INSIGHTS ABOUT CHILDREN'S UNDERSTANDING OF 2-DIGIT AND 3-DIGIT NUMBERS

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Five interpretive place value tasks were added to the *Early Numeracy Interview* (ENI) to gain further insight about students' construction of conceptual knowledge associated with 2-digit and 3-digit numbers. The researchers hypothesised that even though some students were successful at reading, writing and ordering numbers, interpreting multi-digit numbers for problem solving remained a struggle for them. Analyses of students' responses showed that the new tasks distinguished students who previously were assessed as understanding 2-digit or 3-digit numbers, but who could not identify 50 or 150 on a number line or state the total of collections reduced or increased by ten. The new tasks assist teachers to identify students who need further instruction to fully understand 2-digit and 3-digit numbers.

Introduction

Most children learn to read and write 2-digit and 3-digit numbers fairly easily, but interpreting the cardinal value of these numbers is the greater challenge. Research during the *Early Numeracy Research Project (ENRP)* in Australia (Clarke et al., 2002) found that being able to read, write, order and interpret 2-digit numbers was a difficult growth point for young children to reach. In a later study involving over 7000 Victorian primary students, Gervasoni, Turkenburg, & Hadden (2007) also highlighted the number of students in Grades 2–4 who were yet to fully understand 2-digit numbers. If we are to improve young children's whole number learning it is important to understand the challenges children face in coming to understand multi-digit numbers. This is the issue explored in this paper that reports on the refinement of the *ENRP Early Numeracy Interview (ENI)* and framework of Growth Points (Clarke et al., 2002) as part of the *Bridging the Numeracy Gap Project* (Gervasoni et al., 2010). The research team aimed to refine and extend the *ENI* and associated Growth Points, originally designed for use in the first three years of schooling, to address issues such as the Place Value dilemma, and so that they were more appropriate for assessing students across all primary school years. The aspect of the research reported here is the refinement of the assessment tasks for Place Value Growth Point 2 (GP2) — reading, writing, ordering and interpreting 2-digit numbers, and Place Value Growth Point 3 (GP3) — reading, writing, ordering and interpreting 3-digit numbers.

Early Numeracy Interview and Growth Points

The *Early Numeracy Interview* (ENI*)* developed as part of the *Early Numeracy Research Project* (Clarke, Sullivan, & McDonough, 2002), is a clinical interview with an associated research-based framework of Growth Points that describe key stages in the learning of nine mathematics domains. Teachers reported that the *ENI* provided insights about students that might otherwise remain hidden (Clarke, 2001). The data discussed in this paper were drawn from the ENI and Growth Point Framework, so both need to be understood.

The principles underlying the construction of the Growth Points were to: describe the development of mathematical knowledge and understanding in the first three years of school in a form and language that was useful for teachers; reflect the findings of relevant international and local research in mathematics (e.g., Steffe, von Glasersfeld, Richards, & Cobb, 1983; Wright, Martland, & Stafford, 2000); reflect, where possible, the structure of mathematics; allow the mathematical knowledge of individuals and groups to be described; and enable a consideration of students who may be mathematically vulnerable. The processes for validating the Growth Points, the interview items and the comparative achievement of students are described in full in Clarke et al. (2002). The following are the growth points for the domain of Place Value.

1. Reading, writing, interpreting and ordering single-digit numbers.
2. Reading, writing, interpreting and ordering two-digit numbers.
3. Reading, writing, interpreting and ordering three-digit numbers.
4. Reading, writing, interpreting and ordering numbers beyond 1000.
5. Extending and applying Place Value knowledge.

Each growth point represents substantial expansion in knowledge along paths to mathematical understanding (Clarke, 2001). The whole number tasks in the interview take between 15-25 minutes for each student and are administered by the classroom teacher. There are about 40 tasks in total, and given success with a task, the teacher continues with the next tasks in a domain (e.g., Place Value) for as long as the child is successful. Children's responses are recorded on a detailed record sheet.

The challenge of understanding multi-digit numbers

Many studies have provided insight about the challenges involved in understanding and using multi-digit numbers. One important finding is that children who have not constructed grouping and Place Value concepts often have difficulty working with multi-digit numbers (Baroody, 2004). Another finding is that being able to interpret numerals to order them from smallest to largest is another difficulty for some children. Griffin, Case, and Siegler (1994) observed that this involves integrating the ability to generate number tags for collections, and make numerical judgments of quantity based on the construction of a mental number line (Griffin & Case, 1997; Griffin et al., 1994).

Grouping and place value concepts

Studies have found that successful problem solving with 2-digit numbers depends on children's ability to construct a concept of ten that is both a collection of ones and a single unit of ten that can be counted, decomposed, traded and exchanged for units of different value (e.g., Cobb & Wheatley, 1988; Fuson et al., 1997; Ross, 1989; Steffe, Cobb & von Glasersfeld, 1988). Cobb and Wheatley (1988) found that some children develop a concept of ten that is a single unit that cannot be decomposed, and proposed that this type of concept is constructed when children learn by rote to recognise the number of tens and ones in a numeral, but do not recognise that the face value of a numeral represents the cardinal value of a group.

Fuson et al., (1997) identified five different correct conceptions of 2-digit numbers and one incorrect conception that children use, several of which may be available to a given child at a particular moment and used in different situations. These six conceptions provide researchers with a detailed model to analyse children's use of 2-digit numbers and were considered by researchers when developing the *ENRP* Place Value framework of growth points and the associated *ENI*. However, for the *ENRP*, researchers opted for a less complex model than the Fuson et al. model that they hoped would be more user-friendly for teachers. Ten years on, in refining the *ENRP* assessment interview and framework of growth points as part of the research reported in this paper, it seems important to consider whether the Fuson et al. model better explains the difficulties that some children experience in coming to understand 2-digit, and consequently 3-digit numbers. The six conceptions of 2-digit numbers are explained in detail in Fuson et al. (1997). They are the: Unitary Multi-Digit Conception; Decade and Ones Multi-digit Conception (noticing word parts); Sequence of tens and ones conception (noticing the advantage of counting by tens associated with partitioning in tens); Separate Tens and Ones conception (noticing the number of tens and the number of ones); Integrated sequence-separate tens conception (noticing that the number of tens is linked to the number name); and the Incorrect Single-Digits Conception (viewing each digit as representing ones).

Fuson et al. (1997) contend that for full understanding of number words and their written symbols, children need to construct all five of the correct multi-digit conceptions, with the Integrated Sequence-Separate Tens Conception being the most sophisticated understanding. This requires considerable experience and time. Thus, we believe that the refinement of the *ENI* needs to ensure that teachers can identify students who can use the Integrated Sequence-Separate Tens Conception of 2-digit numbers. To this end we included three new tasks that require students to demonstrate this understanding when increasing or decreasing a given quantity by ten.

Constructing a mental number line

Another important characteristic of number learning is the forming of a mental number line. Griffin, Case and Siegler (1994) proposed that success in early arithmetic depends on the formation of a mental number line in association with understanding the generative rule that relates adjacent cardinal values (i.e., each adjacent number in the number line is one more or one less than its neighbour); and understanding the consequence of the previous idea: that each successive number represents a set which contains more objects, and thus has a greater value along any particular dimension.

One way to help children develop a mental number line for use in problem solving is to engage them in activities involving an empty number line. This is a strategy widely used in the Netherlands and aims to link early mathematics activities to children's own informal counting and structuring strategies. "The choice of the empty number line as a linear model of number representation up to 100 (instead of grouping models like arithmetic blocks) reflects the priority given to mental counting strategies as informal knowledge base" (Beishuizen & Anghileri, 1998, p. 525). This emphasis in the research literature on the importance of the mental number line and empty number line as a means of interpreting numbers is not reflected in the *ENI* until Place Value Growth Point 5 (GP5). When refining the *ENI* we included two new number line tasks earlier in the interview to determine whether students who reach Growth Point 2 (GP2) and Growth Point 3 (GP3) are able to interpret numbers on a 2-digit and 3-digit number line.

Refining assessment tasks for 2-digit and 3-digit numbers

This paper examines students' place value knowledge and the effect of the five new tasks designed to identify students who were assessed at GP2 or GP3, but who may not interpret successfully the quantitative value of 2-digit and 3-digit numbers. These tasks were added to the ENI as part of a refinement process. The data examined are drawn from 2011 assessment interviews with approximately 2000 Grade 1 to Grade 4 students (5-9 years old) from 42 low SES school communities in Victoria and Western Australia who are part of the *Bridging the Numeracy Gap Project* (Gervasoni et al., 2010). This is a Federal Government funded project aiming to bridge the numeracy gap for low SES and Aboriginal and Torres Strait Islander students, and is collaboration between the 42 school communities, Catholic Education Offices in the regions of Ballarat, Sandhurst, Sale, and Western Australia, and Australian Catholic University. The new tasks are shown in bold in Figure 1 (GP2 tasks) and Figure 2 (GP3 tasks).

Pop-Sticks Bundling Tasks – Interpreting 2-Digit Numbers

Ask the child to unpack the icy pole sticks.

Here are some icy pole sticks in bundles of ten. *(Offer the chance to check a bundle if it seems appropriate).*

Here are some more loose ones. (*Show white card for 36.)*

a) Get me this many (icy pole) sticks. *(If the child starts to count all in ones, interrupt and ask them if they can do it a quicker way with the bundles).*

Tell me how you worked that out.

b) Please put one bundle back. How many sticks are there now? How do you know that?

2-Digit Number Line – Interpreting 2-digit Numbers

Show the child the mauve 2-digit number line card. **Look at this number line. Please tell me the largest number (100).** ***Point to the little mark*****. What number would go here?** ***(50 – acceptable number range is 45-55).*** **Please explain.**

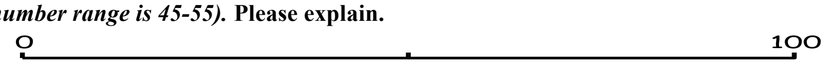

Figure 1. *New Growth Point 2 tasks (in bold). Students' place value knowledge.*

Part b of the *Pop-Sticks Bundling Task* (2-digit), and the *Ten More* and *Ten Less* questions (3-digit) were designed to distinguish those students who use the Integrated Sequence-Separate Tens Conception strategy when interpreting multi-digit numerals. Inclusion of the 2-digit and 3-digit number line tasks reflects the emphasis in the research literature of the importance of students developing a mental number line to interpret quantities when problem solving.

3-Digit Number Line – Interpreting 3-Digit Numbers
(Show the child the white 3-digit number line card.) **Look at this number line. Please tell me the largest number** ***(200.)*** *Point to the little mark.* **What number would go here?** *(150 – acceptable number range is 130-170).* **Please explain.**

Ten More – Interpreting 3-digit numbers
Show the child the white 592 card. Pause for a couple of seconds for the child to look at the number.
Tell me the number that is ten more than this number.

Ten Less – Interpreting 3-digit numbers
Show the child the white 408 card. Pause for a couple of seconds for the child to look at the number.
Tell me the number that is 10 less than this number.

3-digit Chart Task
Show the child the white 3-digit chart card. This is a number chart. Look at the way the numbers go on this number chart. *Point to the shaded square.* Tell me which number goes in this square *(540).* Please explain.

Figure 2. New Growth Point 3 tasks (in bold). Students' place value knowledge.

A key issue for the research reported in this paper was to determine students' Place Value Growth Points, and whether the new GP2 and GP3 tasks identified students who were not successfully interpreting the quantitative value of 2-digit and 3-digit numbers. Figure 3 shows the distributions of ENI Place Value Growth Points at the beginning of the 2011 school year for 1920 Grade 1–4 students. Each student was assessed by their classroom teacher, and the growth points were calculated independently by trained coders to increase the validity and reliability of the data.

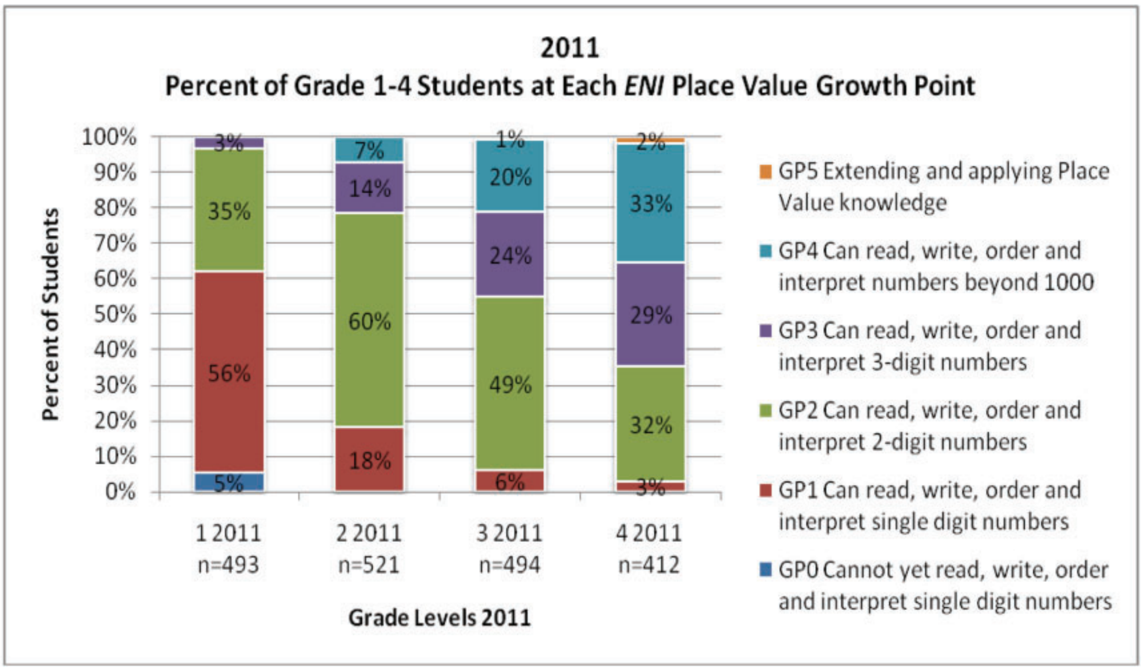

Figure 3. Place Value growth point distribution for Grade 1– 4 students.

An issue highlighted in Figure 3 is the spread of growth points at each level. This has been noted elsewhere (e.g., Gervasoni & Sullivan, 2007; Bobis et al., 2005) and confirms the complexity of the teaching process and the importance of teachers identifying each student's current knowledge and knowing ways to customise learning to meet each student's needs.

The *ENI* data indicate that more than half the Grade 1 students are at GP1, so the initial focus for Place Value instruction for most students is GP2—reading, writing, ordering, and interpreting 2-digit numbers. By the beginning of Grade 2, most students reach GP2. However, by Grade 3, half the students remain on GP2. Examination of the assessment tasks for GP3 and GP4 indicate that students cannot reach these growth points unless they interpret the quantitative value of numbers. We also noted that with the ENI tasks, students could reach GP2 and GP3 successfully using only procedural knowledge to read, write, and order numbers, collect 36 pop-sticks, and identify a 3-digit number on a number chart. The original tasks did not actually require conceptual knowledge to interpret quantities, although conceptual knowledge was assumed.

Analysis of new assessment tasks

Next we examined the data to assess the ability of the new GP2 and GP3 tasks to identify any students who were not interpreting the quantitative value of numbers in the tasks. As the majority of students in Grades 2, 3, and 4 had reached GP2 at least, students in these grades who were assessed at GP2 and GP3 respectively were selected for further examination, and their responses to the two new tasks analysed.

The first new 2-digit task required students to identify the value of a quantity that was reduced by ten (Pop-stick Bundling task). Only students who were judged to be using Fuson et al.'s (1997) Integrated Sequence-Separate Tens Conception strategy were deemed to be successful. This provided confidence that students were able to use all five correct conceptions of 2-digit numbers. The second task required students to interpret a 2-digit number line by identifying the number that was half way between 0 and 100, where a number between 45 and 55 was deemed to be successful.

The data presented in Figure 4 demonstrate that these tasks did identify students who were assessed at GP2, but who did not successfully interpret 2-digit numbers in the *'One Less' Bundle* and *Number Line* tasks.

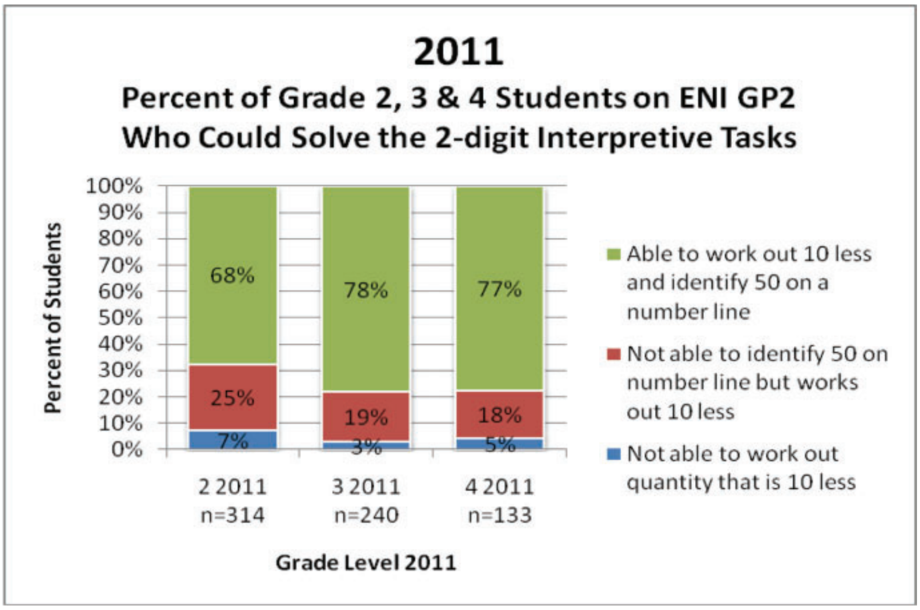

Figure 4. Percent of Gr 2, 3, & 4 students on ENI GP2 who could solve the new 2-digit tasks.

About one third of the Grade 2 students and one quarter of the Grade 3 and Grade 4 students on GP2 were not able to solve both new tasks. This highlights that interpreting 2-digit quantities is an issue for a significant number of students. The number line task was the more difficult of the new tasks. The most common incorrect response was 10, with students counting by ones along the number line until they reached the half-way mark. Of the remaining students who were successful, analysis of their responses to the 3-digit assessment tasks showed that none of these students were successful with the 3-digit interpretive tasks, although many could read, write and order 3-digit numbers. This inability to interpret quantities was the reason why students did not progress to GP3.

Data presented in Figure 5 show that the 3-digit tasks also identified considerable numbers of students who were assessed at GP3, but who could not successfully interpret 3-digit numbers in the 3-digit number line and 10 more/10 less tasks. Only a quarter of the Grade 3 students and 20% of the Grade 2 and Grade 4 students on GP3 were able to solve all 3-digit interpretive tasks. Further analysis showed that the 3-digit *Number Line* task and the *10 Less than 408* tasks were the most difficult of the new tasks. For those students who got three out of the four 3-digit interpretive tasks correct, half were unable to complete the *Number Line* task, and just under half were unsuccessful with the *10 Less* task. Of those students who got only two 3-digit interpretive tasks correct 88% were unsuccessful with the *Number Line* task and 75% were unsuccessful with the *10 Less task*. All these students could read, write and order 3-digit numbers, and all but 5% of these students could successfully complete the original 3-Digit Number Chart task.

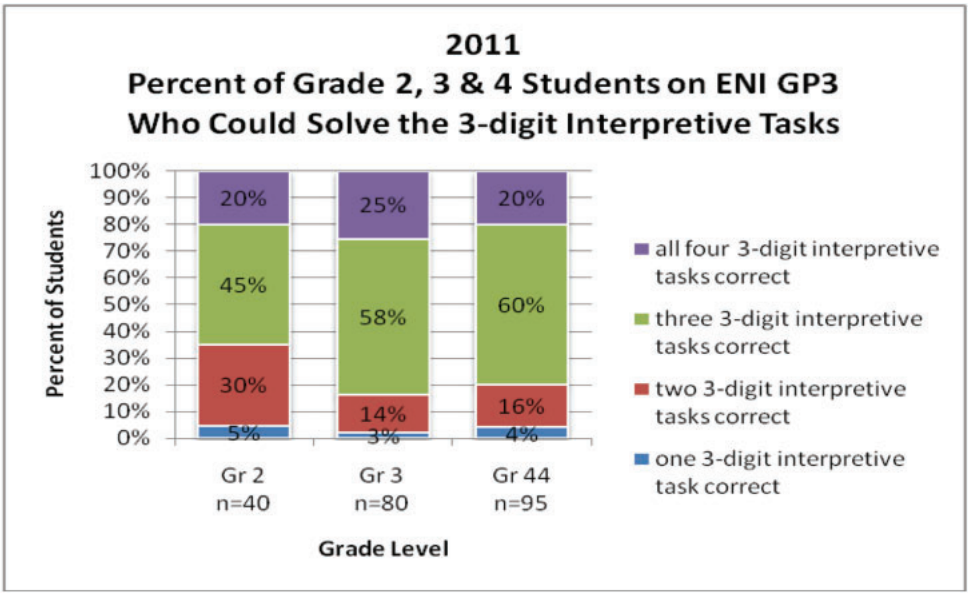

Figure 5. Percent of Gr 2, 3, & 4 students on ENI GP3 who could solve the new 3-digit tasks.

Conclusion

Analysis of 687 Grade 2–4 students' *ENI* responses to the new 2-digit interpretive tasks, and 215 Grade 2–4 student's responses to the new 3-digit interpretive tasks showed that these tasks distinguished students who were assessed as understanding 2-digit and 3-digit numbers respectively, but who in fact could not reliably identify numerals on a number line or state the total of a collection reduced or increased by ten. These additional tasks assist teachers to identify students who need further experience with multi-digit numbers to construct full conceptual understanding, and highlight the importance of teachers focusing instruction on interpreting quantities and developing a mental number line, and not simply reading, writing and ordering numerals. Most children learn to read and write 2-digit and 3-digit numbers easily, but interpreting the cardinal value of these numbers is the greater challenge. Interpretation of quantity and relative quantity are essential for conceptual understanding and problem solving with multi-digit numbers. Perhaps the fact that the *ENI* has not included tasks that identify students who do not fully interpret 2-digit and 3-digit quantities has given teachers an inflated impression of some Place Value GP2 and GP3 students' understanding. We argue that a significant number of these students need further instruction focused on their development of 2-digit and 3-digit number conceptions, including an understanding of quantity, relative quantity and the development of a mental number line.

An implication of these findings is that learning trajectories associated with Place Value and the development of whole number concepts need to adequately account for students' interpretations of quantities. We believe that the *ENRP* Place Value growth points and the associated assessment interview (*ENI*) needs to be modified accordingly, and recommend that the new tasks that were piloted are now included in the *ENI*. Such a refinement will give teachers more certainty about students' current number knowledge and assist them to design more precise instruction.

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