BER-Minimized Space-Time-Frequency Codes for MIMO Highly Frequency-Selective Block-Fading Channels

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Abstract—In this paper, we present bit error rate (BER)-minimized space-time-frequency (STF) block codes for multi-input multi-output (MIMO) highly frequency-selective block-fading channels. We consider the IEEE 802.15.3a ultra-wide band (UWB) channel models (CM) 1–4. Based on a new STF block codes design criterion with the objective of minimizing BER, we develop an efficient searching algorithm for the design of the optimal STF block codes which maximize the coding gain. For 128 subcarriers with two subcarriers jointly encoding with 2–4 transmitting antennas, we find that the optimal STF block codes for all the IEEE 802.15.3a UWB channel models CM 1–4 can be found. Furthermore, the designed STF block codes outperform the recently published high-rate full-diversity STF codes [1] by 1 dB. Last, the proposed STF codes can be decoded by maximum likelihood decoding approach, which is simpler than the sphere decoding principle used in [1].

Index Terms—Bit error rate (BER), block-fading channels, IEEE 802.15.3a channel model, multi-input multi-output (MIMO), space-time-frequency (STF) block codes, ultra-wideband (UWB).

I. INTRODUCTION

The space-time-frequency (STF) coding is a technique which provides error control ability in multi-input multi-output (MIMO) systems, which are usually combined with the orthogonal frequency-division multiplexing (OFDM) technology. The main purpose of using the STF coding is to achieve the full diversity gain. For example, in [1], the authors proposed STF codes which achieve the diversity gain of \(N_t N_r K L\), where \(N_t\) is the number of transmit antennas, \(N_r\) is the number of receive antennas, \(K\) is the number of independent fading blocks in one codeword, and \(L\) is the number of taps of channel impulse response (CIR) between any pair of transmit and receive antennas. The space diversity, time diversity, and frequency diversity are \(N_t N_r K\), and \(L\), respectively.

However, in a highly frequency-selective fading channel, the number of taps of CIR could be very large. For example, in the IEEE 802.15.3a UWB channel model [2], the number of taps of CIR is infinity theoretically and about 1000 to 2000 practically. Thus, it is difficult to achieve the full frequency diversity under the highly frequency-selective fading channel. Thus, it motivates us to turn to a more fundamental problem: How to design BER-minimized STF codes for MIMO highly frequency-selective block-fading channels? Here the block-fading channel is defined as follows: The channel remains the same within one fading block and is independent from one block to another one [1].

The difficulties of design BER-minimized STF block codes for the MIMO highly frequency-selective block-fading channels can be discussed in three aspects. Note that we take the IEEE 802.15.3a UWB channel model as an example in this paper. 1) First, the IEEE 802.15.3a channel model has four different sets of parameters, named CM1, CM2, CM3, and CM4. For different channels, we have to design different codes to reflect the channel characteristics. One challenging issue arises: Is there a universal code which is optimal for all the four channel models CM 1–4 for given numbers of subcarriers and transmit antennas? 2) As the numbers of subcarriers and transmit antennas increase, the number of all possible codes becomes astronomical. Thus, the second challenge is how to search the optimal codes efficiently. 3) Because traditional STF coding methods focus on linear codes, it will be challenging to examine if there exist nonlinear optimal STF block codes.

To our best knowledge, the design of STF block codes for the MIMO-OFDM systems under the IEEE 802.15.3a channel models considering all the three aforementioned challenges has not been seen in the literature.

Here, we introduce some related works about space-frequency (SF) codes and STF codes for the MIMO-OFDM systems. In [3], the authors analyzed the rate-diversity tradeoff for the MIMO-OFDM channels and presented two asymptotically optimal SF code constructions. In [4], the authors investigated STF codes for MIMO-OFDM and found an equivalence between antennas and subcarriers. The authors then suggested a complexity-reduced scheme with coding across subcarriers only. In [5], the authors proposed an adaptive STF coding scheme according to the space-frequency water-filling procedure for MIMO-OFDM systems. In [6], the authors considered STF codes over MIMO-OFDM block-fading channels and derived a sphere packing lower bound on the average word error probability and an upper bound.

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for pairwise word error probability, but they did not show how to design the optimal codes to achieve these bounds. In [1], authors proposed a systematic design method for high-rate full-diversity STF codes for broadband MIMO block-fading channels. In [7], authors presented rate-two STF block codes for multiband UWB-MIMO communication systems using rotated multidimensional modulation. We will show by simulation that our proposed STF codes have better BER performance than the codes in [1] and [7] do.

The objective of this paper is to design the universally optimal STF block codes for the MIMO-OFDM systems under four kinds of IEEE 802.15.3a UWB channel models, i.e., CM 1–4. The rest of this paper is organized as follows. In Section II, we introduce our system model. In Section III we describe the design criterion, an efficient searching algorithm, and the optimal codes in some examples. In Section IV, we discuss the properties of our proposed optimal STF block codes. We show the numerical results in Section V and give our concluding remarks in Section VI.

II. SYSTEM MODEL

Figure 1 shows our system block diagram. First, we divide the information bits into groups. Each group has two bits. Then we pass the bits to our STF block encoder. For example, if we want to encode across two transmit antennas and two subcarriers, then the codeword can be expressed as a matrix. Then we use an OFDM modulator to allocate every elements in the codeword to corresponding subcarriers and transmit antennas. That is, \( d_{ij} \) is allocated on the \( i \)-th subcarrier and \( j \)-th antenna, for \( i = 1, 2 \) and \( j = 1, 2 \). The transmitted signals pass the IEEE 802.15.3a UWB channel. The receiver recovers the original information bits via inverse operations as in the transmitter: We first use an OFDM demodulator to find the codewords. Then we use a maximum likelihood (ML) STF block decoder to find the original information bits.

III. THE UNIVERSALLY OPTIMAL STF BLOCK CODES DESIGN

In this section, we describe a criterion and a efficient searching algorithm of the universally optimal STF block codes.

A. The Optimum Criterion

Our goal is to design the STF block codes to minimize \( P_e \) in [8, (40)]. For given SNR \( \rho \), number of transmit antennas \( N_t \), number of receive antennas \( N_r \), number of OFDM blocks jointly encoded \( K \), and number of OFDM subcarriers jointly encoded \( M \), it is equivalent to maximize the term \( q = \prod_{n=1}^{r} \text{eig}_{\text{max}}(S \circ R_M) \) by designing the matrix \( S = (D - D^*) (D - D^*)^T \), where \( D \) and \( D^* \) are two distinct STF block codes codewords, \( \circ \) denotes the Hadamard product \([9]\), and \( R_M \) is the auto-covariance matrix of which definition can be found in [8]. Similar to the rank and determinant criteria of the space-time block coding (STBC) \([10]\), we have to maximize the minimal \( q \) along the pairs of distinct codewords.

We first consider the simplest case. Let \( N_t \) be the number of input information bits for each codeword \( D \). Let \( M = N_t = N_r = 2 \). Let \( b_1, b_2 \in \{0, 1\} \) be the two input bits. We use the binary phase shift keying (BPSK) modulation. Let \( s_1 \) and \( s_2 \) be the two corresponding symbols, then \( s_i = \text{mod}(b_i) \) for \( i = 1, 2 \), where \( \text{mod}(x) = \begin{cases} 1, & \text{if } x = 1, \\ -1, & \text{if } x = 0. \end{cases} \) The codeword \( D \) is a 2 \( \times \) 2 matrix with each element being 1 or \(-1\), i.e., \( D \in \{-1, 1\}^{2 \times 2} \). Then there are \( 2^{2^2} = 16 \) different codewords. Since there are two input bits, there are \( 2^2 = 4 \) possible inputs, i.e., \( b_1b_2 \in \{00, 01, 10, 11\} \). Hence, we have to choose four distinct codewords for these four different inputs.

For the convenience of expression, let us define the demodulation function \( \text{dem}(x) \triangleq \text{mod}^{-1}(x) \) and the multiple digits version of \( \text{dem}(\cdot) \) is defined as \( \text{dem}(x) \triangleq \{\text{dem}(x_1), \text{dem}(x_2), \ldots, \text{dem}(x_m)\} \), where the vector \( x \) stands for an \( m \)-digit number and the \( i \)-th digit is \( x_i \) for \( 1 \leq i \leq m \).

The following equation gives each codeword \( D \) a unique positive integer \( n \) as its subscript: \( D_n = \left\{ d_{11}, d_{12}, d_{21}, d_{22} \right\} : \text{bd}(\text{dem}(d) + 1) = n \), where the function \( \text{bd}(x) \) is to transform a binary number \( x \) into its decimal form and \( d = [d_{11}, d_{12}, d_{21}, d_{22}] \). Now, the set that contains all the codewords is \( C = \{D_1, D_2, \ldots, D_{16}\} \). Let \( B \) be a subset of \( C \) and \( B \) contains four codewords. Now, our problem can be mathematically described as finding a set \( B^* \) such that

\[
B^* = \arg \max_{B \subseteq C, |B| = 4} \min_{D \in B, D \neq D} \prod_{n=1}^{r} \text{eig}_{\text{max}}(S \circ R_M),
\]

where \( |B| \) is the number of elements of \( B \).

B. An Efficient Searching Algorithm for the Optimal STF Block Codes

In order to simplify the representation of our problem and provide more insight, we introduce a graph representation to our code space. We represent each codeword as a vertex with number \( n \), and between any two distinct vertices there is an undirected edge with metric \( q \). Then, for the case \( N_t = N_r = 2 \) case, we can use a complete graph \([11]\) with 16 vertices which is denoted by \( K_{16} \) to represent our code space. Then our problem becomes to find the optimal \( K^*_4 \) in \( K_{16} \) such that the minimal metric in \( K^*_4 \) is the largest one among that of all \( K_4 \) in \( K_{16} \). There are \( \binom{16}{4} = 1820 \) distinct \( K_4 \) in \( K_{16} \). To find the minimal metric within each \( K_4 \) we need to search for \( \binom{4}{2} = 6 \) metrics. Thus, we need to do \( 1820 \cdot 6 = 10920 \) times of searching to find \( K^*_4 \).

For general \( M, N_t \), and \( N_r \), there are \( 2^{MN_t} \) vertices for the BPSK case. The complexity of the complete search becomes \( O \left( \binom{2^{MN_t}}{2} \right) \). The complexity grows rapidly as \( M, N_t, \) and \( N_r \) increase. Thus, it is necessary to find a more efficient algorithm to search for the optimal STF block codes.

For the case \( M = N_t = N_r = 2 \) and CM1, we find that the metric \( q \) takes only on eight different values. Sorting these values in the decreasing order, we then have \( q \in \{64, 16, 3314, 16, 8, 4, 1.32562, 0.331406, 0\} \). We find that it will save
many searching steps if we search $K_4$ subject to the largest $m$ metrics for $m = 1, 2, \ldots$, until we find all $K_4^*$ for a certain value of $m$. Let use take the $M = N_l = N_r = 2$ case as an example. For $m = 1$, we only consider the edges with the largest metric 64. Obviously, it does not contain any $K_4$. For $m = 2$, we consider the edges with the largest two metrics: 64 and 16.3314. After searching, we also find that there is no $K_4$ in this graph. For $m = 3$, we consider the edges with the largest three metrics: 64, 16,3314, and 16. We find that there are totally eight $K_4$ in this graph, they are: \{1, 7, 12, 14\}, \{1, 8, 10, 15\}, \{2, 8, 9, 15\}, \{2, 8, 11, 13\}, \{3, 5, 12, 14\}, \{3, 6, 12, 13\}, \{4, 5, 11, 14\}, and \{4, 6, 9, 15\}. Note that we do not need to search for the case $m > 3$, because we already find that the max-min value of $q$ is 16.

We use the same method to search the optimal STF block codes for CM2, CM3, and CM4. CM2 and CM3 both have the same optimal STF block codes as CM1 does, but there are nine optimal STF block codes for CM4. These nine codes contain the eight optimal codes which are the same as that of CM1 and an additional codes \{4, 6, 11, 13\}. This is an interesting discovery that for different channel model, the optimal codes may be different. Thus, in order to design the optimal codes, we have to take the channel model into account.

In order to design the codes that are optimal for all channel model, we choose the eight optimal STF block codes for CM1 out of \(\binom{2^4}{4} = 1820\) candidates. The next step is to transform these codes to a code structure. Take the code \{3, 5, 12, 14\} as an example. According to the matrix-indexing procedure we defined in Section III-A, we find these four integers correspond to the following codewords: $D_3 = [-1 -1], D_5 = [-1 1], D_{12} = [1 -1], D_{14} = [1 1]$. We assign these four codewords to the information bits 00, 01, 10, and 11, respectively. Note that we can choose another assignment and the max-min value of $q$ will not change. To discover the code structure from these codewords, we first consider the element in the first row and first column of them. They are $D_i[1, 1] = \{-1, -1, 1, 1\}$, where \(i \in \{3, 5, 12, 14\}\). Since each position can take values on \(-1\) or \(1\), there are totally $2^4 = 16$ possibilities. We establish a truth table of these 16 values, as a function of $s_1$ and $s_2$. For some cases, we find it is more convenient to express the function in terms of $b_1$ and $b_2$. Use this table to check the function $f(s, b)$ for all the elements of $D_i$, we finally find the code structure is $s_1 s_2 s_2 s_1$. It is the Alamouti coding scheme [12]. The other seven optimal code structures are $s_1 s_2 s_2 s_1, s_1 s_2 s_2 s_2, s_1 s_2 s_2 s_1, s_1 s_2 s_2 s_1$.

The pseudo code of our proposed searching algorithm for the optimal STF block codes can be found in Algorithm 1. Note that in the ninth line we only consider the vertices with degree being at least three because any vertex in a $K_4$ must satisfy this condition.

### Algorithm 1: The searching algorithm for the optimal STF block codes.

**Input:** $M, N_l, N_r, \text{and CM.}

**Output:** $B^*.$

1. $B^* \leftarrow \emptyset$
2. $G \leftarrow K_{2MN_l}$
3. found $\leftarrow$ False
4. **foreach** $1 \leq i, j \leq 2^{MN_l}$ **do**
5. \[ S \leftarrow (D_i - D_j)(D_i - D_j)^T \]
6. \[ E(G)_{i,j} \leftarrow \prod_{t=1}^{s} e_{G_t}(S \circ R_M(CM)) \]
7. metric $\leftarrow$ list of distinct values of $E(G)$ in decreasing order
8. **for** $m \leftarrow 1 \text{ to Length(metric)}$ **do**
9. \[ F \leftarrow \{(c : e \in E(G), e \geq \text{metric}[m])\}, \{v : v \in V(G), \text{deg}(v) \geq 3\} \]
10. **foreach** $B, \{B \subset F, |V(B)| = 2^{N_l}\}$ **do**
11. **if** $B$ is $K_{2^{N_l}}$ **then**
12. \[ B^* \leftarrow B^* \cup \{B\} \]
13. **if** found **then**
14. **return** $B^*$

### C. Optimal STF Block Codes for the Other Cases

We use the algorithm described in Section III-B to find the optimal STF block codes for the case $M = N_l = 2$ and $N_r = 3$. We find that there are 54 different optimal STF block codes for all the four CM out of $\binom{2^4}{4} = 635376$ candidates. In order to simplify the expression of the code matrix, we define $s_3 \triangleq s_1 s_2$, $s_i \triangleq -s_i$ for $i = 1, 2, 3, 4$. $s_1 k_2 \triangleq [s_i s_j s_k]$. Then, among these 54 optimal STF block codes, 28 of them have the form of $[s_{T_3} s_{T_5} s_{T_7}]$ and the other 26 codes have the form of $[s_{T_3} s_{T_5} s_{T_7}]$, where $a \in \{3', 12', 23', 13'2', 23'1', 3'21', 23', 1'3', 213', 3'12', 231', 3'21', 23', 1'32', 213', 3'12', 231', 3'21', 23', 1'32', 213', 3'12', 231', 3'21', 23', 1'32', 213', 3'12', 231', 3'21', 23', 1'32', 213', 3'12', 231', 3'21', 23', Given the case $M = N_l = 2$ and $N_r = 4$, we find 5148 different optimal STF block codes for all the four CM out of $\binom{2^4}{4} = 174792640$ candidates. Due to the space limit, we do not list all codes here. One of the optimal STF block codes is $s_1 s_2 s_2 s_1$. For the case $M = 3$ and $N_l = N_r = 2$, there is an interesting fact. We find that there does not exist any optimal STF block codes for all the four CM out of $\binom{2^4}{4} = 2829877120$ candidates. For the case $M = 3, N_l = 4$, and $N_r = 2$, we find that the set of optimal
STF block codes for all the four CM are the same. There are totally 1464 optimal codes out of \(24^3\) = 11710951848960 candidates and they are all nonlinear. For the case \(M = 4\) and \(N_i = N_i = 2\), we find that there does not exist any optimal STF block codes for all the four CM out of \(24^3\) = 174792640 candidates.

IV. PROPERTIES OF THE OPTIMAL STF BLOCK CODES

A. Coding Gain

The coding gain of a code can be computed via \(CG = \frac{1}{N_i} \left( N_i = 1 \hspace{1pt} \text{eig}_r(S \circ R_M) \right)^{1/r}\). In Table I we list the coding gain of the optimal codes we have found in Section III. For the \(N_i = 2\) case, the optimal STF block code is Alamouti code. Its coding gain is one [10]. For the \(N_i = 3\) and \(N_i = 4\) cases, we find that the coding gain is greater than 0 dB by a little amount. Thus, we can predict that the BER performance of these three codes will be very close.

B. Diversity Order

The diversity order of a code is \(rKN_i\). The optimal codes we found above all have the same value of \(r\) for the same values of \(M, N_i, N_i\), and CM and the same modulation. Thus, for the same values of \(K\) and \(N_i\), the optimal codes achieve the same diversity order under the same condition. The values of \(r\) for different kinds of optimal codes are listed in Table II. From this table, we find a interesting fact. Sometimes the optimal codes achieve different diversity order for different values of CM. For example, when \(M = 3, N_i = 2, N_i = 2\), and the modulation is BPSK, the diversity order is two for CM1 and CM2 and three for CM3 and CM4.

V. NUMERICAL RESULTS

Our simulation environment is an MIMO-OFDM system. The number of total subcarriers is 128 and the sub-band bandwidth is 528 MHz. We apply the IEEE 802.15.3a UWB channel model CM 1−4 [2].

A. BER Comparison with STF Codes in [1] and [7]

Figure 2 shows the BER comparison of our code with Chusing’s code [7] and Zhang’s code [1] for the \(M = 4, N_i = 2, N_i = 2\) case in the IEEE 802.15.3a UWB channel model CM4. We can see that the diversity gains of the three codes are the same, but our code has better BER performance than Chusing’s and Zhang’s codes do. At \(BER = 10^{-4}\), the coding gain between our code and Chusing’s code is about 8 dB and the coding gain between our code and Zhang’s code is about 1 dB.

B. Impact of Number of Transmit Antennas Jointly Encoded (\(N_i\)) for Two Subcarriers Jointly Encoded (\(M = 2\))

Figure 3 shows the impact of number of transmit antennas jointly encoded on the BER for CM1, CM2, CM3, and CM4 for the optimal STF block codes for the \(M = N_i = 2\) case. Figs. 3(a), 3(b), and 3(c) are for the cases \(N_i = 2, 3,\) and 4, respectively. For each sub-figure, the BER decreases as CM increases. This phenomenon can be explained by the coding gain. In Table I, the coding gain increases as CM increases for the cases \(N_i = 3\) and 4, thus the BER decreases.

Moreover, we find a surprising fact. The BER in Figs. 3(a), 3(b), and 3(c) are almost the same for the same CM. In other words, the BER for a certain CM does not change as the number of transmit antennas increases. This result is quite different from the STBC case. In STBC, increasing the number of transmit antennas will decrease the BER performance [10]. Thus, we may conclude that in the MIMO-UWB systems, using multiple transmit antennas does not provide significant improvement to the BER performance, because the UWB channels already possess rich diversity inherently. In the uncoded UWB systems using multiple antennas, there exists the same phenomenon [13].

VI. CONCLUSIONS

In this paper, we study the BER-minimized STF block codes designed for the MIMO highly frequency-selective block fading channels. We consider the IEEE 802.15.3a UWB channel model. Based on the BER analysis under the aforementioned environment in [8], we provide a BER-minimized design criterion, an efficient searching algorithm for the optimal STF block codes, and optimal BER performance curves. Compared with other space-frequency-time codes [1], [7] for MIMO-OFDM communication systems under the UWB channel, our code has about 1 and 8 dB coding gain at \(BER = 10^{-4}\), respectively. On the other hand, increasing the number of transmit antennas does NOT improve the BER performance for the MIMO-UWB systems when \(M = 2\).
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multiple transmit and receive antennas in pulse-based ultrawideband

Fig. 1. The system block diagram.

Fig. 2. The BER comparison of our code versus Zhang’s code [1] and
Chusing’s code [7] for four subcarriers jointly encoded, two input information
bits for each codeword, one receive antenna, and two transmit antennas jointly
encoded in the IEEE 802.15.3a UWB channel model CM4. The modulation
is BPSK.

Fig. 3. The effect of different number of transmit antennas jointly encoded
(NT) on the BER for CM1, CM2, CM3, and CM4 for the optimal STF block
codes for two subcarriers jointly encoded and two input information bits for
each codeword. The modulation is BPSK. (a) NT = 2. (b) NT = 3. (c) NT = 4.