BER Analysis in A Generalized UWB Frequency Selective Fading Channel With Randomly Arriving Clusters and Rays

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Abstract— In this paper, we present an analytical method to evaluate the bit error rate (BER) of the ultra-wideband (UWB) system in the IEEE 802.15.4a standardized channel model. The IEEE 802.15.4a UWB channel model is more general and based on more measurements than the earlier IEEE 802.15.3a model. It also poses new challenge in analyzing UWB performance in such a channel. First, the power delay profile become a function of randomly arriving cluster and ray arrival time. Second, the signal amplitude in the IEEE 802.15.4a channel is modeled by a Nakagami random variable of which the Nakagami fading parameter is log-normally distributed. Thus, the signal amplitude is a nonlinear function of a log-normally distributed random variable. By means of counting integral of Lebesgue measure theory, the analytical expression for the BER performance in the IEEE 802.15.4a UWB channel is presented. We apply this analytical model to investigate the impacts of various UWB channel parameters on the system performance and provide some useful insights into the design of UWB transceiver.

Index Terms—Ultra-wideband (UWB), IEEE 802.15.4a channel model, bit error rate (BER).

I. INTRODUCTION

The ultra-wideband (UWB) communications is a promising technique to achieve the objectives of higher data rates and better quality for the modern wireless systems. The complexity of the UWB channel has posed new challenges on performance analysis. Thus, for a more sophisticated channel model such as the IEEE 802.15.4a [1], the performance evaluation of UWB system is mainly performed by simulations. Compared to traditional narrow-band channel, the UWB channel has two important properties. First, the bandwidth of the UWB signals is much wider than the coherence bandwidth of the channel. Thus, the severely frequency selective fading occurs in the frequency domain. Second, The large bandwidth of a UWB signal results in a arrival time with high resolution. Thus, the UWB waves arrive in many clusters and contain some non-Rayleigh multipath components.

A. Motivation and Challenges

Based on measurement results, the IEEE 802.15.4a has captured these important characteristics, but also pose new challenges for performance analysis, which will be discussed next.

• Unlike the fixed number of impulse responses in a narrow-band channel, the UWB signals may arrive with random number of clusters and rays. In the IEEE 802.15.4a, the number of clusters is modeled as the Poisson random variable, and the inter-arrival time of the rays within a cluster is modeled by a hyperexponential random variable. Thus, computing the collected signal energy in a UWB channel with random number of both clusters and rays is complicated since one may need to find a joint distribution of infinite number of random variables.

• When the number of arrival rays in a very narrow time bin (or chip duration) is not large enough, the central limit theorem is no longer applicable to model the distribution of fading signal. Unlike a traditional Rayleigh random variable as in the narrow-band case, the multipath fading signal in the IEEE 802.15.4a UWB channel is characterized by a Nakagami m random variable of which the parameter m is a log-normal random variable related to the ray arrival time. Thus, the UWB signal amplitude become a multidimensional random variable and is a nonlinear function of another variable. The analysis of such a signal is rarely seen in the currently literature.

Based on the IEEE 802.15.4a UWB channel model, a UWB signal is mathematically characterized by a joint continuous Nakagami m random variable for fading amplitude, a discrete Poisson random variable for cluster’s number, and a discrete counting random variable with a hyperexponential distributed inter-arrival time. In [2], the author compared the IEEE 802.15.3a [3] and 802.15.4a channel model and concluded that the 4a model is more general and based on more measurements than the earlier 3a channel model. Thus, we are motivated to develop an analytical model to evaluate the impact of different UWB channel parameters on the system performance. There are nine sets of UWB channel parameters specified in the IEEE 802.15.4a for various environments. These channel parameters include the inter-cluster arrival rate, ray arrival rates (mixed Poisson model parameters), inter-cluster decay.
constant, intra-cluster decay time constant parameters, the mean and variance of Nakagami $m$ parameter, Nakagami $m$ factor for the strongest components, and the alternative power delay profile (PDP) shape.

B. Related Work

Here we provide a literature survey on the related works in performance analysis of the UWB system. In [4], the authors derived the analytical BER formula for the binary and M-ary UWB systems with Walsh codes under the AWGN channel with multiple access interference (MAI). In [5], the authors studied the impacts of the interference from the universal mobile telecommunications system (UMTS)/wideband code division multiple access (WCDMA) system on the UWB systems. In [6], the BER formula of the UWB system under the flat and dispersive Rayleigh fading channels with timing jitters was derived. In [7], the authors analyzed the performance of a transmit-reference (TR) UWB system with an autocorrelation receiver under a slowly fading channel of which the amplitude is characterized by an appropriate moment generating function approach.

The BER formula in the IEEE 802.15.3a UWB channel model [3] was shown as a function of finite window size without considering the RAKE receiver in [8]. The statistics of the output signal-to-noise ratio (SNR) with the RAKE receiver under the IEEE 802.15.3a UWB channel were also obtained [9]. However, the explicit BER formula was unavailable and the shadowing effects are ignored. In [10], we derived the BER analytical formula with a coherent RAKE receiver in the complete IEEE 802.15.3a UWB channel model where shadowing is taken into account. In [11], an analytical expression for the SNR of the pulse position modulated (PPM) signal in a multi-input multi-output (MIMO) system was provided in a UWB channel characterized by Gamma distributed signal power with Poisson distributed arrival clusters and rays. In [12], the error performance of a zero-forcing (ZF) RAKE receiver in a MIMO system was analyzed in the frequency-selective UWB lognormal fading channels with a fixed number of clusters and rays.

C. Objective and Outline of This Paper

The objective of this paper is to evaluate BER performance with various UWB channel parameters specified in the IEEE 802.15.4a model. The rest of this paper is organized as follows. In Section II, we provide the required mathematical background about counting integrals to analyze the IEEE 802.15.4a channel. In Section III, we derive the analytical expression for the BER performance of the antipodal and orthogonal binary signals under the IEEE 802.15.4a UWB channel. We show the numerical results in Section IV and give our concluding remarks in Section V.

II. MATHEMATICAL BACKGROUND

The IEEE 802.15.4a channel model can be viewed as a random process. The random variables associated with the random process can be categorized into two kinds. First, the time domain random variables, which contains the arrival time of all the clusters and rays, i.e., the variables $\{T_i\}$ and $\{\tau_{k,l}\}$, where the indices $k$ and $l$ can be any nonnegative integers. Second, the amplitude domain random variables, which contains the amplitudes of all the channel impulse, i.e., the variables $\alpha_{k,l}$. Thus, in order to investigate the characteristics of the channel, we must consider infinite number of random variables. This makes the analysis very complicated.

Let us treat the problem in another way. Let us rewrite the channel impulse response $h(t)$ to (15) in [1] in a simpler form:

$$h(t) = \sum_k G_k \delta(t - T_k), \quad (1)$$

where $T_k$ is the arrival time of the $k$-th multipath component, regardless it is a cluster or ray. We arrange $\{T_k\}$ such that it is a nondecreasing sequence. $G_k$ is the gain of the $k$-th multipath component. In [14], the author focused on finding the characteristic function of the random variable $\Phi$, which is the sum of path gains that arrive in the time window $[a, b]$. The mathematical expression of the random variable $\Phi$ can be written as

$$\Phi = \sum_k G_k I_{[a,b]}(T_k), \quad (2)$$

where

$$I_{[a,b]}(x) = \begin{cases} 1, & \text{if } x \in [a, b], \\ 0, & \text{if } x \notin [a, b]. \end{cases} \quad (3)$$

The purpose of defining $I_{[a,b]}(x)$ here is to consider the multipath components arriving within the time interval $[a, b]$. The key idea to compute the statistics of $\Phi$ is the use of the counting integral [14]

$$\Phi = \sum_k \varphi(T_k, G_k) = \int_{-\infty}^{\infty} \int_{0}^{\infty} \varphi(s, g)N(ds \times dg), \quad (4)$$

where $N(\cdot)$ is the counting measure [15], $\varphi(s, g) = gI_{[a,b]}(s)$. The counting integral is the Lebesgue integral [16] defined based on the counting measure. That is, we first consider the value of the function $\varphi$ within a small time interval $ds$ and a small gain interval $dg$. Second, we integrate over all possible values of $s$ and $g$, i.e., $s \in [0, \infty)$ and $g \in (-\infty, \infty)$. It is equivalent to sum up the value of the function $\varphi(T_k, G_k)$ for all $k$, which is the summation term in (4). By exploiting the counting integral, the original probability problem with infinitely many random variables has been transformed into a integral with only two dummy variables $s$ and $g$. The remained problem is to find the characteristic function of $\Phi$. The detail derivation can be found in [14].

III. BER ANALYSIS

A. Receiver Structure

Consider a coherent RAKE receiver with $L_{RAKE}$ fingers. The received SNR $\gamma_b$ is

$$\gamma_b = \frac{E_b}{N_0} \sum_{k=1}^{l} C_k^2, \quad (5)$$
where $E_b/N_0$ is the bit SNR, $c_k$ is the channel amplitude that appears at the $k$-th finger of the RAKE receiver. From [17] we know that the conditional error probability for binary signals for the coherent RAKE receiver is

$$P_2(\gamma_b) = Q \left( \sqrt{\frac{\gamma_b}{2}} \left( 1 - \rho_r \right) \right)$$

where $\rho_r = -1$ for antipodal signals and $\rho_r = 0$ for orthogonal signals. Next we will derive the characteristic function of the received energy $E \triangleq \sum_{k=1}^{L} c_k^*$ in the IEEE 802.15.4a UWB channel.

**B. Characteristic Function of the Received Energy ($E$)**

In the following theorem, we give the formula of the characteristic function of $E$. We modify and extend the result in [10] to the case of the IEEE 802.15.4a UWB channel.

**Lemma 1:** Let $\mathcal{L}_{T,t}(\nu)$ be the characteristic function of the squared gain of each path in the IEEE 802.15.4a UWB channel, where $T$ and $t = T + \tau$ denotes the cluster arrival time and the ray arrival time, respectively. Also, let $e^{-\psi_\nu(T)}$ and $e^{-\lambda_j^\nu(T)}$ be the characteristic functions of a shot-noise random variables related to the ray arrival process with parameter $\lambda$ and the cluster arrival process with parameter $\Lambda$, respectively. Then, it can be proved that the characteristic function of the received energy ($E$) in the IEEE 802.15.4a UWB channel can be computed by

$$\Psi(\nu) = \mathcal{L}_{0,0}(\nu)e^{-\psi_\nu(0)-\Lambda_j^\nu(\nu)}, \quad (7)$$

**Proof:** In the IEEE 802.15.4a UWB channel, the ray arrival rate $\lambda(\tau)$ is a function of $T$. Thus, different from the 3a case, we have to redefine the function $\psi_\nu(T)$ by putting $\lambda(T)$ into the integral:

$$\psi_\nu(T) = \begin{cases} \int_{\max(a,T)}^{b} (1 - \mathcal{L}_{T,t}(\nu)) \lambda(t - T) dt, & T \leq b, \\ 0, & T > b, \end{cases}$$

The term $\lambda(\tau)$ can be found in Theorem 2. \hfill \square

**Theorem 1:** Consider a RAKE receiver with $L_{\text{RAKE}}$ fingers in the IEEE 802.15.4a UWB channel. The characteristic function $\mathcal{L}_{T,t}(\nu)$ can be computed by

$$\mathcal{L}_{T,t}(\nu) = (1 - j\nu \Omega/m)^{-m}$$

where

$$\Omega = \frac{1}{\gamma_t} \exp \left( - \frac{T}{\Gamma} - \frac{t - T}{\gamma_t} \right)$$

and

$$m = \exp \left( m_0 + \tilde{m}_0^2/2 \right).$$

The parameter $\gamma_t$ is defined in (20) in [1]. \hfill \square

**Proof:** See Appendix I.

**Theorem 2:** The parameter $\lambda$ in Lemma 1 is a function of $\tau$ and can be expressed as

$$\lambda(\tau) = \begin{cases} \frac{2e^{\lambda_1^\nu}z^\nu - 1 - e^{\lambda_1^\nu} - 1 - e^{\lambda_2^\nu}}{1 - e^{\lambda_2^\nu} - 1 - e^{\lambda_2^\nu}}, & \tau \geq 0, \\ 0, & \tau < 0. \end{cases} \quad (12)$$

**Proof:** See Appendix II.

The equations for calculating $\psi_\nu(T)$ and $J(\nu)$ can be found in Theorem 2 in [10].

With characteristic function of $E$, i.e. $\Psi(\nu)$ in (7), the probability density function (PDF) of $E$ can be computed by the Gauss-Hermite formula as follows:

$$f_\nu(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi(\nu)e^{-j\nu x} d\nu \approx \frac{1}{2\pi} \sum_{k=1}^{N} w_k^{(H)} \Psi(\nu)e^{-j\nu x} e^{x^2} \big|_{\nu = x_0}.$$ \quad (13)

Combining (9), (13), and (16) and (17) in [10], the BER of the RAKE receiver in the IEEE 802.15.4a UWB channel can be computed as

$$P_2 = \mathbb{E}_E \left[ Q \left( \sqrt{\frac{1 - \rho_r}{E_b/N_0}} E \right) \right] = \int_{0}^{\infty} Q \left( \sqrt{\frac{1 - \rho_r}{E_b/N_0}} x \right) f_\nu(x) dx \approx \frac{1}{2\pi} \sum_{k=1}^{N} w_k^{(H)} \exp \left( -\frac{1}{2} \lambda(L - 1)T_c \sum_{p=1}^{N_{\mathbb{Z}}} w_{l_p} \right) \left[ 1 - \mathcal{L}_{0,0}(\nu) \right]_{x=\frac{1}{2}(L - 1)T_c(x_{l_p}^0 + 1)} \exp \left( -\frac{1}{2} \Lambda(L - 1)T_c \sum_{l_p=1}^{N_{\mathbb{Z}}} w_{l_p} \right) \int_{0}^{\infty} Q \left( \sqrt{\frac{1 - \rho_r}{E_b/N_0}} x \right) \exp(-j\nu x) dx \exp(\nu^2) \big|_{\nu = x_0^0}. \quad (14)$$

**C. Discussion**

It is possible to apply the above analytical method to the generalized UWB frequency selective fading channel with any given fading distribution, PDP, and the distribution of cluster and ray inter-arrival time. For a give PDP, we only need to express it as a function of $T$ and $t$ and place it in (10) as the procedure in Appendix I. For a given PDF of the fading amplitude, we can first find the PDF of the squared amplitude as in (17) and use (18) to find $\mathcal{L}_{T,t}(\nu)$. For any given PDF of ray inter-arrival time, we can use the same method as in Appendix II to find the intensity function and $\lambda(\tau)$ and applied it in (7). For any given PDF of cluster inter-arrival time, we can also find its intensity function $\Lambda(T)$. Since it is a function of $T$ we need to redefine the function $J(\nu)$ as

$$J(\nu) = \int_{0}^{\infty} \Lambda(T)[1 - \mathcal{L}_{T,T}(\nu)e^{-\psi_\nu(T)}]dT.$$ \quad (15)

That is, we put the function $\Lambda(T)$ into the integrand and integrate it with respect to $T$. On the other hand, the function
According to (18) in [1], we can see that ray interarrival time has the probability 0.095 decided by the parameter \( \lambda_1 \) and has the probability of 0.905 decided by the parameter \( \lambda_2 \). Thus \( \lambda_2 \) has more significant impact on BER than \( \lambda_1 \) does.

**E. Impact of the Ray Arrival Parameter \( \beta \) on BER**

Figure 4 shows the BER v.s. \( E_b/N_0 \) for different values of \( \beta = 0, 1/3, 2/3, \) and 1. As \( \beta \) increases, the BER decreases. According to (18) in [1], when \( \beta \) increases, the ray process has the higher probability to choose the arrival rate \( \lambda_1 \) rather than \( \lambda_2 \). For CM1, \( \lambda_1 = 1.54 \) and \( \lambda_2 = 0.15 \). From the discussion in the last two subsection, we know that the higher the ray arrival rate is, the lower the BER. This phenomenon is similar to the cluster. As the cluster arrival rate increases, the BER decreases, as we have seen in Subsection IV-A.

**F. Impact of the Intra-Cluster Decay Constant \( \gamma_0 \) on BER**

Figure 6 shows the BER v.s. the intra-cluster decay constant \( \gamma_0 \) for \( E_b/N_0 = 5, 10, \) and 15 dB. When \( \gamma_0 \) increases, the BER first increases and then decreases. This phenomenon can be explained as follows. Based on (20) in [1], a larger value of \( \gamma_0 \) also leads to a larger value of \( \gamma_1 \). According to (19) in [1], for a small value of \( \gamma_1 \), the term \( 1/\gamma_1 \) dominates the BER performance, which cause the decrease of \( \Omega_1 \) and the increase of BER. When \( \gamma_1 \) is large, the exponential term dominates and therefore \( \Omega_1 \) increases and BER decreases. The maximum of the BER occurs at \( \gamma_0 = 30, 40, \) and 100 for \( E_b/N_0 = 5, 10, \) and 15 dB, respectively.

**V. CONCLUSIONS**

In this paper, we have derived the BER analytical formula for a coherent RAKE receiver under the IEEE 802.15.4a UWB channel model. Our proposed analytical BER formula can obtain the BER values quickly, compared to the computer simulation. Further, we also discuss the impact of various parameters of the IEEE 802.15.4a UWB channel model on BER. Furthermore, it is worthwhile to emphasize that the suggested analytical method can be applied to other multipath channel models with any given fading distribution, PDP, and cluster and ray inter-arrival time distributions.

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**APPENDIX I**

**PROOF OF THEOREM 1**

From (24) in [1], we can easily find the PDF of \( x = a^2 \) by exploiting the resulting of Example 7b in [18]. That is,

\[
 f_x(x) = \frac{1}{2\sqrt{x}} \left[ f_a(\sqrt{x}) + f_a(-\sqrt{x}) \right] 
\]

\[
 = \begin{cases}
 \exp(-\frac{x}{\Gamma(m)}) \left(\frac{a}{\Gamma(m)}\right)^m, & x \geq 0, \\
 0, & x < 0.
\end{cases}
\]
The characteristic function of $x$ is
\begin{equation}
\mathcal{L}_{T,t}(\nu) = \int_{-\infty}^{\infty} f(x)e^{i\nu x} dx = (1 - j\nu\Omega/m)^{-m}.
\end{equation}

The term $\Omega = E\{x\}$ is defined in (19) in [1]. To fit it into our formula, we substitute $T_1$ by $T$ and $M_{\text{cluster}}$ by its mean, zero, in (21) in [1] and $\tau_{T,t}$ by $(t - T)$ in (19) in [1]. Then we can get (10).

Finally, we set $m$ to its mean and get (11). The mean is given by (4) in [19].

**APPENDIX II**

**PROOF OF THEOREM 2**

According to (18) in [1], the PDF of interarrival time of ray is
\begin{equation}
f(\tau) = \begin{cases} 
\beta \lambda_1 e^{-\lambda_1 \tau} + (1 - \beta)\lambda_2 e^{-\lambda_2 \tau}, & \tau \geq 0, \\
0, & \tau < 0.
\end{cases}
\end{equation}

The cumulative distribution function (CDF) is
\begin{equation}
F(\tau) = \int_{-\infty}^{\tau} f(x) dx = \begin{cases} 
1 - \beta e^{-\lambda_1 \tau} -(1-\beta)e^{-\lambda_2 \tau}, & \tau \geq 0, \\
0, & \tau < 0.
\end{cases}
\end{equation}

The parameter $\lambda$ in Lemma 1 is the intensity function [20] of the interarrival time of rays:
\begin{equation}
\lambda(\tau) = \frac{f(\tau)}{1-F(\tau)} = \begin{cases} 
\frac{\beta \lambda_1 e^{-\lambda_1 \tau} + (1-\beta)\lambda_2 e^{-\lambda_2 \tau}}{\beta e^{-\lambda_1 \tau} +(1-\beta)e^{-\lambda_2 \tau}}, & \tau \geq 0, \\
0, & \tau < 0.
\end{cases}
\end{equation}

The ray arrival rate is a function of time $\tau$, which means the ray arrival is a nonhomogeneous Poisson process. Note that when $\beta = 1$, the PDF in (19) reduces to the exponential PDF and the intensity function reduces to the constant $\lambda_1$. This case corresponds to a homogenous Poisson process.

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Fig. 1. The BER v.s. $E_b/N_0$ for the RAKE receiver with 10 fingers in the IEEE 802.15.4a UWB channel CM1. The inter-cluster arrival rate $\Lambda$ is 0.01, 0.1, 0.5, and 1.

Fig. 2. The BER v.s. $E_b/N_0$ for the RAKE receiver with 10 fingers in the IEEE 802.15.4a UWB channel CM1. The parameter $\lambda_1$ is 0.01, 0.1, 1, and 10.

Fig. 3. The BER v.s. $E_b/N_0$ for the RAKE receiver with 10 fingers in the IEEE 802.15.4a UWB channel CM1. The parameter $\lambda_2$ is 0.01, 0.1, 1, and 10.

Fig. 4. The BER v.s. $E_b/N_0$ for the RAKE receiver with 10 fingers in the IEEE 802.15.4a UWB channel CM1. The parameter $\beta$ is 0, 1/3, 2/3, and 1.

Fig. 5. The BER v.s. $E_b/N_0$ for the RAKE receiver with 10 fingers in the IEEE 802.15.4a UWB channel CM1. The inter-cluster decay constant $\Gamma$ is 0.1, 1, 10, and 100.

Fig. 6. The BER v.s. $\gamma_0$ for the RAKE receiver with 10 fingers in the IEEE 802.15.4a UWB channel CM1. $E_b/N_0$ is 5 dB, 10 dB, and 15 dB.