Multi-leak estimator for pipelines based on an orthogonal collocation model

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Abstract—The orthogonal collocation method (OCM) is used to obtain an approximate solution of the water hammer equations which represent one-phase water flow transients in pipeline systems. The OCM provides solutions over the entire spatial domain, therefore it can be used to obtain an accurate model with possible leaks spatially distributed. An estimator can be designed based on the spatially-discretized model in order to detect multiple leaks by identifying their positions and leak coefficients.

I. INTRODUCTION

Sometimes it is difficult to avoid leakages even if the pipelines are constructed to maintain their integrity since leakage may result, for example, due to sudden changes of pressure not considered in the design, corrosive action, or a poor maintenance. Since leaks are inevitable, every pipeline operation should have a monitoring procedure for leak detection in order to diminish not desirable effects such as: economic loss, damage to the surrounding infrastructure, environmental disasters, etc.

In recent years, the scientific community has focused more its attention to develop more efficient methods for leak detection in pipelines.

For example, in [1] it was formulated a pressure wave method by studying the difference in transient response of a pipeline system with/without a leak. When the transient pressure wave encounters a leak, part of the wave is reflected back. The leak location is determined from the arrival time of the reflected wave. In [2] it was investigated the damping characteristics of the transient pressure wave. It is underlined that the different mode damping by leakage depends on leak location.

In [3]-[4] several components of a hydraulic system can be represented by a transfer matrix. Transient flow is caused by the periodic opening/closing of a valve. A frequency response diagram at the valve is developed based on the transfer matrix. For a system with leaks, this diagram has additional resonant pressure amplitude peaks. From the frequency of the peaks, the location of the leak can be detected. In [5] a similar method is presented, this consists on the generation of a steady-oscillatory flow in pipelines, by the sinusoidal operation of a valve, and the analysis of the system frequency response for a certain range of oscillatory frequencies. A leak creates a resonance effect in the pressure signal with a secondary superimposed standing wave. The pressure measurement and the spectral analysis of the maximum pressure amplitude at the excitation site enable the estimation of the leak approximate location.

In [6], the governing equations for transient flow pipes are solved directly in the frequency domain by means of the impulse response method. Harmonic analysis of the transient pressure is used to identify the location and size of a leak.

As a final example, a good off-line approach is presented in [7] which allows to identify the unknown parameters associated to the existence of multi-leaks in a pipeline based on a combination of transient and steady conditions without requirements of valve perturbation. The method is based on a family $\mathcal{F}$ of lumped parameters nonlinear models.

It’s notable that all the methods previously mentioned are not based on steady-state response, on the contrary, they profit the transient conditions. Therefore, a method based on transient time domain analysis can be applied for the case of multiple leaks, which is proved to be an unachievable task by employing steady state methods since these ones do not satisfy two indispensable requirements: distinguishability and uniqueness (see [8]).

In the present work in order to overcome the singularity of the multi-leak detection problem, we propose the conception of an observer based on a spatial-discretized nonlinear model obtained by means orthogonal collocation method (as in [22] for single leak case). The model is extended to include the various leak positions and their coefficients into the state vector. The proposed observer works for so-called persistent inputs [17] which allow to distinguish between two states from the response of the system- in our case between the leak positions and between the coefficients from the flow rate response.

The ‘water-hammer’ equations for the flow dynamic in a pipeline are first recalled in section II, and from them the proposed orthogonal collocation model is presented. The estimator design for multi-leak detection is discussed and illustrated in section III. Simulation results are shown in section IV. Finally some conclusions and perspectives are given in section V.

II. DYNAMICS PRESENTATION AND COLLOCATION MODEL

The water hammer behavior is the transmission of pressure waves along the pipeline resulting from a change in liquid flow velocity. This phenomenon is described by a set of hyperbolic partial differential equations (PDE) formed by one-dimensional continuity and momentum equations (basis
of hydraulic transients), which are used to solve problems of unsteady flow in pipelines [10], [11].

A. The dynamical model

Assuming convective changes in velocity to be negligible, and that the liquid density and pipe cross-sectional area are constant, the momentum and continuity equations governing the dynamics of the fluid in the pipeline can be expressed as

\[ \frac{1}{gA} \frac{\partial Q(z,t)}{\partial t} + \frac{\partial H(z,t)}{\partial z} + J_S + J_U = 0 \]

(1)

\[ \frac{\partial H(z,t)}{\partial t} + \frac{b^2}{gA} \frac{\partial Q(z,t)}{\partial z} = 0 \]

(2)

where \( H \) is the pressure head (m), \( Q \) the flow rate in the pipeline (m/s), \( b \) the wave speed in the fluid (m/s), \( g \) the gravitational acceleration (m/s²), \( A \) the cross-sectional area of the pipe (m²), \( D \) the diameter of the pipe, \( J_S \) and \( J_U \) the head losses per unit length due to steady and unsteady friction respectively, \( t \) and \( z \) the time (s) and space (m) coordinates respectively. Here \( z \in [0, L] \) where \( L \) is the length of the pipe. The steady component of the friction may be defined as:

\[ J_S = \frac{f Q^2(z,t)}{2gDA^2} \]

The unsteady friction model considered in this work is that given in [25]:

\[ J_U = \frac{k}{gA} \left( \frac{\partial Q}{\partial t} + b \frac{\partial Q}{\partial z} \right) \]

where the coefficients \( k \) is experimentally calibrated or determined numerically from another. Initial conditions for Eqs. (1)-(2) are given by

\[ H(z,0) = H^0(z); \quad Q(z,0) = Q^0(z) \quad \forall z \in [0, L] \]

The boundary conditions in the flow transient equations can represent the end of the pipe in a tank, a valve, the connection between two pipes or a kind of different element, for example a pump, a leak, by-pass valves, etc. In this work the boundary conditions to be handled are: imposed pressures representing constant-level reservoirs at the ends of the pipe and possible leaks along the pipeline. The imposed pressure heads will be determined numerically from another. Initial conditions for \( H(z,0) \) and \( Q(z,0) \) are given by

\[ H(z,0) = H_{in}(t); \quad H(z,L,t) = H_{out}(t) \]

A leak at position \( z_f \) of the pipeline with outflow is represented by

\[ Q_f(t) = \mathbb{H}_{z_f} \lambda_f \sqrt{H(z_f,t)} \]

(3)

where \( \lambda_f = A_f C_f t \geq 0, A_f \) is the sectional area of the leak, \( C_f \) the discharge coefficient and \( \mathbb{H}_{z_f} \) is the Heaviside unit step function associated to the occurrence of the leak at time \( t_f \).

A close-form solutions of these equations is not available. However, several methods have been used to numerically integrate them, such as method of characteristics, finite-difference method, finite element method, and linear element method [13]-[16]. In the present paper, we propose to use orthogonal collocation, previously applied in hydraulic systems [20]-[22]. This method is a special case of the so-called weighted-residual methods, commonly used in computational physics for solving PDE [12]. This method is fairly simple, provides fast convergence and accurate results over the entire spatial domain.

B. The Orthogonal Collocation Method (OCM)

Given two bases of functions \( N_{H_j}(z) \) and \( N_{Q_j}(z) \), the following approximation to the solution of (1)-(2) for both flow rate and pressure head are considered:

\[ Q(t,z) = \sum_{j=1}^{n_Q} Q_j(t)N_{Q_j}(z) \]

(4)

\[ H(t,z) = \sum_{j=1}^{n_H} H_j(t)N_{H_j}(z) \]

(5)

for collocation points \( z_i \), where \( n_Q \) number of flow rate, \( n_H \) number of collocation points for the pressure head. For this model we have chosen Lagrange interpolation functions as basis functions both for \( N_{H_j} \) and \( N_{Q_j} \), which are given by

\[ N_j(z) = \prod_{i=1}^{n} \frac{z - z_i}{z_j - z_i} \quad \text{if} \quad j \neq i \]

(6)

We can consider possible different points \( z_i^Q \) and \( z_i^H \) for \( N_{H_j} \) and \( N_{Q_j} \) respectively and the optimal location of these points is at the roots of the Legendre polynomials (see [23]).

As a consequence of the sudden occurrence of a leak in a pipeline, a transient boundary condition appears in the system. The occurrence of this new boundary condition is associated with the discharge flow given by (3).

An approximation for \( Q(z,t) \) is typically given by Eq.(4), however when a leak or many leaks develop at the system, this approximation must be changed to include the leaks.

Since the Water Hammer model is spatially discretized, the space coordinates \( z_i \) (collocation points) can be used to represent the position of a possible leak. To each collocation point \( i \) corresponds a pressure and a flow, but also now a possible leak.

In the absence of leak, the flow rate admits from (1)-(2) an uniform (constant) steady state. In the presence of leaks (assumed to be constant or with negligible transient) the flow steady state is clearly modified, but depends on the positions. For leaks \( Q_{f_1} \) up to \( Q_{f_i} \) at positions \( z_1 \) up to \( z_i \) for instance
the flow at \( z_i \) becomes \( Q_i + \sum_{k=1}^{j} Q_{fk} \). This suggests a change of variables (see [23]) in Eq.(4) of the form:

\[
\bar{Q}_i = Q_i - \sum_{k=1}^{j} Q_{fk}
\] (8)

namely a new approximation of the flow on \( i \) as follows:

\[
Qa(z, t) = \sum_{j=1}^{nQ} (Q_j(t) + \sum_{k=1}^{j} Q_{fk}) N_{Q_j}(z)
\] (9)

The derivatives w.r.t. space of (9) and (5) are

\[
N'_{Q_j}(z^Q_j) = \sum_{j=1}^{nQ} dN_{Q_j}(z^Q_j)(\dot{Q}_j(t) + \sum_{k=1}^{j} Q_{fk})
\] (10)

\[
N'_{H_j}(z^H_j) = \sum_{j=1}^{nH} dN_{H_j}(z^H_j)(H_j(t))
\] (11)

Then, equations (1) and (2) becomes the following ODE system:

\[
\dot{\bar{Q}}_i(t) = a_1 \left[ \sum_{j=1}^{nH} N'_{H_j}(z^H_j) H_j(t) + J_S + J_U \right]
\] (12)

\[
\dot{H}_i(t) = a_2 \left[ \sum_{j=1}^{nQ} N'_{Q_j}(z^Q_j) (\dot{Q}_j(t) + \sum_{k=1}^{j} Q_{fk}) \right]
\] (13)

with \( a_1 = -gA \) and \( a_2 = \frac{h^2}{2A} \).

III. MULTI-LEAKS DETECTION

A. Persistent Inputs for leak detection

It has previously been shown that the detection of multiple leaks can not be achieved in steady state only using flow rates measurements at the ends of the pipeline, because multiple leaks generates the same steady state flow at the ends with an infinite leaks cases as solution. However, a transient fluid analysis in time domain with respect to the leaks parameters given in [9] concluded that the transient responses allow the leaks identification.

We propose here, persistent inputs to generate transient responses in order to detect multiple leaks from the flow rates at the ends of the pipeline.

Fig. 1 shows the flow rates responses of the model in the \( nth \)-collocation point (\( Q_n(t) = Q(L, t) \)) for two different cases of a single leak given in Table I. The boundary conditions are constant pressures at the ends of the pipe, \( H_{in}(t) = 7m \) and \( H_{out}(t) = 1m \). Since the outflow is similar for both leaks, by conservation of the mass the flow rate on the \( nth \)-collocation point in steady state is the same for both leak cases, \( Q_n(t) = Q_n'(t) \). Under this steady condition, a leak can not be identified by direct estimation.

If one is interested by an estimation from the flow rates in order to locate leaks, the inputs must satisfy some property of persistency [18]-[19], namely here:

\[
\exists \tau > 0: \forall \tau' \neq \tau \text{ or } \forall \lambda \neq \lambda'
\]

\[
Q_{a}(z, t) = \sum_{i=1}^{nQ} (Q_i(t) + \sum_{k=1}^{i} Q_{fk}) N_{Q_i}(z)
\]

\[
H_{a}(z, t) = \sum_{i=1}^{nH} (H_i(t) + \sum_{k=1}^{i} H_{fk}) N_{H_i}(z)
\]

\[
\bar{Q}_i = Q_i - \sum_{k=1}^{i} Q_{fk}
\]

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\]

\[
N'_{Q_j}(z^Q_j) = \sum_{j=1}^{nQ} dN_{Q_j}(z^Q_j)(\dot{Q}_j(t) + \sum_{k=1}^{j} Q_{fk})
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\]

\[
H_{a}(z, t) = \sum_{i=1}^{nH} (H_i(t) + \sum_{k=1}^{i} H_{fk}) N_{H_i}(z)
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\]
TABLE III
BOUNDARY CONDITIONS FOR SIMULTANEOUS LEAK CASES

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>$H_{in}(t)$ (m)</th>
<th>$H_{out}(t)$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC1</td>
<td>$7 + 0.1\sin(t)$</td>
<td>$1 + 0.1\sin(t)$</td>
</tr>
<tr>
<td>BC2</td>
<td>$7 + 0.5\sin(t)$</td>
<td>$1 + 0.5\sin(t)$</td>
</tr>
<tr>
<td>BC3</td>
<td>$7 + 0.5\sin(t)$</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 2. Flow rates of a couple of leaks with different persistent inputs

B. Observer Conception

In order to detect two leaks (for instance) in a pipeline system, the OC model given by equations (12)-(13) can be used to design an observer which can estimate the respective positions and coefficients of both leaks.

Taking the pressure ends as boundary conditions, if the objective is the detection of sequential leaks, it’s enough to use a model with three collocation points for the pressure ($n_H = 3$). Although, if the aim is the detection of simultaneous leaks, it must be $n_H > 3$. Therefore for the observer design, we choose an OC model with $n_H = n_Q = 4$, where the collocation points are: $z_1 = 0$ representing the beginning or left side of the pipeline, $z_4 = L$ representing the end, $z_2 = z_{f_1}$ and $z_3 = z_{f_2}$ representing the positions of the two possible leaks.

We assume the pressures at the ends of the pipeline as inputs of the system, thus the input vector is given by $u = [H_{in}(t), H_{out}(t)]^T$, where $H_{in}(t) = H_1(t)$ and $H_{out}(t) = H_4(t)$ for four collocation points.

Let us denote by $x = [\tilde{Q}_1(t) \ 	ilde{Q}_2(t) \ 	ilde{Q}_3(t) \ 	ilde{Q}_4(t) \ H_2(t) \ H_3(t)]^T$ the state vector, where $Q_1(t) = Q_{in}(t)$ and $Q_4(t) = Q_{out}(t)$ are the flow rates at the ends of the pipeline and are assumed to be the measurable outputs of the system $y = [Q_{in}(t) \ Q_{out}(t)]^T$.

If the unknown leak positions ($z_{f_1}$, $z_{f_2}$) and the unknown leak coefficients ($\lambda_{f_1}$, $\lambda_{f_2}$) are added as “new” state variables to the state vector ($x_7, x_8, x_9, x_{10}$), the obtained extended model can be used to design an observer on the basis of an extended Kalman filter (as in [24] for instance).

The extended model can be represented by the following state representation:

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = h(x(t))$$

then an observer can be designed as follows:

$$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t)) + K(t)[y(t) - h(\hat{x}(t))]$$

where the state estimate is denoted by $\hat{x}(t)$ and the observer gain $K(t)$ is a time-varying $q \times m$ matrix computed as

$$K(t) = P(t)C^T(t)R^{-1}$$

with

$$\dot{P}(t) = (A(t) + \alpha I)P(t) + P(t)(A^T(t) + \alpha I) - P(t)C^T(t)R^{-1}C(t)P(t) + W$$

and a positive real number $\alpha > 0$.

IV. SIMULATION RESULTS

In order to illustrate the observer performance, three simulation tests are realized. The aim of the first test is to show the detection of a single leak, its position as well as its leak coefficient. The second test aims to show the observer performance in the detection of sequential leaks. The final test is realized in order to present the performance of the estimator when two simultaneous leaks occur in the pipeline.

In the simulation tests, the pipeline system is represented by an OC model with $n_H = n_Q = 5$ and the following physical variables: $g = 9.8 m/s^2$, $L = 132.56 m$, $b = 1250 m/s$, $D = 0.105 m$, $f = 0.005$. The observer parameters are tuned with the following values: $W = I$, $R = I$ and $P(0) = I$, where $I$ is the identity matrix. The convergence rate parameter is chosen $\alpha = 0.3$. The inputs applied are BC2 given in Table III. Noise has been added to the outputs to represent the measurement noise of instruments.

A. Single leak detection test

Three single leak cases are given in Table IV, together with initial conditions for the observer. The estimation results of the observer can be seen in Fig.3 and in Fig.4. The observer state $\hat{x}_7(t)$ converges to the true position $z_{f_1}$ simulated by the model in each case while $\hat{x}_9(t)$ converges to the simulated leak coefficients $\lambda_{f_1}(t)$.

If the unknown leak positions ($z_{f_1}$, $z_{f_2}$) and the unknown leak coefficients ($\lambda_{f_1}$, $\lambda_{f_2}$) are added as "new" state variables to the state vector ($x_7, x_8, x_9, x_{10}$), the obtained extended model can be used to design an observer on the basis of an extended Kalman filter (as in [24] for instance).
TABLE IV
SINGLE LEAK DETECTION

<table>
<thead>
<tr>
<th>Position (m)</th>
<th>Coefficient (m²)</th>
<th>Observer I.C. Position (m)</th>
<th>Observer I.C. Coefficient (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>z_{f1} = 40</td>
<td>λ_{f1} = 0.03</td>
<td>\dot{x}<em>{7}(t) = 20 \dot{x}</em>{8}(t) = 90 \dot{x}<em>{9}(t) = 0 \dot{x}</em>{10}(t) = 0</td>
</tr>
<tr>
<td>C2</td>
<td>z_{f1} = 50</td>
<td>λ_{f1} = 0.02</td>
<td>\dot{x}<em>{7}(t) = 20 \dot{x}</em>{8}(t) = 90 \dot{x}<em>{9}(t) = 0 \dot{x}</em>{10}(t) = 0</td>
</tr>
<tr>
<td>C3</td>
<td>z_{f1} = 60</td>
<td>λ_{f1} = 0.01</td>
<td>\dot{x}<em>{7}(t) = 20 \dot{x}</em>{8}(t) = 90 \dot{x}<em>{9}(t) = 0 \dot{x}</em>{10}(t) = 0</td>
</tr>
</tbody>
</table>

B. Sequential leak detection test

From the first test it can be deduced that the estimated leak coefficients are indicators of the presence or absence of a leak. So, in this second test two sequential leaks are simulated in order to emphasize even more this fact. The first leak has a coefficient λ_{f1} = 0.01 m² and the second one has λ_{f2} = 0.03 m², the results of the observer estimation can be appreciated in Fig.5.

C. Simultaneous leak detection test

In Table V are given two cases of simultaneous couple of leaks and the initial conditions of the observer. The results of the simulation are shown in Fig.6 and Fig.7. In both cases the leaks are located and estimated.

V. CONCLUSIONS AND FUTURE WORKS

The presence of multiple leaks in a close time interval or simultaneously are rare cases, but thinkable considering the human factor. Therefore the feasibility to detect multiple leaks is a challenge in the observation field. Here, we have proposed an estimator based on a spatially discretized model to detect multi-leaks using persistent inputs. The persistency of the inputs allows to distinguish the position leaks from the flow rates taken as measurable system outputs, even if the leaks occur at the same time.

However, there are many difficulties to overcome in order to implement this technique in real life as:

- The generation of persistent signals since these must to have a suitable estimation frequency. But frequency in pipelines is a variable limited by the existing instrumentation.
- The validation of a good unsteady friction model for the pipeline. This model must be frequency-dependent such that it represents the dispersion effects in transient pipe flows. If the flow (or pressure) varies sinusoidally...
Consider the friction can be modeled as in [26].

- The selection of a good model dimension which represents accurately the oscillatory behavior of a real pipe but taking into account that the dimension model is the dimension of the observer. A grand dimension model demands a high computational effort.

Considering these difficulties an experimental validation of this technique is the next step to do immediately, as well as: (i) The comparison of this technique using observers based on models solved by another numerical methods, (ii) changes on the input-output topology, (iii) and the design of better performance observers.

![Multi-leak detection with similar leaks coefficients, C1](image1)

![Multi-leak detection with different leaks coefficients, C2](image2)

### References


