}_s) = \frac{(\phi_s - \hat{\phi}_s)^2}{\sigma_\phi^2}$$

The standard deviation $\sigma_\phi^2$ at site $s$ is approximated by the Cramer-Rao bound $\sigma_\phi^2 = \frac{1}{\text{L} \rho_s^2}$ (with $\text{L}$ the number of average samples and $\rho_s$ the coherence at site $s$). For low coherence areas (shadows or smooth surfaces, denoted Shadows in the following), this Gaussian approximation is less relevant and a uniform distribution model is better $p(\phi_s|\hat{\phi}_s) = \frac{1}{\pi \sigma^2}$.

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2.2. Regularization term

The proposed method aims at preserving simultaneously phase and amplitude discontinuities. Indeed, the phase and amplitude information are linked since they reflect the same scene. Amplitude discontinuities are thus usually located at the same place as phase discontinuities and conversely. We propose in this paper to perform the joint regularization of phase and amplitude. The prior model chosen to express the dependency between phase and amplitude is disjunctive max operator:

\[
E(\hat{a}, \hat{\phi}) = \sum_{(s,t)} \max(|\hat{a}_s - \hat{a}_t|, \gamma|\hat{\phi}_s - \hat{\phi}_t|),
\]

with \(\gamma\) a parameter that can be set to 1, or otherwise accounts for the relative importance given to the discontinuities of the phase \((\gamma > 1)\) or of the amplitude \((\gamma < 1)\). Note that the regularization term defined in equation 1 is convex. The global joint energy term is then (with \(\beta_a\) and \(\beta_\phi\) hyper-parameters that balance the relative weight of the regularization term and each likelihood term):

\[
E(\hat{a}, \hat{\phi}|a, \phi) = \frac{1}{\beta_a} \sum_s M[\frac{a^2}{\hat{a}_s^2} + 2 \log \hat{a}_s] + \frac{\gamma}{\beta_\phi} \sum_s \frac{(\phi_s - \hat{\phi}_s)^2}{\hat{\sigma}_\phi^2} + \sum_{(s,t)} \max(|\hat{a}_s - \hat{a}_t|, \gamma|\hat{\phi}_s - \hat{\phi}_t|).
\]

3. ENERGY MINIMIZATION

Minimizing a non-convex energy is a difficult task as the algorithm may fall in a local minimum. Algorithms such as the iterated conditional modes (ICM) require a “good” initialization and then performs local changes to reduce the energy. Graph-cut approach provides a way to explore a combinatorial set of changes involving simultaneously all pixels. Following [1], we denote such changes large moves. Instead of allowing a pixel to either keep its previous value or change it to a given one (\(\alpha\)-expansion), we suggest that a pixel could either remain unchanged or its value be increased (or decreased) by a fixed step. Such an approach has first been described independently in [2, 3, 4] and applied recently with unitary steps in [2]. We however use these large moves in a case of non-convex data term. The trial steps are chosen to perform a scaling sampling of the set of possible pixel values. We express the algorithm in the general case of joint regularization.

4. HYPER-PARAMETER TUNING

Considerable effort has been devoted to hyper-parameter estimation. One of the possible methods to perform hyper-parameter tuning is the analysis of the so-called L-curve [5]. This curve is the graphical representation of the regularization energy term with respect to the likelihood energy term. The corner of the curve corresponds to a good trade-off between under-regularization (steep part of the curve, where the regularization term can be largely improved with minor likelihood modification) and over-regularization (slowly varying part of the curve, where the regularization term can no longer be improved, whatever the likelihood price). Note however that the L-curve method is known to fail in some cases. We successfully apply this method on simulated and real data in the next section and therefore provide a fast and automatic method for InSAR image denoising.

5. RESULTS

Results on real interferometric SAR images of urban areas will be presented. The interest of the joint regularization of phase and amplitude data will be demonstrated.

6. REFERENCES