Fuzzy Judgments and Fuzzy Sets

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ABSTRACT

Using fuzzy set theory has become attractive to many people. However, the many references cited here and in other works, little thought is given to why numbers should be made fuzzy before plunging into the necessary simulations to crank out numbers without giving reason or proof that it works to one’s advantage. In fact it does not often do that, certainly not in decision making. Regrettably, many published papers that use fuzzy set theory presumably to get better answers were not judged thoroughly by reviewers knowledgeable in both fuzzy theory and decision making. Buede and Maxwell (1995), who had done experiments on different ways of making decisions, found that fuzzy does the poorest job of obtaining the right decision as compared with other ways. “These experiments demonstrated that the MAVT (Multiattribute Value Theory) and AHP (Analytic Hierarchy Process) techniques, when provided with the same decision outcome data, very often identify the same alternatives as ‘best’. The other techniques are noticeably less consistent with the Fuzzy algorithm being the least consistent.”

Keywords: Analytic Hierarchy Process, Fuzzy Sets, Eigenvector, Multiattribute Decision Making

1. INTRODUCTION

First linked to decision-making problems by Bellman and Zadeh (1970), the use of fuzzy sets in fuzzy multi-attribute decision-making (FMADM) methods is to deal with fuzzy data. Since the first classic FMADM method developed by Bass and Kwakernaak (1977), various FMADM methods have been developed (see Chen & Hwang, 1992; Triantaphyllou & Lin, 1996; Triantaphyllou, 2000; Figueira et al., 2004). In their testing and review, Triantaphyllou and Lin (1996) and Triantaphyllou (2000) found that each of the fuzzy decision-making methods under review yielded different rates of contradiction. In another review, Rao (2007) found that a majority of FMADM methods require cumbersome computations. Furthermore, they often force all elements, including those with crisp numbers in the decision matrix to be in a fuzzy format. This transformation not only goes against the intention of fuzzy set theory (e.g., no subjectivity introduced into precisely-known data) but also increases the computational burden and makes those FMADM methods hard to use.

Our purpose here is to show that whatever the claim of making numbers fuzzy may be,
2. FUZZY, AHP, EIGENVALUE AND EIGENVECTOR

Fuzzy set theory uses the AHP to drive fuzzy priorities that are already obtained by calculating the eigenvector. It relies on using the eigenvalue to improve inconsistence although it is known that a perfectly consistent matrix does not of necessity yield a valid result in that it is a best estimate of underlying measurements when such measurements are known. It is largely the quality of the judgments that determines the validity of the outcome and not their numerical precision. When the matrix is inconsistent we need the eigenvector to derive priorities.

In the field of decision-making, the concept of priority is quintessential and how priorities are derived influences the choices one makes. Priorities should be unique and not one of many possibilities; they must also capture the dominance of the order expressed in the judgments of the pairwise comparison matrix. The idea of a priority vector has much less validity for an arbitrary positive reciprocal matrix than for a consistent and a near consistent matrix. A matrix is near consistent if it is a small perturbation of a consistent matrix. The custom is to look for a vector $w = (w_1, \ldots, w_n)$ such that the matrix $W = (w_i/w_j)$ is “close” to $A = (a_{ij})$ by minimizing a metric. Metric closeness to the numerical values of the $a_{ij}$ by itself says little about the numerical precision with which one element dominates another directly as in the matrix itself and indirectly through other elements as represented by the powers of the matrix. We now show that with the idea of dominance, the principal eigenvector, known to be unique to within a positive multiplicative constant (thus defining a ratio scale), and made unique through normalization, is the only plausible candidate for representing priorities derived from a positive reciprocal near consistent pairwise comparison matrix.

Let $a_{ij}$ be the relative dominance of $A_i$ over $A_j$ in the paired comparisons process. Let the matrix corresponding to the reciprocal
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