A fuzzy nonlinear model for quality function deployment considering Kano’s concept

Liang-Hsuan Chen*, Wen-Chang Ko

Department of Industrial and Information Management, National Cheng Kung University, Tainan, Taiwan, ROC

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Abstract

Quality function deployment (QFD) is a customer-driven approach for processing new product developments in order to maximize customer satisfaction. Each engineering design characteristic is maximized for product performance according to the level of customer satisfaction. To cope with the vague nature of product development processes, fuzzy approaches are used to represent the importance scores of customer requirements (CRs), the relationship between CRs and design requirements (DRs) and relationship between the DRs themselves. Considering Kano’s category of design requirements, this paper extends Chen and Weng’s model (2003), and presents a fuzzy nonlinear model to determine the performance level of each DR for maximizing customer satisfaction, under the same group of constraints as that in Chen and Weng’s model. The results, from an illustrative example, indicate that the total degree of customer satisfaction achieved by the proposed nonlinear model is greater than that by the original fuzzy model.

Keywords: Mathematical models; Quality function deployment (QFD); Fuzzy set; Fuzzy nonlinear model; Kano’s concept

1. Introduction

Quality function deployment (QFD) is a customer-driven approach that allows the needs of the customer to be communicated through the various stages of product planning, design, engineering and manufacturing into a final product. It has been successfully introduced in many industries to improve design processes, customer satisfaction, and to create a competitive advantage [1]. A QFD team collects and treats a set of customer requirements (CRs) for new product development. A number of design requirements (DRs) that affect CRs are also identified and worked out to maximize customer satisfaction. In practice, a QFD team is organized to determine the importance levels of DRs by computing the relationships between CRs and DRs and relationship between the DRs themselves with the importance score of each customer requirement [2,3].

The importance scores of CRs, the relationships between CRs and DRs and relationship between the DRs themselves are usually determined subjectively by ambiguous or vague judgments. However, they are usually treated as crisp variables [4,5]. For example, in traditional practice, the importance score of each CR is set as a crisp value, although linguistic terms seem more adequate for evaluating the CR’s importance. Furthermore, the degree to which a DR affects a CR is expressed on a scale system such as 1-3-9, or 1-5-9, representing linguistic expressions such as
“weak”, “moderate”, and “strong”. However, design engineers usually do not have sufficient knowledge and information about the influence of engineering responses on CRs, due to the lack of information or language hedge from the customer [6]. These considerations have made the applications of fuzzy approaches significant in addressing diversified and imprecise problems in the importance score of each CR and the relationships between CRs and DRs and relationship between the DRs themselves. In applying fuzzy approaches, several studies [7–11] have attempted to deal with the vague nature of linguistic terms by using fuzzy numbers to express the importance score of CRs. Chen and Weng [6] proposed a fuzzy linear programming model to determine the optimal fulfilment degree of each DR at each $\alpha$-cut for achieving the optimal customer satisfaction. Wang [5] presented a fuzzy outranking approach to model the imprecise preference relationships between DRs to achieve customer satisfaction and balanced product design. Temponi et al. [12] developed a fuzzy logic-based extension to QFD for capturing vague requirements. Some models have also been formulated for determining the levels of CRs or DRs based on fuzzy sets theory [8,13–16]. However, these existing studies still suffer from drawbacks in approaches and methodologies, as mentioned by Chen and Weng [6].

In practice, effective design leads to the achievement of maximum customer satisfaction. DRs are able to achieve better design effectiveness when their performance is highly correlated with customer satisfaction. However, the performance of DRs is not necessarily proportionally consistent with customer satisfaction. Kano et al. [17] proposed that quality performance of a product or service has three different relationships with customer satisfaction. Some researchers have incorporated Kano’s idea into QFD for assigning weights to different CRs to explain how CRs impact customer satisfaction in different ways [1,2,18–20]. Unlike the existing literature, in this study we apply Kano’s concept to classify DRs into three categories based on their importance to customer satisfaction, since the performance of various DRs achieve different levels of customer satisfaction. This approach is mainly used to extend the fuzzy linear QFD model in Chen and Weng’s study [6], and a fuzzy nonlinear mathematical programming model is constructed to increase customer satisfaction under the same conditions, such as budget limitations, market competition and technical difficulty constraints. An example of a semiconductor packing case is given to illustrate the proposed model.

In the following section, the fuzzy linear model is introduced. In Section 3, a fuzzy nonlinear model is developed to figure out the performance degrees of DRs that produce maximum customer satisfaction. The way in which DRs are categorized into three classes based on the Kano’s concept is also described. An example is presented to illustrate our approach in Section 4. Finally, the concluding remarks are provided in Section 5.

2. Fuzzy linear model

In practice, an important task in the QFD processes is to determine the importance levels (or scores) of DRs based on the relationships between CRs and DRs and relationship between the DRs themselves, referring to the importance of each CR. The achievement priority or level of DRs is determined based on their importance levels. The associated relationships in QFD are represented in the matrix form as shown in Fig. 1, which is also called the House of Quality.
(HOQ). In the figure, \( R_{ij} \) denotes the relation level in terms of score between the CR\(_i\) and DR\(_j\), and \( r_{ij} \) is the correlation score between DR\(_j\) and DR\(_j\). The notations \( k_i \) and \( W_j \) represent the importance score and rating for the CR\(_i\) and DR\(_j\), respectively, and \( \sum_i k_i = 1 \).

Considering the correlations among DRs, Wasserman [21] proposed a normalization formula (1) to calculate the normalized relationship value between CRs and DRs:

\[
R_{ij}' = \frac{\sum_{l=1}^{J} R_{il}r_{lj} \sum_{j=1}^{J} \sum_{l=1}^{J} R_{il}r_{lj}}{1},
\]

where \( R_{ij}' \) is the normalized relationship value between CR\(_i\) and DR\(_j\), \( i = 1, 2, \ldots, I; j = 1, 2, \ldots, J \), and \( \sum_{j} R_{ij}' = 1 \) for each \( i \). In existing literature [1–3,6,21], the relationships are quantified through the use of a 1-3-9 or 1-5-9 scale to denote low, medium and high relationship degrees between CRs and DRs and relationship between the DRs themselves. In order to reflect the imprecise nature of the relationships, Chen and Weng [6] proposed a fuzzified quantitative formulation as in the following equation:

\[
\tilde{R}_{ij} = \frac{\sum_{l=1}^{J} \tilde{R}_{il} \otimes \tilde{r}_{lj} \sum_{j=1}^{J} \sum_{l=1}^{J} \tilde{R}_{il} \otimes \tilde{r}_{lj}}{1},
\]

where \( \tilde{R}_{il} \) and \( \tilde{r}_{lj} \) are described by linguistic terms and defined as the fuzzy numbers of \([0,1]\), and the symbol of \( \otimes \) represents the fuzzy multiplication operator.

Actually, a fuzzy set can fully and uniquely be represented by its \( \alpha \)-cuts [22]. The \( \alpha \)-cut of the fuzzy set \( \tilde{R}_{il} \) at the \( \alpha \) level, \( \alpha \in [0,1] \), can be denoted by its lower and upper bounds as \( [(\tilde{R}_{il})^L_{\alpha}, (\tilde{R}_{il})^U_{\alpha}] \), which is defined by

\[
(\tilde{R}_{il})^L_{\alpha} = \inf_{x \in [0,1]} \{x \mid \mu_{\tilde{R}_{il}}(x) \geq \alpha\},
\]

and

\[
(\tilde{R}_{il})^U_{\alpha} = \sup_{x \in [0,1]} \{x \mid \mu_{\tilde{R}_{il}}(x) \geq \alpha\},
\]

where \( \mu_{\tilde{R}_{il}}(x) \) is the membership degree of \( x \) belonging to \( \tilde{R}_{il} \). Based on \( \alpha \)-cuts and the extension principle [22–24], the membership function of the fuzzy normalized relationship can be defined by the lower and upper bounds of each \( \alpha \)-cut of \( \tilde{R}_{il} \) and \( \tilde{r}_{lj} \).

In addition, Chen and Weng [6] further proposed a modified formulation to obtain a more precise representation of the fuzzy normalized relationship, of which the lower and upper bounds of the membership function at each \( \alpha \)-cut are formulated as:

\[
m(\tilde{R}_{ij}'^L_{\alpha}) = \frac{\sum_{l=1}^{J} (\tilde{R}_{il})^L_{\alpha} (r_{lj})^L_{\alpha}}{\sum_{m=1}^{J} \sum_{l=1}^{J} (\tilde{R}_{il})^U_{\alpha} (r_{lmi})^U_{\alpha} + \sum_{l=1}^{J} (\tilde{R}_{il})^L_{\alpha} (r_{lj})^L_{\alpha}},
\]

and

\[
m(\tilde{R}_{ij}'^U_{\alpha}) = \frac{\sum_{l=1}^{J} (\tilde{R}_{il})^U_{\alpha} (r_{lj})^U_{\alpha}}{\sum_{m=1}^{J} \sum_{l=1}^{J} (\tilde{R}_{il})^L_{\alpha} (r_{lmi})^L_{\alpha} + \sum_{l=1}^{J} (\tilde{R}_{il})^U_{\alpha} (r_{lj})^U_{\alpha}}.
\]
The above lower and upper bounds of $\alpha$-cuts of fuzzy normalized relationship are aggregated with the importance of each CR to determine the fuzzy technical importance ratings $\tilde{W}_j$ for the $j$th DR in the form of $\alpha$-cuts. Unlike Chen and Weng’s study [6], considering the importance weight of customer requirements as a crisp number, fuzzy numbers are used in this paper, since the fuzzy version seems more appropriate to deal with the inherent imprecision and ambiguity of QFD processes. In determining the fuzzy importance score of CRs, the consensus of customers is important since the DRs should satisfy the CRs maximally. For doing this, Karsak [8] used the fuzzy Delphi method to decide the fuzzy importance weight. This method is also adopted here to determine the fuzzy importance score of CRs as a triangular fuzzy number. Appendix A simply describes the procedures of the fuzzy Delphi method.

$k_i$ as a fuzzy subset in $[0, 1]$ represents the final average of the fuzzy importance score of CR, by experts’ evaluation in the fuzzy Delphi method. In the $\alpha$-cuts aspect, $k_i$ can also be denoted as $[(k_i)_L^\alpha, (k_i)_U^\alpha]$. Once the lower and upper bounds of $\alpha$-cuts of $m(\tilde{R}_ij')$ and $\tilde{k}_i$ are obtained, the fuzzy technical importance ratings $\tilde{W}_j$ for the $j$th DR can be determined in the form of $\alpha$-cuts as

$$(W_j)_\alpha = [(W_j)_L^\alpha, (W_j)_U^\alpha], \quad (5)$$

based on the division operation of positive fuzzy numbers described by the interval arithmetic [22,25]. The resulting $\tilde{W}_j$ is then employed to figure out the optimal fulfillment level of each DR to maximally fulfill the CRs, since the fuzzy set of $\tilde{W}_j$ represents the overall customer satisfaction that can be achieved by DR$_j$.

Using the fuzzy technical importance ratings of DRs, Chen and Weng [6] proposed a fuzzy linear programming model, in which the decision variables, $x_j$, are defined in percentages to denote the level of fulfillment percentage of the DR$_j$, $j = 1, 2, \ldots, J$, i.e. $x_j \in [0, 1]$. The $x_j = 0$, which implies that the DR has a basic design requirement so that no more effort and cost are needed. In Chen and Weng’s model, the increment unit cost to achieve the fulfillment level is represented as a fuzzy number, $\tilde{C}_j$, to reflect its fuzzy nature at the design stage. With the fulfillment percentage of DR$_j$, a corresponding percentage of the increment unit cost is required to enhance the quality of the product or service. The total incremental unit cost cannot exceed a cost constraint. Besides the cost limitation, the impact on customer satisfaction of various DRs is prioritized, and business competitions and technological difficulties are also considered in the model. If DR$_j$ is preferred to DR$_p$ in terms of customer satisfaction, a constraint of $\tilde{W}_s \cdot x_s - \tilde{W}_p \cdot x_p \geq 0$ is needed. In order to solve the fuzzy linear programming model, the lower and upper bounds of $\alpha$-cuts of $\tilde{W}_j$ and $\tilde{C}_j$ are placed in the model to find the lower and upper bounds of $x_j$ and the total customer satisfaction at each $\alpha$-cut. Thus, a fuzzy linear programming model is formulated as:

$$(Z)_L^\alpha = \max \sum_{j=1}^{J} (W_j)_L^\alpha \cdot x_j,$$

s.t. $\sum_{j=1}^{J} (C_j)_U^\alpha \cdot x_j \leq B,$

$$(W_s)_L^\alpha \cdot x_s - (W_p)_U^\alpha \cdot x_p \geq 0,$$

$0 \leq \varepsilon_j \leq x_j \leq \eta_j \leq 1, \quad \forall j,$

$s, p \in \{1, 2, \ldots, J\},$ (6a)

$$(Z)_U^\alpha = \max \sum_{j=1}^{J} (W_j)_U^\alpha \cdot x_j,$$

s.t. $\sum_{j=1}^{J} (C_j)_L^\alpha \cdot x_j \leq B,$

$$(W_s)_U^\alpha \cdot x_s - (W_p)_L^\alpha \cdot x_p \geq 0,$$

$0 \leq \varepsilon_j \leq x_j \leq \eta_j \leq 1, \quad \forall j,$$

$s, p \in \{1, 2, \ldots, J\}.$
where $B$ is the budget limitation, and $(Z)^L_\alpha$ as well as $(Z)^U_\alpha$ represent the lower and upper bounds of objective values, i.e. total customer satisfaction, at each $\alpha$-cut, respectively. In addition, $\varepsilon_j$ and $\eta_j$ denote the possible range of the fulfillment level of one DR, indicating the minimum required level due to the business competition and the maximum level due to technical difficulty.

### 3. A fuzzy nonlinear model

The solutions of Model (6) are the fulfillment levels of DRs in order to maximize total customer satisfaction. From this perspective, the fulfillment levels of DRs are related to product performance, and DRs with the higher technical importance rating will have a greater contribution to total customer satisfaction. In other words, the DRs with equal fulfillment levels achieve different levels of customer satisfaction. For classifying DRs, this paper employs Kano’s concept to categorize DRs into different classes. Kano et al. [13] proposed a useful method in quantifying the relationship between quality performance of a product or service and customer satisfaction. Three kinds of conceptual relationships were presented: attractive, one-dimensional, and must-be relationships. Fig. 2 demonstrates these relationships. For the DRs with an attractive relationship, increasing performance of the product in terms of the fulfillment levels can produce more customer satisfaction than the other DRs. The one-dimensional relationship is a linear function between performance and satisfaction. The must-be relationship is described as when an essential performance of product is enough to satisfy the customers’ need.

Since the fulfillment level of each DR is related to product performance and its fuzzy technical importance rating $\tilde{W}_j$ is closely associated with customer satisfaction, it is reasonable to categorize DRs into three Kano’s classes based on the fuzzy technical importance ratings. The membership function of the fuzzy technical importance rating of each DR is determined by (5). Therefore, the categorization of DRs can be achieved by ranking the fuzzy technical importance ratings $\tilde{W}_j$ of DRs. $\tilde{W}_j$ is a fuzzy number, since it is convex and normal based on (4) and (5).

Accordingly, for the categorization, a fuzzy number ranking approach, proposed by Chen and Lu [26], is employed by this paper to rank the fuzzy ratings. The approach uses the lower and upper bounds of several $\alpha$-cuts of two fuzzy numbers to calculate the average difference between them. For ranking the $j$ and $l$ fuzzy technical importance ratings, the measure for the average difference $D_{j,l}$ by $(n + 1)$ $\alpha$-cuts is formulated as follows [26]:

$$D_{j,l}(\beta) = \frac{1}{n + 1} \left\{ \left[ \beta \sum_{m=0}^{n} (W_j)^U_m + (1 - \beta) \sum_{m=0}^{n} (W_j)^L_m \right] - \left[ \beta \sum_{m=0}^{n} (W_l)^U_m + (1 - \beta) \sum_{m=0}^{n} (W_l)^L_m \right] \right\},$$

where $(W_j)^L_m$ and $(W_j)^U_m$ are the lower and upper bounds of the $m$th $\alpha$-cut of $\tilde{W}_j$; the index $\beta$ is used to reflect a decision maker’s attitude to the lower or upper bound. The value of $\beta$ equal to 0.5 is adopted for a neutral decision. The greater the measure $D_{j,l}(>0)$ is, the greater the average difference that the fuzzy number $j$ is larger than $l$. In
that the possible range of the performance level of one DR can be changed in accordance with EMC effects between adjacent traces; however, it also produces some problems due to the more heat dissipation and good ground shielding to reduce the electromagnetic interference (EMI) and electromagnetic compatibility (EMC) effects between adjacent traces; however, it also produces some problems due to the more

After ranking the fuzzy technical importance ratings \( \tilde{W}_j \), we categorize all DRs into three classes in accordance with Kano’s concept. Let 1-class DRs be ones with the higher measure value with respect to the reference, and 3-class be the reverse. Obviously, the performance achieved by the 1-class DRs can increase customer satisfaction by more than the DRs in the other classes. In other words, increasing the performance level by 1-class DRs, and even reducing that by 3-class DRs due to budgetary limitations, can augment the total customer satisfaction. Based on this consideration, we treat the 1-class DRs as attractive requirements, while 3-class DRs are must-be ones. As to the 2-class DRs, they are considered as one-dimensional, playing the same role as in (6).

In order to incorporate the above concepts into constructing the fuzzy model, the decision variable \( x_j \) representing the performance level of DR\( j \) is modified as \( x_j^{q_j} \), where \( q_j = 0.5, 1, \) and 2, when the DR has the attractive, one-dimensional and must-be feature, due to \( x_j \in [0, 1] \). Referring to the model (6), a fuzzy nonlinear programming model to maximize customer satisfaction can be formulated as

\[
\begin{align*}
(Z')^L_\alpha &= \max \sum_{j=1}^{J} (W_j)^L_\alpha \cdot x_j^{q_j}, \\
\text{s.t.} \quad &\sum_{j=1}^{J} (C_j)^U_\alpha \cdot x_j^{q_j} \leq B, \\
\quad &\sum_{j=1}^{J} (C_j)^L_\alpha \cdot x_j^{q_j} - \sum_{j=1}^{J} (C_j)^U_\alpha \cdot x_j^{q_j} \geq 0, \\
\quad &0 \leq \varepsilon_j^{q_j} \leq \eta_j^{q_j} \leq 1, \quad \forall j, \\
\quad &s, p \in \{1, 2, \ldots, J\},
\end{align*}
\]

\[
\begin{align*}
(Z')^U_\alpha &= \max \sum_{j=1}^{J} (W_j)^U_\alpha \cdot x_j^{q_j}, \\
\text{s.t.} \quad &\sum_{j=1}^{J} (C_j)^L_\alpha \cdot x_j^{q_j} \leq B, \\
\quad &\sum_{j=1}^{J} (C_j)^U_\alpha \cdot x_j^{q_j} - \sum_{j=1}^{J} (C_j)^L_\alpha \cdot x_j^{q_j} \geq 0, \\
\quad &0 \leq \varepsilon_j^{q_j} \leq \eta_j^{q_j} \leq 1, \quad \forall j, \\
\quad &s, p \in \{1, 2, \ldots, J\},
\end{align*}
\]

It is noted in (8) that the possible range of the performance level of one DR can be changed in accordance with the modified decision variable. From the application end, the possible performance level of one DR with the attractive feature can be increased, while that with the must-be feature can be reduced for the increment of the total customer satisfaction under the budgetary limitation. After solving the above model at various \( \alpha \in [0, 1] \), the lower and upper bounds of the objective function and each decision variable \( x_j^{q_j} \) can be obtained, and then the associated membership functions are determined, which is useful for the QFD team in decision making.

4. An illustrative example

In this section, a semiconductor packing case of the turbo thermal ball grid array (T²-BGA) package is used to exemplify the applicability of the proposed models. T²-BGA is a member of the ball grid array (BGA) family in microelectronics packaging, and it can provide excellent thermal and electrical performance since a heat slug is inserted into the moulding compound of a plastic BGA (PBGA). The heat slug has the advantages of high heat dissipation and good ground shielding to reduce the electromagnetic interference (EMI) and electromagnetic compatibility (EMC) effects between adjacent traces; however, it also produces some problems due to the more
complex structure of package when compared to PBGA [27]. Fig. 3 displays the cross-sections of PBGA and $T^2$-BGA.

Based on the above considerations of $T^2$-BGA, a QFD team collects five customer requirements (CRs) and proposes five design requirements (DRs) in the earlier design process. The fuzzy relations $\tilde{R}_{ij}$ between CRs and DRs and the relevant relations $\tilde{r}_{ij}$ among DRs are shown in the house of quality (HOQ) in Fig. 4. In HOQ, the CRs are “package profile” (CR$_1$), “thermal performance” (CR$_2$), “electrical performance” (CR$_3$), “reliability” (CR$_4$), and “co-planarity” (CR$_5$), and the five DRs are “heat slug exposed area” (DR$_1$), “heat slug attached material” (DR$_2$), “height of heat slug” (DR$_3$), “copper pattern” (DR$_4$), and “molding flow” (DR$_5$). The importance levels of CRs and relationships are denoted as “very low (VL)”, “low (L)”, “medium (M)”, “high (H)” or “very high (VH)”, which are translated into the triangular fuzzy numbers as a 3-element set, $(0, 0, .2)$, $(0, .2, .4)$, $(.3, .5, .7)$, $(.6, .8, 1)$, $(.8, 1, 1)$, respectively, with membership functions in Fig. 5. As an example, $\tilde{R}_{il} = (.6, .8, 1)$ is formulated as

$$
\mu_{\tilde{R}_{il}}(R_{il}) = \begin{cases} 
(R_{il} - .6), & .6 \leq R_{il} \leq .8, \\
(.8 - .6), & .8 \leq R_{il} \leq 1.
\end{cases}
$$

The middle value in the 3-element set representing the membership function of a fuzzy number is the most likely one with the membership degree equivalent to 1, such as $\mu_{\tilde{H}}(.8) = 1$. In this study, the system of 0-.2-.5-.8-1 is used to express the most likely value of the linguistic terms on VL-L-M-H-VH. Based on this system, the fuzzy Delphi method is used to obtain the fuzzy importance score $\tilde{k}_i$, and finally they are calculated as $(.038, .238, .438), (.263, .463, .663), (.113, .313, .513), (.55, .75, .925), (0, .15, .35)$ for the five CRs, respectively. The relevant data and computational procedures are listed in Appendix B. Furthermore, considering the linguistic relationships in Fig. 4, the fuzzy normalized relationship at each $\alpha$-cut can be calculated using (4). For doing this, the lower and upper bounds of
Fig. 5. The membership functions of linguistic variables in the HOQ.

Table 1
The lower and upper bounds of fuzzy technical importance ratings at different \( \alpha \) levels

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( W^L_1 )</th>
<th>( W^U_1 )</th>
<th>( W^L_2 )</th>
<th>( W^U_2 )</th>
<th>( W^L_3 )</th>
<th>( W^U_3 )</th>
<th>( W^L_4 )</th>
<th>( W^U_4 )</th>
<th>( W^L_5 )</th>
<th>( W^U_5 )</th>
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<tbody>
<tr>
<td>1.0</td>
<td>0.168</td>
<td>0.168</td>
<td>0.233</td>
<td>0.233</td>
<td>0.052</td>
<td>0.052</td>
<td>0.315</td>
<td>0.315</td>
<td>0.094</td>
<td>0.094</td>
</tr>
<tr>
<td>0.8</td>
<td>0.116</td>
<td>0.280</td>
<td>0.182</td>
<td>0.354</td>
<td>0.037</td>
<td>0.092</td>
<td>0.238</td>
<td>0.517</td>
<td>0.063</td>
<td>0.159</td>
</tr>
<tr>
<td>0.6</td>
<td>0.074</td>
<td>0.419</td>
<td>0.138</td>
<td>0.498</td>
<td>0.025</td>
<td>0.140</td>
<td>0.169</td>
<td>0.745</td>
<td>0.039</td>
<td>0.243</td>
</tr>
<tr>
<td>0.4</td>
<td>0.040</td>
<td>0.583</td>
<td>0.102</td>
<td>0.668</td>
<td>0.015</td>
<td>0.198</td>
<td>0.110</td>
<td>0.998</td>
<td>0.025</td>
<td>0.349</td>
</tr>
<tr>
<td>0.2</td>
<td>0.016</td>
<td>0.775</td>
<td>0.073</td>
<td>0.862</td>
<td>0.008</td>
<td>0.264</td>
<td>0.059</td>
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</tr>
<tr>
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<td>0.050</td>
<td>1.081</td>
<td>0.002</td>
<td>0.339</td>
<td>0.017</td>
<td>1.584</td>
<td>0.000</td>
<td>0.625</td>
</tr>
</tbody>
</table>

Fig. 6. The membership functions of fuzzy technical importance ratings.

each \( \alpha \)-cut of \( \tilde{R}_{il} \) and \( \tilde{r}_{ij} \) should be determined beforehand based on their membership functions. Actually, the lower and upper bounds of each \( \alpha \)-cut can be represented as the functions of \( \alpha (\in [0, 1]) \). As an example, the \( \alpha \)-cut of the membership function \((0.6, 0.8, 1) \) of “H” is expressed as \([ (R_{il})^{L}_{\alpha}, (R_{il})^{U}_{\alpha} ] = [0.6 + 0.002, 0.8 - 0.2\alpha, 1 - 0.2\alpha] \). Without any biases, the \( \alpha \) levels in this paper are evenly distributed in \([0, 1]\). Let \( \alpha_m \) denote the \( m \)th \( \alpha \) level, and \( \alpha_m = m/n, m \in \{0, 1, \ldots, n\} \), such that the distance between each two adjacent \( \alpha \) levels is equal, i.e. \( \alpha_m - \alpha_{m-1} = 1/n, m \geq 1 \).

The lower and upper bounds of the fuzzy technical importance rating \( (\tilde{W}_j)_{\alpha} \), \( j = 1, 2, \ldots, J \), at each \( \alpha \) level, is obtained by (5) to determine the importance priority of each DR. The lower and upper bounds of \( (\tilde{W}_j)_{\alpha} \) at different \( \alpha \) levels are listed in Table 1, and their membership functions are shown in Fig. 6. They are applied to (6) and (8) to solve fuzzy linear and nonlinear models.

As described previously, the DRs are categorized into three basic classes based on Kano’s concept. For categorizations, (7) is applied to calculate the measure of difference between two fuzzy technical importance ratings. Visually, \( \tilde{W}_3 \) has the least contribution to customer satisfaction, so it is treated as the reference in the comparisons. Suppose that the neutral decision factor is set, i.e. \( \beta = 0.5 \). After applying (7) based on 11 \( \alpha \)-cuts that \( \alpha \) levels are distributed evenly in \([0, 1]\), the values of the difference measure \( D_{j,3}, j = 1, \ldots, 5 \), are 0.198, 0.269, 0.0, 0.424, and 0.077, respectively. In order to assign each DR into the associated class based on the measure, a relative scale is used.
Table 2
The cost, business competition, and technical difficulty for each DR

<table>
<thead>
<tr>
<th>DR_j</th>
<th>ĉ_j</th>
<th>α-cut</th>
<th>ε_j</th>
<th>η_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>DR_1</td>
<td>(2, 3, 4)</td>
<td>[2 + .1α, 4 − .1α]</td>
<td>.5</td>
<td>1.0</td>
</tr>
<tr>
<td>DR_2</td>
<td>(8, 9, 1)</td>
<td>[8 + .1α, 1 − .1α]</td>
<td>.3</td>
<td>1.0</td>
</tr>
<tr>
<td>DR_3</td>
<td>(4, 5, 6)</td>
<td>[4 + .1α, 6 − .1α]</td>
<td>.1</td>
<td>1.0</td>
</tr>
<tr>
<td>DR_4</td>
<td>(3, 4, 5)</td>
<td>[3 + .1α, 5 − .1α]</td>
<td>.1</td>
<td>1.0</td>
</tr>
<tr>
<td>DR_5</td>
<td>(1, 2, 3)</td>
<td>[1 + .1α, 3 − .1α]</td>
<td>.2</td>
<td>1.0</td>
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</table>

in the following:

\[ D_{j,l}^I = \frac{D_{j,l} - \min D_{j,l}}{\max D_{j,l} - \min D_{j,l}}, \quad j = 1, \ldots, J, \]

where \( l \) is set as the reference. In this case, the minimum among the difference measures is zero, so that the resulting scales are the percentages relative to the maximum, i.e. 46.7%, 63.4%, 0%, 100%, 18.2%. For the assignments, a three-interval system, i.e. 0-33.3%-66.7%-100%, is adopted in this paper based on the assumption that the quantity of the relative difference is closely related to a DR’s feature in the interval manner. Following this rule, the fourth requirement DR_4 is considered the one with the attractive feature; DR_1 and DR_2 have one-dimensional features; DR_3 and DR_5 have must-be features.

In addition to incorporating the Kano’s concept, the fuzzy nonlinear model also considers the business competition and technical difficulty of each DR as well as the required cost when one DR’s performance level is fully achieved, i.e. 100%. As with the fuzzy linear model [6], the required increment unit cost is represented as a fuzzy number. For comparisons, Table 2 lists the associated data for the fuzzy linear model. To be in accordance with the above Kano’s categorization, the corresponding business competition is changed as \( ε'_1 = ε_1 = .5, \quad ε'_2 = ε_2 = .3, \quad ε'_3 = .01, \quad ε'_4 = .32, \quad \text{and} \quad ε'_5 = .04 \), in constructing the fuzzy nonlinear model. The budgetary total increment cost is limited to 1 unit, i.e. \( B = 1.0 \) in (6) and (8). Furthermore, we suppose that the QFD team has prioritized DRs as \( DR_4 > DR_1 > DR_2 \) and \( DR_3 > DR_5 \) for the design preference, where “>” means “is more preferred than”.

Applying fuzzy technical importance ratings, fuzzy increment cost, and the associated constraints to (6) and (8) at several \( α \) levels, the performance level of each DR and the total customer satisfaction degree (the objective function value) can be obtained from each model.

Both models (Model (6) and (8)) are solved by Lingo 9.0 software in a Microsoft Windows XP environment running on a laptop with a 1.5 GHz processor as well as a 512 MB random access memory. The efficiency is satisfactory due to the software runtime close to zero seconds for each \( α \)-cut. In this example, the settings of \( α \) levels less than 0.9 and 0.85 will produce infeasible regions in solving fuzzy linear and nonlinear models, respectively. From the property of a fuzzy number, this means that the confidence level of fuzzy inputs should be high to avoid producing the wider ranges of \( α \)-cuts. Fig. 7 shows the membership functions of the total customer satisfaction degree for the two models based on three \( α \) levels (0.9, 0.95, 1). Obviously, the membership function constructed from the fuzzy nonlinear model is greater than that from the fuzzy linear model, indicating that incorporating the Kano’s concept into the model can enhance the total customer satisfaction. The range of the performance level of each DR at the corresponding \( α \) level can also be obtained from (6) and (8). However, for application purposes, the crisp performance level should be taken as the action by the QFD team in the design.

For doing this, a crisp performance level belonging to the \( α \)-cut of each DR at a particular \( α \) level that the QFD team accepts can be adopted. In fact, with the fuzzy sense, the \( α \) level can be interpreted as the confidence degree [28–31]. Based on the acceptable confidence degree, the crisp performance level of \( DR_j \) from the fuzzy nonlinear model can be determined as

\[ (x_j^q)_{α} = β(x_j^q)^{L} + (1 − β)(x_j^q)^{U}, \]

where \((x_j^q)^{L}\) and \((x_j^q)^{U}\) are the lower and upper bounds of \( DR_j \) at the \( α \) level, respectively; the index \( β \) is used to reflect the QFD team’s attitude to the lower or upper bound. Suppose that the QFD team’s attitude is neutral and \( α \) level = 0.9 is accepted for the design purpose. Then the crisp performance levels of the five DRs at \( α = 0.9 \), \((x_j^q)_{α=0.9} \), \( j = 1, \ldots, 5 \), are 0.905, 0.314, 0.075, 1.0, and 0.04, respectively; while, based on the fuzzy linear model (6), they are...
0.65, 0.3, 0.392, 0.751, and 0.2. It is obvious that the crisp performance levels of DR\(_3\) and DR\(_5\) from (8) are less than those from (6) by 0.317 and 0.16 respectively, since they are classified as the must-be features. With the attractive feature, the DR\(_4\) level is increased by around 0.25 from 0.751 to 1.0. As to DR\(_4\) and DR\(_5\) with the one-dimensional feature, they are also increased, in comparison with those from (6), to make up the less performance levels of DR\(_3\) and DR\(_5\) in order to maximize the total customer satisfaction, even though the DR\(_4\) level is larger than that from (6). It is worth noting that DR\(_1\) is increased more than DR\(_2\), because DR\(_1\) has higher priority in design preference and a lower increment unit cost than DR\(_2\). In fact, the change of the performance levels of DRs with the one-dimensional feature is dependent on the quantity change of those DRs belonging to the other two Kano’s categories in maximizing the total customer satisfaction. Based on the modification of the performance level, the total customer satisfaction degree is increased, shown in Fig. 7.

5. Concluding remarks

Determining the performance levels of DRs is an important decision problem in QFD applications. Considering the Kano’s concept, this paper proposes a fuzzy nonlinear model to determine the performance levels of DRs based on the existing fuzzy linear model. The DRs can be categorized into three classes based on the Kano’s concept. The attractive class has the higher impact on customer satisfaction, so that the performance level of one DR in this class is preferably increased. Meanwhile, the must-be class makes less contribution to customer satisfaction, and therefore the performance level can be reduced. As to the DRs in the one-dimensional class, the change of their performance levels is dependent on the quantity change of those DRs belonging to the other two Kano’s categories in maximizing total customer satisfaction. Through the modification of the decision variables denoting the performance level, a fuzzy nonlinear model can be constructed. The illustration of the proposed model with a numerical example indicates that the total customer satisfaction degree is greater than that from the fuzzy linear model. By comparing the performance levels of DRs from the two models, the changes actually reflect the categorizations based on Kano’s concept. Besides Kano’s concept, in future research, the other considerations, such as design risk concept, the integration of more phases of QFD activity, and so on, could be modelled in the QFD problems.

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Appendix A

The fuzzy Delphi method consists of four steps for determining the importance score of CRs, and is described as follows [8,32].
Step 1. The current and potential customers, acting as an expert role $\text{Cust}_m$, $m = 1, 2, \ldots, n$, provide the possible importance score of $\text{CR}_i$, $i = 1, 2, \ldots, I$. The importance score assigned by each expert $\text{Cust}_m$ is presented in the form of a triangular fuzzy number as

$$\tilde{k}_{i,m} = \left[(k_{i,m})^a, (k_{i,m})^b, (k_{i,m})^c\right], \quad m = 1, 2, \ldots, n; \quad i = 1, 2, \ldots, I,$$

where $\tilde{k}_{i,m}$ denotes the importance score of $\text{CR}_i$ by the expert $m$, the indexes “$a$”, “$b$”, “$c$” represent the lowest, the most likely and the highest level of $\tilde{k}_{i,m}$.

Step 2. Computing the average of all $\tilde{k}_{i,m}$, the formulation is expressed as

$$\tilde{k}_i = \left[(\tilde{k}_i)^a, (\tilde{k}_i)^b, (\tilde{k}_i)^c\right] = \left[\frac{1}{n} \sum_{m=1}^{n} (k_{i,m})^a, \frac{1}{n} \sum_{m=1}^{n} (k_{i,m})^b, \frac{1}{n} \sum_{m=1}^{n} (k_{i,m})^c\right],$$

where $i = 1, 2, \ldots, I$. And then, for each expert $\text{Cust}_m$ the differences between $\tilde{k}_{i,m}$ and $\tilde{k}_i$ are calculated and returned to each expert for the next reevaluation run.

Step 3. Each expert takes the differences in the previous step as a reference, and then provides a revised linguistic variable (if necessary) of the importance scores as

$$\tilde{k}'_{i,m} = \left[(k'_{i,m})^a, (k'_{i,m})^b, (k'_{i,m})^c\right], \quad m = 1, 2, \ldots, n; \quad i = 1, 2, \ldots, I.$$ 

Then the average $\tilde{k}'_{i,m}$ can also be obtained from (A.2) and its difference to $\tilde{k}_i$ is calculated using a distance measure, proposed by Bojadziev and Bojadziev [33], as follows:

$$d(\tilde{k}_i, \tilde{k}'_i) = 0.5 \max \left[|\tilde{k}_i^a - \tilde{k}'_i^a|, |\tilde{k}_i^b - \tilde{k}'_i^b|, |\tilde{k}_i^c - \tilde{k}'_i^c|\right].$$

This process could be repeated until two adjacent averages become reasonably close. The consensus criterion is assumed to be a small distance, such as $d \leq 0.2$.

Step 4. If there is any change or update information that may lead to a reevaluation of the importance scores of CRs at a later time, Step 1 should start the next-round process.

Appendix B

The fuzzy Delphi method is applied to determine the importance score of CRs in $T^2$-BGA package development. The computational procedure is described as follows:

Step 1. It is assumed that a QFD team forwards a questionnaire with linguistic evaluation levels, “very low (VL)”, “low (L)”, “medium (M)”, “high (H)” or “very high (VH)”, to eight current and potential customers in order to obtain the importance scores of five CRs, as mentioned in Section 4. As the experts, the eight customers use those five linguistic terms to express the importance score of each CR. Their first evaluations are listed in Table B.1. Then, the QFD team translated those linguistic terms into the triangular fuzzy numbers as a 3-element set, namely (0, 0, .2), (0, .2, .4), (.3, .5, .7), (.6, .8, 1), and (.8, 1, 1), respectively. For example, the importance score of the fifth CR evaluated by the second expert is expressed as $\tilde{k}_{5,2} = (0, .2, .4)$.

Step 2. Computing the average of all $\tilde{k}_{i,m}$ by (A.2), the producing results of $\tilde{k}_i$ to each CR are (.075, .225, .425), (.263, .463, .663), (.113, .263, .463), (.438, .638, .813), and (.038, .138, .338), respectively. Then, the differences between $\tilde{k}_{i,m}$ and $\tilde{k}_i$ are calculated and sent back to each corresponding expert for reevaluation.

Step 3. As Step 1, each expert provides a revised evaluation, as listed in Table B.2. The average of $\tilde{k}'_{i,m}$ can also be obtained from (A.2), and then the $\tilde{k}'_i$ of each CR is determined as (.038, .238, .438), (.263, .463, .663), (.113, .313, .513), (.55, .75, .925), and (0, .15, .35), respectively. The distance between $\tilde{k}'_i$ and $\tilde{k}_i$ is calculated by (A.4) as .025, .05,.113,.052, respectively. Since the distances between two adjacent averages are less than or equal to 0.2, this means that the consensus condition is achieved. The resulting fuzzy number $\tilde{k}'_i$ will be applied to determine the fuzzy technical importance ratings. If the consensus condition was not obtained, the reevaluation process would be asked to perform the next round until the consensus condition is achieved.
Table B.1
The first round evaluation of importance score of each CR

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<tr>
<th>Expert</th>
<th>CR₁</th>
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Table B.2
The second round evaluation of importance score of each CR

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References