On Design of Noise Resistant Complexity Reduced Decision Feedback Equalizer for Large Delay Sparse Echoes

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Abstract—A modified decision feedback equalizer (DFE) is proposed for efficient equalization of sparse channels with large delay echoes. This new structure can both avoid noise accumulation in large number of filter taps and reduce hardware implementation complexity effectively. Brief analysis and the corresponding symbol rate is in Section II and Section III, respectively. Base on noise accumulation effects and DFE architecture are presented in Section II and Section III, respectively. After pulse shaping, the signal will be transmitted through the multi-path channel and consider time-domain delay-line based multi-path model for simplicity with impulse response of

\[ h(t) = \sum a_{i} \delta(t - \tau_{i}) e^{-j\theta_{i}} \]  \hspace{1cm} (1)

where the \( a_{i} \), \( \tau_{i} \) and \( \theta_{i} \) are the attenuation, delay, and phase rotation of the \( i \)-th echo, respectively and \( \delta(t) \) is the unit impulse response. Under some non-static channel conditions, phase of the echoes may vary with time, which can be represented as \( \theta_{i} = 2\pi f_{d} t \), \( f_{d} \) is the Doppler rotation rate in Hz. The received signal is then match filtered against the (square root Nyquist) transmitted pulse and get complex base-band equivalent received signal as

\[ r(t) = \sum_{\{i\}} I(t - kT_{s}) + n(t) \]  \hspace{1cm} (2)

Where \( c(t) \) includes effects of pulse shaping filter, multi-path channel and the receiver filter; and \( \{i\} \) are independent M-array complex symbols. The component \( n(t) \) is the independent background noise (always modelled as AWGN noise), whose variant is \( E[v(t)^{2}] / 2 = N_{0} \).

For high-speed wireless channels, \( c(t) \) can be divided into pre-echoes (that reach the receiver earlier than the main signal) and post-echoes (reach later than main signal), which will both be reflected in the DFE taps’ complex coefficients after convergence. It often occurs that the pre-echo is strong (which induces a series of large tap coefficients in module) and the post-echo is long and sparse [1]. Besides, the echoes’ delay may not be accurate integer multiple of the symbol period, which acquire the T-spaced equalizer to adapt its nearby one or more taps to counteract them. For example, the HDTV channel will have echoes whose arriving intervals are larger than 700 symbol-intervals, but only less than ten of them are larger than –10dB (relative to the main signal) according to the recommended channel model and the in-field investigation results [5,6].

III. DFE STRUCTURE AND ADAPTIVE ALGORITHM

Typical DFE is composed of feed forward filter (FFF) that is a FIR filter used to cancel pre-echoes and near post-echoes and a feedback filter (FBF) that is an IIR filter to cancel long post-echoes as shown in Fig 1.
A. Equalizer Model Description

Input vector of DFE is represented as \( r_k \), while the output vector is represented as \( z_k \). Before slicing, the equalizer output \( z_k \) of \( k \)-time satisfies

\[
z_k = \sum_{i=0}^{M-1} f_i \cdot r_{k+i} + \sum_{j=0}^{N} g_j \cdot \hat{z}_{k-j}
\]

where \( f_i \) and \( g_j \) are the FFF and FBF coefficients (either real or complex), \( M \) and \( N \) are the corresponding filter tap number. \( \hat{z}_k \) is the transformed version of \( z_k \): slicer output \( \hat{z}_k \), or even \( z_k \) itself. Actually, selection of \( \hat{z}_k \) depends on the mode under which the equalizer is working. For example, \( \hat{z}_k = z_k \) brings with better performance under decision-directed LMS mode than under CMA mode, and vice versa.

B. Adaptive Algorithm

There are many adaptive algorithms that can be employed to adjust the filter taps. Theoretical performance bounds of each algorithm are discussed in contributions as [2,4]. Difference between these algorithms mainly lies in the cost function they used, which ultimately decides the error item used to adjust the equalizer taps to convergence. Selection of adaptive algorithm depends on factors like channel condition, data frame formats, coding mode, system requirements and etc. Actually, the constant modulus algorithm (CMA) and the least mean square (LMS) algorithm are the most popular ones and are employed here in our discussion [3]. The error items are described as

\[
e_k = \begin{cases} 
  z_k - \hat{z}_k & \text{CMA} \\
  \hat{z}_k - z_k & \text{LMS} \\
  \hat{z}_k - z_k & \text{DD-LMS}
\end{cases}
\]

where \( \gamma \) is the Godard’s constant that is decided by \( \gamma = E[|z_k|^2]/E[|\hat{z}_k|^2] \) and DD-LMS referred to the decision direct LMS algorithm, in which the slicer output will be used as the “training sequence” in LMS algorithm.

Feed forward filter (FFF) and the feedback filter (FBF) are both updated by the error item as

\[
\begin{align*}
  f_{k+1} &= f_k + \mu_1 \cdot e_{k-d} \cdot r_{k-d}^{*} / (r_{k-d}^{*} r_{k-d}) \\
  g_{k+1} &= g_k + \mu_2 \cdot e_{k-d} \cdot \hat{z}_{k-d}^{*} / (\hat{z}_{k-d}^{*} \hat{z}_{k-d})
\end{align*}
\]

where \( d \) is the symbol delay of the updating processing that is set for hardware implementation convenience. \( \mu_1 \) and \( \mu_2 \) are the step-sizes adopted for the updating procedure. They may be adopted various values within different taps’ region with considering both convergence speed and convergence precision. Scalars \( (r_{k-d} r_{k-d}^{*}) \) and \( (\hat{z}_{k-d} \hat{z}_{k-d}^{*}) \) are two normalization items, which entitles the adaptive algorithm with finite feasibility in the equivalent updating step-sizes under different signal level according to some contributions.

![Fig.1. Typical DFE Architecture](image)

It can be found that the CMA algorithm is a blind one that needs no training sequence, which is always used in the system that is lack of training symbols, especially in the acquiring phase. The LMS algorithm always has good convergence performance and is with high adaptability if the training information is rich. However, training symbols will also use the expensive bandwidth. Decision direct LMS algorithm uses the slice output to substitute the training symbol, whose effectiveness is based on the assumption that the symbol error ratio (SNR) of slicer output is low enough. Otherwise the algorithm will fail because of the error spread affects.

In the systems like ATSC (Advanced Television Systems Committee) based digital TV receiver, practical option is that CMA algorithm is employed during the first acquiring phase or under conditions that the background noise is severe. The DD-LMS will be start up only when the symbol error ratio is low enough. With considering data frame structure of ATSC standard, the segment synchronization symbols and first 511 symbols of each field synchronization structure of ATSC standard, the segment synchronization symbols and first 511 symbols of each field synchronization segment can be used as training symbols for LMS algorithm (although their ratio is small) to get better convergence under static channel and higher stability for dynamic ones.

IV. NOISE ACCUMULATION IN EQUALIZER COEFFICIENTS

A. Noise Accumulation Analysis

According to (3), the equalizer output of \( k \)-time can be further divided into more components as

\[
z_k = \sum_{i=0}^{M-1} \bar{f}_i \cdot r_{k+i} + \sum_{i=0}^{M-1} (f_i - \bar{f}_i) \cdot r_{k+i} - \sum_{j=0}^{N} (g_j - \bar{g}_j) \cdot \hat{z}_{k-j}
\]

where \( \bar{f}_i \) and \( \bar{g}_j \) are the ideal (or theoretic, if exist) solutions for specific multi-path channel. If possible to reach, this ideal coefficients combination will reward us with best performance. However, we can only approach it with help of adaptive algorithms like CMA and LMS algorithm in practice. It is quite similar in discussion the FFE coefficients.
and the FBF ones. So we only refer to the FFF case in the
next part of discussions with some aggressive assumptions,
which is helpful for analysis.

Differential component \( f_{i,\Delta} = (f_i - \tilde{f}_i) \) is the noise
components caused by adaptive coefficients updating
algorithm or background noise. First, specific \( f_{i,\Delta} \) is
regarded as a random process that satisfies normal
distribution with parameters of \( N \left( \varepsilon_{f,i}, \delta^2_{f,i} \right) \). Under most
conditions that the equalizer is working properly, it is
reasonable to assume that \( f_{i,\Delta} \) is zero mean as
\( \varepsilon_{f,i} = \mathbb{E}[f_{i,\Delta}] = 0 \). However, the variation \( \delta^2_{f,i} \) is decided by
factors like adaptive algorithm type, updating step-sizes,
background noise level and etc.

Secondly, input vector element \( r_{k+M-i} \) of FFF is a decided
value to a specific \( i \). However, it can be seen as satisfying
normal distribution \( r \sim N \left( \varepsilon_{r,i}, \delta^2_{r,i} \right) \) to different samples \( r_i \).
Then specific product \( f_{i,\Delta} \cdot r_{k+M-i} \) is a process satisfying

\[
\sum_{i=0}^{M-1} f_{i,\Delta} \cdot r_{k+M-i} \sim N \left( \varepsilon_{f,i}, \delta^2_{f,i} \right) \tag{7}
\]

Thirdly, the products \( f_{i,\Delta} \cdot r_{k+M-i} \) are in-dependant to each
other for different \( i \). Summation \( \sum_{i=0}^{M-1} f_{i,\Delta} \cdot r_{k+M-i} \) is also a
random process with complicated characters, which is
difficult for accurate induction. However, it can be
simplified here (from view of engineering) as

\[
\sum_{i=0}^{M-1} f_{i,\Delta} \cdot r_{k+M-i} \sim N \left( \varepsilon_{f}, \delta^2_{f} \right) \tag{8}
\]

where \( \varepsilon_{f} = 0 \) and \( \delta^2_{f} \approx M \cdot \mathbb{E}[r_{j,i}^2] \cdot \mathbb{E}[\delta^2_{f,i}] \) \( i \in [0,M) \) and it is
also an approximate normalized process. Here, the channel
background noise effects (variants) are included in the
received signal components \( r_i \).

When it comes to the FBE, similar results are that

\[
\sum_{j=0}^{N} g_{j,\Delta} \cdot \tilde{z}_{k-j} \sim N \left( \varepsilon_{g}, \delta^2_{g} \right) \tag{9}
\]

where expectation \( \varepsilon_{g} = 0 \), \( \delta^2_{g} \approx N \cdot \mathbb{E}[\tilde{z}_{j,i}^2] \cdot \mathbb{E}[\delta^2_{g,j}] \) \( j \in [1,N] \)
and \( \delta^2_{g,j} \) is variant of \( g_{j,\Delta} = (g_j - \bar{g}_j) \).

Now back to (6), ideal solution decided parts of \( \bar{f}_i \) and
\( \bar{g}_j \) result in a decided component of \( z_k \), which is equal to
the transmitted signal without distortion and the background
noise if the definite length effects of FIR filter are not
included. Here, this decided component is recorded as

\[
Z_k = \sum_{i=0}^{M-1} \bar{f}_i \cdot r_{k+M-i} + \sum_{j=1}^{N} \bar{g}_j \cdot \tilde{z}_{k-j} \tag{10}
\]

Based on (8)–(10), the equalizer output \( z_k \) can also be
seen as a random variable for each sample \( k \), which
approximately satisfy

\[
z_k \sim N \left( \varepsilon_k, \delta^2_{dfe} \right) \tag{11}
\]

where \( \varepsilon_k \) and \( \delta^2_{dfe} \) satisfy: \( \varepsilon_k \approx \mathbb{E}[\tilde{Z}_k] \), \( \delta^2_{dfe} = \delta^2_{B} + \delta^2_{R} \approx
M \cdot \mathbb{E}[r_{j,i}^2] \cdot \mathbb{E}[\delta^2_{f,i}] \mathbb{E}[\delta^2_{g,j}] \mathbb{E}[\delta^2_{B,j}] \mathbb{E}[\delta^2_{R,j}] \). In different
system, \( Z_k \) means not the same, e.g. \( Z_k \) will be quite similar
to the transmitted pulse in PAM system while \( Z_k \) has more
distortion to transmitted version in VSB system.

To most commonly used modulation methods, Symbol
Error Ratio (SER) of equalizer output \( z_k \) satisfies

\[
P_e \approx Q \left( \alpha \sqrt{\beta} \frac{1}{\varepsilon_k / \delta^2_{dfe}} \right) \tag{12}
\]

where \( \alpha, \beta \) and \( \chi \) are constants that is decided by the
modulation way. \( \alpha, \beta \) and \( \chi \) are all larger than zero,
which implies that \( P_e \) has same trend with \( (\varepsilon_k / \delta^2_{dfe}) \).

It should be noted that inductions above are not strict ones
and some assumptions are even not quite right under some
conditions. However, it is these assumptions that reward us
with simplification in analysis and reasonable qualitative
(instead of quantitative) conclusions that we need.

B. Large Numbers of Small Coefficients Degradation

As discussed above in (11) and (12), the equalizer
performance (mainly reflected by SER) has tight
relationship with the distribution parameters of \( z_k \). To
reduce the SER, we have three options: 1) set \( Z_k \) to the
expected point (which is always decided by the system
factors like modulation way and etc.); 2) Reduce the level of
each \( \delta^2_{f,i} \) and \( \delta^2_{g,j} \) , which is decided by adaptive updating
algorithm mainly; 3) Reduce the number of FFF and FBF
taps to reduce \( M \) and \( N \) (however, these parameters
decide the equalizer capability, which can not be changed
easily). Besides, items \( \mathbb{E}[r_{j,i}^2] \) and \( \mathbb{E}[\tilde{z}_{j,i}^2] \) are also decided for
a specific system under objective channel condition.

In discussion of section II, sparse channel with strong pre-
echoes and long delay post-echoes requires the DFE to be
equipped with large number of taps. Noise component \( \delta^2_{dfe} \) of (11) will be large (because \( M \) and \( N \) are large) and
the SER will be high according to (12).
C. Vestigial Noise in Dynamic Coefficients Updating

The above two sections only consider static (or slow fading) channel condition. When the channel is with dynamic echoes that are caused by reflection of moving objects like flying plane and passing buses, solution of DFE coefficients is then also time-variant. Under this condition, it is found in our research that the variants $\delta_{F,i}^{2}$ and $\delta_{G,j}^{2}$ will become larger, the mean $\varepsilon_{F,i}$ and $\varepsilon_{G,j}$ may deviate from zero for some specific taps that is used to cancel the dynamic echoes. We call this phenomenon Vestigial Noise Magnification (VNM). The VNM phenomenon is quite adverse to the whole system and can be relieved (not removed) by well-chosen combination of DFE parameters.

V. NOISE RESISTANT COMPLEXITY REDUCED DFE

Based on analysis above, new design of DFE is presented in this section, which can remove noise accumulation effects in filter taps effectively. Besides, this novel algorithm also rewards us with complexity reduction in hardware implementation. So we call this equalizer noise resistant complexity reduced DFE (NRCR-DFE).

A. Noise Resistant Design Mechanism

Refer to (12), one effective option to reduce the SER is to reduce to number of filter taps ($M$, $N$), whose selection is based on system requirement and can’t be cut easily. However, actual number of coefficients employed in $z_k$ calculation can be reduced, which brings us similar gifts.

Filter taps are divided into two groups: “active” ones with relative large coefficients and “inactive” ones with noise like small coefficients. Only the active tap coefficients are included in output calculation

$$z_k = \sum_{i \in \Omega} f_i \cdot r_{k-M+i} + \sum_{j \in \Psi} g_j \cdot \tilde{z}_{k-j}$$

(13)

where $\Omega$ and $\Psi$ are the active sets for FFF and FBF taps with numbers of elements $M_i$ and $N_i$, respectively. Under sparse channel condition, $M_i$ and $N_i$ can be much smaller than $M$ and $N$. Now the problem lies in designing a smart robust mechanism for active taps selection.

Initialization: 15%~30% of total filter taps are set to active sets randomly. This percent selection is based on the scale of the equalizer and channel condition. Larger number of filter taps and sparser channel admit lower percentage.

Tap exchanging between active and inactive groups: during coefficients updating, some taps in the active group will be smaller (in module) than some in the inactive one. So we get the smallest and largest tap in the active and inactive groups, respectively. If the former is smaller than the later, the smaller active tap will be substituted by the larger inactive tap and abandoned to the inactive group. This exchange is easily performed by setting the corresponding active/inactive flags. To reduce the complexity of module comparison, the taps can be divided sub-groups and only comparison of one sub-group is performed for each updating circle. If no exchange occurred for one circle, the comparison results will be added to the next circle’s comparison progress until exchange occurs.

Abandoned tap coefficients’ flushing: abandoned tap (from active to inactive group) is flushed to zero (for each of its’ components) and is “clear” for next circle’s updating. This flushing processing can be seen as a dynamic noise removal procedure and is an inseparable part of algorithm.

B. Brief analysis for the algorithm

Noise removal ability: if a specific tap is with small ideal solution $f_i$ ($g_j$), the differential component $f_{i,\Delta}$ ($g_{j,\Delta}$) is a random variable whose module changes frequently. If $f_i$ ($g_j$) belongs to the active group, there must be time that it becomes smaller than some of that in the inactive group. Then it is abandoned to the inactive group with flush (noise removal). When vestigial noise occurs (for dynamic channel) in adaptive updating, similar is the case. Thus, the “noise” coefficients in the active group can periodically cleared and smaller number of active ones take part in the equalizer output calculation ($M_i < M$ and $N_i < N$), which reduces the effects of noise accumulation greatly.

Adaptability: the in-group comparison, exchanging and flushing processes operate during all DFE working phase. When the channel is fast fading (with dynamic echoes) or the background noise is severe, exchanging and flushing operation occurs frequently during each circle of comparison, which can adapt to the change of channel condition. However, it rarely occurs if the channel is static and is with high signal-to-noise ratio (SNR), which entitles the algorithm with high robustness and stability.

Complexity reduction: number of “expensive” multipliers needed in (13) is obviously much smaller than that in (3). This is quite important when equalizer scale is large. Why not enjoy the “free lunch” with only definite cost of additive control module that is quite cheap compared to multipliers.

VI. SIMULATION RESULTS

In this section, ATSC [5] based 8-VSB terrestrial broadcasting system (10.76M-Hz symbol-rate on 6MHz
bandwidth) is considered and the receiver is equipped with either conventional or NRCR-DFE with same number of filter taps $M=N=512$. Adaptive algorithm parameters are both optimized for comparison objectivity.

**A. NRCR-DFE configuration and channel selection**

The number of active taps in NRCR-DFE is set as $M=112$ and $N=80$ for FFF and FBF, respectively. Both FFF and FBF taps are divided into 32 sub-groups and only one group of 16 taps is involved in module comparison for one symbol interval, which needs small resource costs.

The multi-path channel selection is based on ATSC and ATTC test recommend guidelines documents [5,6], which satisfies the large delay sparse echoes character.

**B. Numeric results**

Fig.2 presents performance comparison for conventional and NRCR DFE under AWGN and single static echo multi-path condition, i.e. static echo is with 3dB D/U ratio, 20us delay and 0.05Hz Doppler rate for all phase coverage.

![Performance comparison under AWGN and static echo channels](image)

Fig.2. Performance comparison under AWGN and static echo channels

When it comes to multi-path with more than one echo, similar performance gains are reached. To the ATSC R2.1 assembles model with several echoes at 25dB AWGN level, conventional DFE can work properly (for TOV [5]) under 1# and 2# assembles with the variable echo index degraded, while NRCR-DFE can work properly for all 1#–4# assembles with no index degradation. For ATTC assemble and single dynamic echo cases, NRCR-DFE can remove dynamic ghosts with higher Doppler rate and has higher robustness in long period of operation because VNM effects are greatly relieved in NRCR-DFE architecture. Qualitative result lists are shown in TABLE.I.

Besides, multiplier numbers in NRCR-DFE are about 20% and 16% for FFF and FBF output calculation than conventional ones, which both reduces the hardware cost and power needed greatly.

**TABLE.I**

<table>
<thead>
<tr>
<th>ATSC R2.1</th>
<th>ATTC</th>
<th>3us Dynamic echo with D/U ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1# 2# 3# 4# 5#</td>
<td>2# 1.5x 2# 1.8x 1.4x 1.3x 1.5x 3x</td>
<td></td>
</tr>
<tr>
<td>TA</td>
<td>X</td>
<td>Maximum Doppler rate (in Hz) that the conventional DFE can counteract</td>
</tr>
<tr>
<td>TB</td>
<td>○ ○ × × X</td>
<td>Notes: TA for NRCR-DFE and TB for conventional DFE</td>
</tr>
</tbody>
</table>

Notes: 1.TA for NRCR-DFE and TB for conventional DFE 2.○: index degraded satisfaction; ×: not working; √: working

**VII. CONCLUSION**

In this paper, an equalizer algorithm is presented, in which a group division based noise removal mechanism is carried out with considering long delay sparse multi-path character. Analysis and simulation results show that this NRCR-DFE can both remove (at least relieve) the noise accumulation effects in large-scale equalizers (needed in high-speed wireless communication systems) and reduce the implementation complexity effectively. Benefit from its superiority to conventional DFE, this new structure DFE can surely find good applications in systems like HDTV and wideband mobile communication system.

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