Spatial Modulation: Optimal Detection and Performance Analysis

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Abstract—In this letter, we derive the optimal detector for the so-called spatial modulation (SM) system introduced by Mesleh et al. in [1]. The new detector performs significantly better than the original (~4 dB gain), and we support our results by deriving a closed form expression for the average bit error probability. As well, we show that SM with the optimal detector also achieves performance gains (~1.5 – 3 dB) over popular multiple antenna systems, making it an excellent candidate for future wireless communication standards.

Index Terms—Antenna modulation, spatial modulation, maximum likelihood detection, MIMO.

I. INTRODUCTION

Using multiple antennas in wireless communications allows unprecedented improvements over current systems. Large spectral efficiency is obtained by using transmission techniques designed for multiple input multiple output (MIMO) systems, such as the Bell Laboratories layered space-time (BLAST) architecture [2]. Due to inter-channel interference (ICI) caused by coupled multiple symbols in time and space, maximum likelihood (ML) detection increases exponentially in complexity with the number of transmit antennas. Consequently, avoiding ICI greatly reduces receiver complexity, and contributes in attaining performance gains.

The so-called spatial modulation (SM), introduced by Mesleh et al. in [1], [3], is an effective means to remove ICI and the need for precise time synchronization amongst antennas. SM is a pragmatic approach for transmitting information, where the modulator uses well known amplitude/phase modulation (APM) techniques such as phase shift keying (PSK) and quadrature amplitude modulation, but also employs the antenna index to convey information. Only one antenna remains active during transmission so that ICI is avoided. As well, inter-antenna synchronization (IAS) during transmission is no longer needed as in the case of Vertical-BLAST (V-BLAST) [4], in which all antennas transmit symbols at the same time.

Contribution: In [1], a sub-optimal detection method is presented and only valid under some constrained assumptions about the channel. For conventional channels, their detector fails and even with their assumption, detection is not optimal.

In this paper, we present the optimal detector for SM and show that the detection is a joint optimization problem that cannot be separated as in [1]. We analyze the performance of the SM system and derive a closed form expression for the bit error probability when real constellations are used. As well, prior to this work, SM’s advantages lied in removing ICI and IAS from the communication systems, where gains in performance over other schemes in the literature was not present. With optimal SM however, we show that performance gains over maximum ratio combining (MRC) and V-BLAST is observed, making the use of SM in practical systems more attractive.

Organization: This paper is organized as follows. Section II introduces the basic SM system model, the mapper and the original detector. In Section III, we derive the optimal detector and provide a performance analysis for the SM system. Section IV presents some simulation results, and we conclude the paper in Section V.

II. SPATIAL MODULATION

A. System Model

The general system model is shown in Fig. 1, which consists of a MIMO wireless link with \( N_t \) transmit and \( N_r \) receive antennas. A random sequence of independent bits \( b \) enters the SM mapper, which groups \( B^1 \) bits and maps them to a constellation vector \( x = [x_1 \ x_2 \ \cdots \ x_{N_t}]^T \), where we assume a power constraint of unity (i.e. \( E_x[x^H x] = 1 \)).

In SM, only one antenna remains active during transmission and hence, only one of the \( x_i \) in \( x \) is nonzero. The signal is transmitted over an \( N_r \times N_t \) wireless channel \( H \) and experiences an \( N_r - \dim \) additive white Gaussian (AWGN) noise \( \eta = [\eta_1 \ \eta_2 \ \cdots \ \eta_{N_r}]^T \). We express the received signal as

\[
y = \sqrt{\rho} H x + \eta \tag{1}
\]

where \( \rho \) is the average signal to noise ratio (SNR) at each receive antenna, and \( H \) and \( \eta \) have independent and identically distributed (iid) entries according to \( \mathcal{CN}(0, 1) \).

1. \( B \) is defined in Section II-B.
2. The following notations are used throughout the paper. Italicized symbols denote scalar values while bold lower/upper case symbols denote vectors/matrices. We use \((\cdot)^T\) for transpose, \((\cdot)^H\) for conjugate transpose, and \(\binom{n}{k}\) for the binomial coefficient. We use \(|\cdot|\) for absolute value of a scalar, and \(\|\cdot\|_F\) for the Frobenius norm of a vector/matrix. We use \(\mathcal{CN}(\mu, \sigma^2)\) for the complex Gaussian distribution of a random variable, having independent Gaussian distributed \( \mathcal{N}(\mu, \frac{\sigma^2}{2}) \) real and imaginary parts with mean \( \mu \) and variance \( \frac{\sigma^2}{2} \). We use \( p_Y(y) \) for the probability density function (PDF) of a random variable \( Y \), and \( E_Y[\cdot] \) for the statistical expectation with respect to \( Y \). We use \( \text{Re}\{\cdot\} \) for the real part of a complex variable, and \( X_{\\text{AR}} \) represents a constellation of size \( M \).
where \( h \) represents the activated antenna and \( x_q \) is the \( q \)th symbol from the constellation \( X_M \). Hence, only the \( j \)th antenna remains active during symbol transmission. Figure 2 in [1] illustrates an example of the mapper and is omitted here due to space constraints. For example, in 3 bits/s/Hz transmission the \( j \)th antenna is transmitted from the four available antennas. The output of the channel when \( x_q \) is transmitted from the \( j \)th antenna is expressed as

\[
y = \sqrt{\frac{P}{2}} h_j x_q + \eta
\]

where \( h_j \) denotes the \( j \)th column of \( H \).

C. SM Detection

In [1], assuming constant modulus signaling such as PSK, a sub-optimal detection rule is given by

\[
\hat{j} = \arg \max_j |h_j^H y| \quad \hat{q} = \arg \max_q \text{Re}\left\{ (h_j x_q)^H y \right\}
\]

where \( \hat{j} \) and \( \hat{q} \) represent the estimated antenna and symbol index, respectively. Since the mapping is one to one, the demapper obtains an estimate of the transmitted bits by taking \( \hat{j} \) and \( \hat{q} \) as inputs. However, this detector only works for transmission over normalized channels (i.e. for each channel realization, \( H^H H = I_{N_t} \)). Conventionally though, each entry of \( H \) is distributed according to \( CN(0, 1) \) and hence, \( E_h [H^H H] = N_t I_{N_t} \) where there is no control over each channel realization. In Section IV, we see that without such an assumption, the detector fails, and even with their assumption, the detection is sub-optimal. We now derive the optimal detector for SM and show that we cannot separate the problem of estimating the antenna and symbol index.

III. OPTIMAL DETECTION AND PERFORMANCE ANALYSIS

A. Optimal Detection

Since the channel inputs are assumed equally likely, the optimal detector is ML, which is given by

\[
[j_{ML}, q_{ML}] = \arg \max_j p(y | x_{jq}, H) = \arg \min_j \sqrt{\frac{\rho}{2}} ||g_{jq}||_F^2 - 2 \text{Re} \left\{ y^H g_{jq} \right\}
\]

where \( g_{jq} = h_j x_q, 1 \leq j \leq N_t, 1 \leq q \leq M, \) and \( p_y (y | x_{jq}, H) = \pi^{N_r} \exp\left(-\frac{1}{2} ||y - \sqrt{\rho} H x_{jq}||_F^2\right) \) is the PDF of \( y \), conditioned on \( x_{jq} \) and \( H \). It can be seen that detection is a joint optimization problem which cannot easily be separated. Even with normalized channels and constant modulus signaling (i.e. \( ||g_{jq}||_F^2 = 1 \)), the detector reduces to

\[
[j_{ML}, q_{ML}]_{PSK} = \arg \max_j \text{Re} \left\{ y^H g_{jq} \right\}
\]

which is different from [1].

B. Performance Analysis

The performance of the SM system will be derived using the well known union bounding technique [5, p. 261-262]. The average bit error rate (BER) in SM is union bounded as

\[
P_{b, bit} \leq \mathbb{E}_x \left[ \sum_{j=1}^{N_t} \sum_{q=1}^{N_r} \sum_{\hat{q}=1}^{M} \frac{N(q, \hat{q}) P(x_{jq} \rightarrow x_{\hat{q}j})}{N_t M} \right]
\]

where \( N(q, \hat{q}) \) is the number of bits in error between the symbol \( x_q \) and \( x_{\hat{q}} \), and \( P(x_{jq} \rightarrow x_{\hat{q}j}) \) denotes the pairwise error probability (PEP) of deciding on the constellation vector \( x_{\hat{q}j} \) given that \( x_{jq} \) is transmitted. By simplifying (2), the PEP conditioned on \( H \) is given by

\[
P(x_{jq} \rightarrow x_{\hat{q}j} | H) = P(d_{jq} > d_{\hat{q}j} | H) = Q\left(\sqrt{\frac{\rho}{2}}\right)
\]

where \( d_{jq} = \left( \frac{1}{2} ||g_{jq}||_F^2 - 2 \text{Re} \left\{ y^H g_{jq} \right\} \right) \) and \( Q(x) = \int_x^{\infty} \frac{1}{2\pi} e^{-\frac{t^2}{2}} dt \). We define \( \kappa \) as

\[
\kappa \triangleq \frac{\rho}{2N_t} \frac{\sum_{n=1}^{N_r} ||g_{jq} - g_{\hat{q}j}||_F^2}{P} = \sum_{n=1}^{N_r} [A(n) + iB(n)]^2
\]

where \( i = \sqrt{-1} \) and

\[
A(n) = \sqrt{\frac{\rho}{2N_t}} (h_{n_j}^R x_{j}^R - h_{n_j}^I x_{j}^I - h_{n_j}^R x_{\hat{q}}^R + h_{n_j}^I x_{\hat{q}}^I)
\]

\[
B(n) = \sqrt{\frac{\rho}{2N_t}} (h_{n_j}^R x_{j}^I + h_{n_j}^I x_{j}^R - h_{n_j}^R x_{\hat{q}}^I - h_{n_j}^I x_{\hat{q}}^R)
\]

The superscript \( R \) and \( I \) denote the real and imaginary part, respectively, and \( h_{n_m} \) is the element of \( H \) in the \( n \)th row, and \( m \)th column. The distribution of the random variable \( \kappa \) in (4) is not easily obtained since \( A(n) \) and \( B(n) \) are not, in general, independent. In this case, the performance can be evaluated numerically. However, for symbols \( x \) drawn from a real constellation \( X_M \), this independence is satisfied and (4)
reduces to \( \kappa = \sum_{n=1}^{2N_r} \alpha_n^2 \) where \( \alpha_n \sim \mathcal{N}(0, \sigma^2_{\alpha}) \) with \( \sigma^2_{\alpha} = \rho(\|x_n\|^2 + \|x_d\|^2) \). Hence, \( \kappa \) is a chi-squared random variable with \( s = 2N_r \) degrees of freedom and PDF \( p_{\kappa}(v) \) given in [5, p. 41]. The PEP can then be formulated as

\[
    P(\mathbf{x}_{ij} \rightarrow \mathbf{x}_{ij}) = E_{\kappa} [P(\mathbf{x}_{ij} \rightarrow \mathbf{x}_{ij} | H)] = \int_{0}^{\infty} Q(\sqrt{v}) p_{\kappa}(v) dv = \int_{t=0}^{\infty} \exp\left(-\frac{t^2}{2}\right) F_{\kappa}(t^2) dt
\]

(5)

where the last line follows from a simple change of integration order and \( F_{\kappa}(y) = \int_{0}^{y} f_{\kappa}(v) dv \) is the chi-squared cumulative distribution function (CDF). We use the expression for \( F_{\kappa}(y) \) given in [5, p. 42, Eq. (2.1 – 114)], and a closed form integral expression from [6, p. 337, Eq. (3.326 – 2)] to simplify Equation (5) as

\[
    P(\mathbf{x}_{ij} \rightarrow \mathbf{x}_{ij}) = \frac{1 - \sum_{k=0}^{N_r-1} \Gamma(k) \left(\frac{2\sigma^2_{\alpha}}{\sigma^2}\right)^k} {2 \mu_\alpha}
\]

where \( \mu_\alpha = \sqrt{\frac{\sigma^2_{\alpha} + \frac{\sigma^2}{\sigma^2}}{\sigma^2}} \), \( m = \frac{N_r}{2} \), and \( k' = k + \frac{1}{2} \). Using [6, p. 897, Eq. (8.339 – 2)], with some straightforward algebra, we get the PEP expression as

\[
    P(\mathbf{x}_{ij} \rightarrow \mathbf{x}_{ij}) = \frac{\mu_\alpha - \sum_{k=0}^{N_r-1} \binom{2k}{k} \left(\frac{2\mu_\alpha \sigma_{\alpha}}{\sigma_{\alpha}}\right)^{-2k}} {2 \mu_\alpha}
\]

(6)

Plugging (6) into (3), we obtain

\[
    P_{e,bit} \leq \sum_{q=1}^{M} N_t \sum_{q'=1}^{M} N_r \left(\frac{\mu_\alpha - \sum_{k=0}^{N_r-1} \binom{2k}{k} \left(\frac{2\mu_\alpha \sigma_{\alpha}}{\sigma_{\alpha}}\right)^{-2k}} {4M \mu_\alpha} \right)
\]

(7)

IV. Simulation Results

In this section, we present some examples to compare the optimal SM’s detector over Mesleh’s SM detection scheme [1]. We perform Monte Carlo simulations for at least \( 10^6 \) channel realizations and plot the average BER performance versus \( \rho \), the average SNR per receive antenna. Three bits/s/Hz transmission with \( N_r = 4 \) antennas are assumed in all schemes. Fig. 2 illustrates the simulation results for both constrained (dotted line) and conventional (solid line) channel assumptions (see Section II-C). The MRC scheme is essentially a single input multiple output (SIMO) communication system using APM and employing an ML receiver, where we use 8-QAM to achieve the spectral efficiency requirement. V-BLAST using BPSK with \( N_t = 3 \) antennas and ordered successive interference cancellation (OSIC) using the minimum mean squared error (MMSE) receiver [7] is also compared. SM with BPSK and \( N_t = 4 \) antennas is shown for both sub-optimal [1] and optimal receivers (derived in Section III-A), along with the SM BER bound of (7).

Let us first consider the case of constrained channels (dotted lines). As shown, the optimal SM detector gains 4 dB at \( P_{e,bit} = 10^{-5} \) over Mesleh’s SM detector. Higher gains are noticed when simulations over conventional channels (solid lines) are performed, in which Mesleh’s detector fails. As well, prior to this work, SM was inferior in terms of performance over V-BLAST and MRC, and its advantages mainly lied in enabling simple detection as well as removing the need for ICI and IAS. However, it is shown here that optimal SM also provides performance improvements over these popular schemes (3 dB gain over MRC at \( P_{e,bit} = 10^{-5} \)), motivating its use in practical systems.

V. Conclusion

In this letter, we derive the optimal detector for SM for which, significant performance gains are observed over the detector in [1]. To support the results, we also derive a closed form expression for the average BER of SM when real constellations are used. As well, optimal SM is shown to outperform V-BLAST and MRC, making it an excellent candidate for future wireless communication systems.

REFERENCES