Performance of STBC with Unequal-Power Co-Channel MIMO Interferers Under Path Loss and Rayleigh Fading

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Abstract — In a cellular system, when multiple-input multiple-output (MIMO) techniques are employed, the interference scenario is quite complicated. In this paper, considering realistic propagation conditions including both path loss and Rayleigh fading, and considering the existence of unequal-power interferers, we analyze the statistical distributions of the post-processing SIRs for downlink space-time block coding (STBC) transmission with different co-channel MIMO interferers. Simulation results verify the validity of the theoretical analyses.

I. INTRODUCTION

Due to their potential for achieving higher link reliability and data rates, MIMO techniques utilizing multiple antennas at the transmitter and/or receiver have emerged as a milestone in modern wireless communications. Typical MIMO techniques include STBC, open-loop spatial multiplexing (OLSM), closed-loop spatial multiplexing (CLSM), MIMO beamforming (MBF) and maximal ratio transmission (MRT). A well-designed cellular system is inherently interference-limited; when MIMO techniques are incorporated, the interference scenario will become more complicated [1-3]. As one of the important and promising MIMO techniques, STBC [4-5] has been adopted in several emerging wireless standards. In addition, STBC is a good option for communicating with users on the cell edges, due to its ability to combat fading and its robustness to co-channel interference.

Analyzing the post-processing signal-to-interference-plus-noise ratio (SINR) or signal-to-interference ratio (SIR) is very useful in investigating the performance of cellular MIMO systems. Some work has been done in this area. In [6] and [7], the focus is on the distribution of post-processing SINR for maximal ratio combining (MRC), and in [8], optimum combining is considered. For equal-power interfering base stations (BS), the distributions of the post-processing SIRs for several multi-antenna techniques are derived in [9]. In all of the above work, it is assumed that all the cells adopt the same MIMO mode; this is unrealistic and would require significant coordination and complexity. To relax this constraint, in [10], the impact of interfering MIMO schemes on other MIMO schemes is studied; however, in this work, only a simulation-based approach is used.

In [11], assuming all the wireless channels are i.i.d. complex Gaussian random variables with zero mean and unit variance, the probability density functions (pdf) of the post-processing SIRs for STBC with various equal-power/unequal-power interfering MIMO schemes are analyzed. The assumption of i.i.d. channels in [11] can be viewed as the case with only pure Rayleigh fading. If the channels are modeled to include the effects of both path loss and Rayleigh fading, so that generally they are independent but non-identical complex Gaussian variables, even for the case of equal-power MIMO interferers, the derivation for the distribution of the post-processing SIR is much more difficult.

In this paper, we analyze the distributions of the post-processing SIRs for STBC with various co-channel MIMO interferers under more realistic propagation conditions and with unequal-power interferers. Closed-form expressions for the pdfs of the interference in the post-processing SIRs are provided. Further, the distributions of the post-processing SIRs are analyzed. The validity of theoretical analyses is verified by simulation results. The rest of the paper is organized as follows: The system model is described in Section II. The distribution of the SIR is analyzed in Sections III and IV. Simulation results and conclusions are provided in Sections V and VI, respectively.

II. SYSTEM MODEL

In this paper, focusing on the downlink of a cellular-like system, we consider an interference-limited system using STBC in the desired link with $K$ co-channel interfering BSs. Each BS employs $M$ ($M \geq 2$) transmit antennas and the desired receiver employs $N$ ($N \geq 2$) receive antennas. Denote $h_{k,ij} (k = 0, 1, \ldots, K; i = 1, \ldots, N; j = 1, \ldots, M)$ as the instantaneous channel coefficients between the $i$-th receiving antenna at the desired receiver and the $j$-th transmitting antenna at the $k$-th BS. The index $k = 0$ indicates the desired BS. The channel coefficients include the effects of path loss and quasi-static flat Rayleigh fading. Since the distances from the antennas of a co-located array to the antennas of another co-located array can be considered equal, for any one given $k$, $h_{k,ij}$ ($i = 1, \ldots, N; j = 1, \ldots, M$) have the same mean channel gain.
\[ \mu_k. \] It is assumed that the system is located in a rich scattering environment and the antennas are sufficiently separated, so that the channels \{h_{k,ij}\} can be modeled as independent complex Gaussian variables with zero mean and variance \( \mu_k. \) The transmit power of each antenna at the \( k \)-th BS is denoted as \( P_k. \) For simplicity, we include the transmit powers into the channel power gains; so, we consider an equivalent system in which (i) the channels \{h_{k,ij}\} are independent complex Gaussian random variables with zero mean and variance \( P_k \mu_k \) and (ii) each antenna at each BS uses unit transmit power.

At the desired receiver, the received signal is corrupted by co-channel interference and additive white Gaussian noise (AWGN). Since we only consider an interference-limited environment, the effect of noise will be neglected in the subsequent analysis. We also assume that each BS only has knowledge about its own channel state information (CSI); thus, no knowledge of the interferers can be used by the desired receiver to mitigate the impact of the co-channel interference. This can be viewed as a “worst-case” scenario.

When the desired link uses STBC, as in [11] we denote the “first category” of interfering MIMO schemes as STBC, OLSM and CLSM, and the “second category” as MBF and MRT. It has been shown in [11] that the co-channel MIMO interfering schemes of the same category have the same effect on the desired received signal and the same expression for the effective post-precessing SIR. Thus, in this paper, under more realistic propagation conditions and with the existence of unequal-power interferers, we analyze the distributions of the post-precessing SIRs for downlink STBC transmission with the two categories of co-channel MIMO interferers.

The analytical derivations will be presented by using a (2Tx, 2Rx) antenna configuration (i.e., \( M = N = 2 \)) as an example. Under this antenna configuration, in the case of STBC, OLSM or CLSM, for every two consecutive symbol periods, during the first symbol period, \( s_{k,1} \) and \( s_{k,2} \) are preprocessed and transmitted from the two antennas of the \( k \)-th BS; in the next symbol period, \( s_{k,3} \) and \( s_{k,4} \) are sent simultaneously. In particular, for BSs employing an STBC mode (e.g., the desired BS), an Alamouti code [4] is used so that \( s_{k,3} = -s_{k,2}^{*} \) and \( s_{k,4} = s_{k,1}^{*} \). In contrast, in the case of MBF or MRT, only one symbol is transmitted during one symbol period.

### III. SIR DISTRIBUTION FOR STBC WITH DIFFERENT CO-CHANNEL MIMO INTERFERERS

### A. Interfering Links Use First-Category MIMO Schemes

Because the co-channel MIMO interferers of the same category have the same effect on the desired received signal [11], in the subsequent derivations, we use STBC to stand for the first category of interfering MIMO schemes. For the (2Tx, 2Rx) antenna configuration which we use to illustrate the analysis, the signal matrix at the \( k \)-th BS is

\[
\begin{bmatrix}
  s_{k,1} & s_{k,2} \\
  -s_{k,2}^* & s_{k,1}^*
\end{bmatrix}
\]

where the row of the matrix indicates the time index and the column indicates the antenna index. Denote

\[
\begin{align*}
a_0 &= [h_{0,11}^*, h_{0,12}, h_{0,21}, h_{0,22}]^T \\
b_{k1} &= [h_{k,11}^*, h_{k,12}, h_{k,21}, h_{k,22}]^T \\
b_{k2} &= [h_{k,12}^*, -h_{k,11}, h_{k,22}, -h_{k,21}]^T
\end{align*}
\]

Then, similar to the related result in [11], the post-processing SIR at the desired receiver is

\[
\text{SIR} = \frac{\frac{1}{2} \sum_{k=1}^{K} \left( |a_0^H b_{k1}|^2 / |a_0^H a_0| + |a_0^H b_{k2}|^2 / |a_0^H a_0| \right)}{\frac{1}{2} \max_{k=1}^{K} |a_0^H b_{k2}|^2 / |a_0^H a_0|}
\]

Note that, different from [11], here, the transmit powers \( \{P_k\} \) have been included into the channel power gains. In (1), \( X = |a_0^H a_0| \) has a central chi-squared distribution and the pdf is

\[
f_X(x) = \frac{x^{(\nu-2)/2}e^{-x/2}}{(\nu/2)^{\nu/2}2^{\nu/2} \Gamma(\nu/2)}
\]

Further, by exploiting the result in [6], \( y_{ki} = [a_0^H b_{ki}]^2 / |a_0^H a_0| (k = 1, \ldots, K; i = 1, 2) \) has an exponential distribution with pdf \( f_{y_{ki}}(y_{ki}) = \frac{1}{\mu_{ki}}e^{-y_{ki}/\mu_{ki}} \).

For \( k', k'' \in \{1, \ldots, K\} \) with \( k' \neq k'' \), \( y_{ki} \) and \( y_{k'i} \) (\( i = 1, 2 \)) are independent. For any given \( k \in \{1, \ldots, K\} \), it is reasonable to assume that \( y_{k1} \) and \( y_{k2} \) are mutually independent [9,11]; this assumption can be justified from the fact that, in \( y_{k1} \) and \( y_{k2} \), the common terms related to \{h_{0,ij}\} are multiplied by independent random variables so that \( y_{k1} \) and \( y_{k2} \) become nearly independent. Later, the assumption will be further justified through Monte Carlo simulations. In general, for \( k' \neq k'' \), \( y_{ki} \) and \( y_{k'i} \) (\( i = 1, 2 \)) have different distributions, except for the special case of \( P_k \mu_{k'} = P_k \mu_{k''} \). In contrast, for any given \( k, y_{k1} \) and \( y_{k2} \) have the same distribution.

Thus, generally, \( Y = \sum_{k=1}^{K} (y_{ki} + y_{k2}) \) is the sum of 2K independent but non-identically exponentially distributed random variables. Only (i) when \( K = 1 \) or (ii) when \( K > 1 \) and \( P_k \mu_1 = \ldots = P_k \mu_K \), will \( Y \) be simplified as the sum of i.i.d. exponentially distributed random variables; in this special case, \( Y \) follows a central chi-squared distribution, equivalent to what is analyzed in [11]. In the following, we will first derive the closed-form pdf expression for \( Y \), then analyze the distribution of the post-processing SIR, \( X/Y \).

Denote the 2K independent exponentially distributed random variables \( y_{ki} \) (\( k = 1, \ldots, K; i = 1, 2 \)) as \( \beta_1 = y_{11}, \beta_2 = y_{12}, \ldots, \beta_{2K-1} = y_{K1}, \beta_{2K} = y_{K2}. \) Then, \( \beta_i \) (\( i = 1, \ldots, 2K \)) are independent exponentially distributed random variables with the parameters \( 1/\mu_{ki} \) (\( k = \lfloor i/2 \rfloor \)), where \( \lfloor i/2 \rfloor \) means “rounding \( i/2 \) to the the next largest integer”. Let \( \alpha_i \) (\( i = 1, \ldots, 2K \)) be 2K independent complex Gaussian random variables with zero mean and variance \( P_k \mu_k \) (\( k = \lfloor i/2 \rfloor \)) per complex dimension. Since \( |\alpha_i|^2 \) has the same distribution as \( \beta_i \), for any \( i \in \{1, \ldots, 2K\} \), denote \( \beta_i = |\alpha_i|^2 \). Next, we derive the pdf of \( Y = \sum_{k=1}^{K} (y_{k1} + y_{k2}) = \sum_{k=1}^{2K} \beta_i = \sum_{i=1}^{2K} |\alpha_i|^2 \) using the properties of quadratic forms in complex Gaussian variables [12].

1) Closed-Form pdf for Interference Term in Post-Processing SIR: Denote \( |\alpha_1, \ldots, \alpha_{2K}|^2 \) as \( \alpha \) and the \( 2K \times 2K \) identity ma-
trix as I. Because the identity matrix I is a semi-positive definite Hermitian matrix, and \( \alpha \) is a zero-mean complex Gaussian random vector, \( Y = \sum_{k=1}^{K} (y_{k1} + iy_{k2}) = \sum_{i=1}^{2K} |\alpha_i| = \alpha^H \alpha \) is a central quadratic form in Gaussian random variables [12]. The mean value of \( |\alpha_i|^2 \) is equal to \( P_k \mu_k \) with \( k = [i/2] \) \((i = 1, ..., 2K)\). Further, denote \( V_{2K \times 2K} \) as \( E[\alpha \alpha^H] = \text{diag}(P_1 \mu_1, P_1 \mu_1, P_2 \mu_2, P_2 \mu_2, ..., P_K \mu_K, P_K \mu_K) \), which is the covariance matrix of \( \alpha \). Because \( V \) is a semi-positive definite real symmetric matrix, there exists a real symmetric matrix \( Q_{2K \times 2K} \) for which \( QQ = V \). We can easily obtain \( Q = \text{diag}(\sqrt{P_1 \mu_1}, \sqrt{P_1 \mu_1}, \sqrt{P_2 \mu_2}, \sqrt{P_2 \mu_2}, ..., \sqrt{P_K \mu_K}, \sqrt{P_K \mu_K}) \). Denote \( B \) as \( QIQ \), which is a semi-positive definite Hermitian matrix. Since \( B = QIQ = V \), the eigenvalue decomposition of \( B \) is equal to the eigenvalue decomposition of \( V \). That is to say, the 2K eigenvalues of \( B \) are \( \lambda_1 = P_1 \mu_1, \lambda_2 = P_1 \mu_1, \lambda_3 = P_2 \mu_2, \lambda_4 = P_2 \mu_2, ..., \lambda_{2K-1} = P_K \mu_K, \lambda_{2K} = P_K \mu_K \).

Without loss of generality, denote \( M_p \) \((M_p \leq 2K)\) as the total number of distinct positive eigenvalues and \( v_m \) as the multiplicity of \( \lambda_m \) \((m = 1, ..., M_p)\). For central quadratic forms in complex Gaussian random variables, the moment generating function of \( Y \) can be expressed as

\[
\Phi_Y(s) = E[e^{sY}] = 1 / \prod_{m=1}^{M_p} (1 - s \lambda_m)^{v_m}
\]  

(2)

In (2), \( E[\cdot] \) represents expectation.

Denote \( f_Y(y) \) as the pdf of \( Y \), which is the inverse Laplace transform of \( \Phi_Y(s) \). Since the value of \( Y \) is definitely non-negative, we derive \( f_Y(y) \) for \( y \geq 0 \). By utilizing the Laplace Inversion Theorem, the Jordan’s Lemma, and the Residue Theorem [13], we get

\[
f_Y(y) = \sum_{m=1}^{M_p} f_m(y) = \sum_{m=1}^{M_p} \text{Res}[\Phi_Y(s)e^{-sy}, 1/\lambda_m], \quad y \geq 0
\]  

(3)

where \( f_m(y) \) is the residue of \( \Phi_Y(s)e^{-sy} \) at the pole \( 1/\lambda_m \).

Then, we calculate \( f_m(y) \) \((m = 1, ..., M_p)\) as follows

\[
f_m(y) = \text{Res}[\Phi_Y(s)e^{-sy}, 1/\lambda_m]
= \frac{1}{v_m} \lim_{s \rightarrow -1/\lambda_m} \frac{d}{ds} \Phi_Y(s)e^{-sy}
= (-\lambda_m)^{-v_m}e^{-sy} \sum_{m=0}^{v_m-1} C_{v_m}^r (-y)^{v_m-r} \phi^{(r)}(\lambda_m)
\]  

(4)

In the third equality in (4), \( C_i^j = t!/i!(t-i)! \) denotes the number of ways of selecting \( i \) elements from \( t \) elements; \( C_i^j = 1 \) for \( i = 0 \) and \( t \geq 0 \). Moreover, \( \phi^{(r)}(\lambda_m) \) is the \( r \)-th order derivative of \( \phi(\lambda_m) \), and \( \phi(\lambda_m) \) is specified as

\[
\phi(\lambda_m) = \begin{cases} 1, & M_p = 1 \\ \prod_{n \not= m} (\lambda_n - \lambda_m)^{v_n}, & M_p > 1 \end{cases}
\]  

(5)

Using the logarithmic derivative [12] to evaluate the derivatives of \( \phi(\lambda_m) \), we have

\[
\phi^{(r)}(\lambda_m) = G^{(r)}(\lambda_m) \phi(\lambda_m)
\]  

(6)

In (6), \( G^{(r)}(\lambda_m) \) \((m = 1, ..., M_p)\) is equal to 1 when \( r_m = 0 \), and is equal to 0 when \( r_m \geq 1 \) and \( M_p = 1 \); when \( r_m \geq 1 \) and \( M_p > 1 \), it is given as

\[
G^{(r)}(\lambda_m) = \sum_{r_m=0}^{r_m-1} \sum_{r_{m'}=0}^{r_{m'}-1} \sum_{r_{m''}=0}^{r_{m''}-1} G^{(r_{m'')}(\lambda_m) \times ... \times G^{(r_{m''}(\lambda_m)}
\]  

(7)

In (7), \( g^{(t)}(\lambda_m) \) \((m = 1, ..., M_p)\) is further specified as

\[
g^{(t)}(\lambda_m) = t! \times \sum_{n=1}^{M_p} v_n \times \prod_{n \not= m} \lambda_n^{t+1} \times \phi^{(t)}(\lambda_m)
\]  

(8)

By combining (3)-(8), the closed-form expression for the pdf of \( Y \) is given as

\[
f_Y(y) = \sum_{m=1}^{M_p} \left\{ -\frac{e^{-wy}}{-\lambda_m} \phi(\lambda_m) \times \sum_{r_m=0}^{v_m-1} C_{v_m}^r (-y)^{v_m-r} \phi^{(r)}(\lambda_m) \right\}
\]  

(9)

Recall that the 2K eigenvalues of \( B \) are \( \lambda_1 = P_1 \mu_1, \lambda_2 = P_1 \mu_1, \lambda_3 = P_2 \mu_2, \lambda_4 = P_2 \mu_2, ..., \lambda_{2K-1} = P_K \mu_K, \lambda_{2K} = P_K \mu_K \). For the special cases in which (i) the number of interfering BSs, \( K \), is equal to 0 or (ii) \( K > 1 \) with \( P_{j_1} = ... = P_{j_l} \mu_j \), clearly, \( M_p = 1 \) and \( \lambda_j = P_{j_1} \mu_j \). Then, using (5) and (7)-(9), we get

\[
f_Y(y) = \sum_{k=1}^{K} \left\{ \frac{K!}{P_{k} \mu_k^{M_p}} \prod_{j=1}^{K} (\varepsilon_{k,j})^2 \left( -y + K \right) \right\}
\]  

(11)

In (11), \( \varepsilon_{k,j} = \frac{P_{k} \mu_k - P_{j} \mu_j}{P_{k} \mu_k - P_{j} \mu_j} \).

2) Distribution of Post-Processing SIR: Next, we consider the distribution of the post-processing SIR given in (1), which is equal to \( X/Y \). Although \( X = \frac{|a_m^H a_0|^2}{|a_m^H b_0|^2} \) and \( Y = \sum_{k=1}^{K} (|a_m^H b_k|^2 + |a_m^H b_{k,j}|)^2 \) may not be independent since both include terms related to \( a_0 \), we initially assume that they are independent. This assumption can be justified from the fact that, in \( X \) and \( Y \), the common terms are multiplied by independent random variables so that they become nearly independent. The assumption is further justified later through Monte Carlo simulations. (In [11], this...
assumption was verified for equal-power interferers with pure Rayleigh fading channels.)

Then, the pdf of the post-processing SIR, \( \frac{X}{Y} = \gamma (\geq 0) \), is given as [14]

\[
f_{y}(r) = \int_{-\infty}^{+\infty} f_{X}(yr)f_{Y}(y) \mid y \mid dy = \int_{0}^{+\infty} f_{X}(yr)f_{Y}(y)dy
\]

(12)

For the special cases with the \( f_{y}(y) \) shown in (10), because \( X \) and \( Y \) are two independent central chi-squared random variables, (12) can be simplified as a canonical \( F_{C} \)-distribution, consistent with the result in [11] for equal-power STBC interferers with i.i.d. channels. However, for the general scenarios, due to the complicated mathematical form of \( f_{y}(y) \) shown in (11), (12) can only be calculated via numerical evaluations.

Remarks: For a general (MTx, NRx) antenna configuration, using the same mathematical techniques, the closed-form pdf expression of \( Y \) can be obtained in a similar fashion. When the \( K \) interfering links use OLSM (or CLSM), exploiting the results in [11], it can be found that the approximate expression for the post-processing SIR has the same form as using STBC at the interfering BSs. Thus, under realistic propagation conditions and with the existence of unequal-power interferers, the closed-form pdf expression for the denominator of the post-processing SIR and the distribution of the post-processing SIR can be analyzed using the same mathematical techniques.

B. Interfering Links Use Second-Category MIMO Schemes

When the second category of interfering MIMO schemes is used by the interfering BSs, we can use MFB [15] to represent this category. For the (2Tx, 2Rx) antenna configuration which we use to illustrate the analysis, at the \( k \)-th (\( k = 1, ..., K \)) interfering BS, the signal matrix is

\[
\begin{bmatrix}
k_{1,1} & k_{1,2} \\
n_{1,2} & n_{2,2}
\end{bmatrix}
\]

The precoding matrix of MFB is given by

\[
\begin{bmatrix}
\sqrt{2}k_{1,1} & 0 \\
0 & \sqrt{2}k_{2,2}
\end{bmatrix}
\]

with \( |w_{k,1}|^2 + |w_{k,2}|^2 = 1 \)

where \( w_{k,1} \) and \( w_{k,2} \) come from the singular vector \( [w_{k,1} \ w_{k,2}] \) corresponding to the maximum singular value of the channel matrix from the \( k \)-th BS to the desired receiver. Denote

\[
\begin{align*}
h_{k,1} &= h_{k,11}w_{k,1} + h_{k,12}w_{k,2} \\
h_{k,2} &= h_{k,21}w_{k,1} + h_{k,22}w_{k,2} \\
x_{0} &= |h_{0,11}|^2 + |h_{0,12}|^2 + |h_{0,21}|^2 + |h_{0,22}|^2 \\
y_{k} &= \frac{|h_{0,11}|^2 + |h_{0,12}|^2}{|h_{0,11}|^2 + |h_{0,12}|^2}
\end{align*}
\]

Here, again, the transmit powers \( \{P_{k}\} \) are included in the channel power gains. Since \( |w_{k,1}|^2 + |w_{k,2}|^2 = 1 \), \( h_{k,1} \) and \( h_{k,2} \) are still independent complex Gaussian random variables with zero mean and variance \( P_{k} \).

Similar to the related result in [11], the post-processing SIR is approximately expressed as

\[
\text{SIR}_{\text{STBC-MFB}} \approx \frac{x_{0}}{2} \cdot \sum_{k=1}^{K} \frac{y_{k}}{y_{k}}
\]

(13)

In (13), \( x_{0} \) has a central chi-squared distribution with \( 2 \times (2 \times 2) \) degrees of freedom, and the pdf of \( X = x_{0}/2 \) is \( f_{X}(x) = 2^{-2}x^{2-1} \exp(-x/2) \). Further, \( y_{k} (k = 1, ..., K) \) has an exponential distribution with pdf \( f_{y_{k}}(y_{k}) = \frac{1}{P_{k} \mu_{y_{k}}} \exp(-y_{k}/P_{k} \mu_{y_{k}}) \). As in Section III-A, generally, \( y_{k} = \sum_{k=1}^{K} y_{k} \) is the sum of \( K \) independent but non-identically exponentially distributed random variables.

1) Closed-Form pdf for Interference Term in Post-Processing SIR: The closed-form expression for the pdf of \( Y \) can be obtained using similar mathematical derivations as in Section III-A. In this case, denote \( \mathbf{I} \) as the \( K \times K \) identity matrix and \( \alpha \) as \( [\alpha_{1}, ..., \alpha_{K}] \) such that \( \alpha_{i} (i = 1, ..., K) \) are independent central chi-squared random variables with zero mean and variance \( P_{k} \mu_{y_{k}} \) per complex dimension. Then \( Y = \sum_{k=1}^{K} y_{k} = \alpha \mathbf{I} \) is a central quadratic form in complex Gaussian random variables. The \( K \) eigenvalues of \( \mathbf{B} \) are \( \lambda_{1} = P_{1} \mu_{1}, \lambda_{2} = P_{2} \mu_{2}, ..., \lambda_{K-1} = P_{K-1} \mu_{K-1}, \lambda_{K} = P_{K} \mu_{K} \). For the special cases in which (i) \( K = 1 \) or (ii) \( K > 1 \) with \( \mu_{1} = 1 \) and \( \lambda_{K} = K \mu_{K} \), we get \( M_{p} = 1 \) and \( \mu_{1} = 1 \), \( \gamma_{1} = 1 \). By exploiting (5) and (7)-(9), the pdf of \( Y \) is

\[
f_{y}(y) = \frac{y^{K-1} \exp(-y \mu_{y})}{(\mu_{y})^{K}}
\]

(14)

The result here is similar to (10); the only difference is that the factor \( 2K \) in (10) is replaced by \( K \). In particular, this is a central chi-squared distribution with \( 2K \) degrees of freedom, consistent with what we derived in [11] for equal-power MFB interferers with i.i.d. channels.

For the general scenarios where \( K > 1 \) and \( P_{1} \mu_{i} \neq P_{j} \mu_{j} \) for \( i \neq j \) with \( i, j \in \{1, ..., K\} \), clearly, \( M_{p} = K \) and \( y_{k} = 1 (k = 1, ..., M_{p}) \). Then, utilizing (5) and (7)-(9), we have

\[
f_{y}(y) = \sum_{k=1}^{K} \left\{ \frac{e^{-y \mu_{y}}}{\mu_{y}^{K}} K \prod_{j=1}^{K} \frac{P_{k} \mu_{k}}{P_{j} \mu_{j}} \right\}
\]

(15)

2) Closed-Form CDF for Post-Processing SIR: As in Section III-A, we assume \( X \) and \( Y \) are independent. To facilitate the analysis for the distribution of the post-processing SIR, we denote \( Z = 2X = 2x_{0}/2 = x_{0} \); clearly, \( Z \) has a central chi-squared distribution with \( f_{Z}(z) = \frac{z^{K-1} \exp(-z/2)}{(\mu_{y})^{K} \Gamma(K)} \), and \( Z \) is independent of \( Y \). Then, we re-express \( \gamma \) as \( \frac{\gamma}{\lambda} = \frac{\gamma}{\lambda} = \frac{\gamma}{\lambda} \). For the special cases with the \( f_{y}(y) \) shown in (14), \( f_{z}(r) = 2f_{r}^{2}(2r) \) and \( f_{z}(r) \) follows a canonical \( F_{C} \)-distribution, consistent with the result in [11] for equal-power MFB interferers with i.i.d. channels.

For the general scenarios which have the \( f_{y}(y) \) shown in (15), \( f_{z}(r) \) needs to be calculated via numerical evaluations of the integral expression in (12). However, in this case, we can
use some mathematical manipulations to obtain a closed-form expression for the cumulative distribution function (CDF) of the post-processing SIR shown in (13). The derivation is presented next.

The CDF of the post-processing SIR, $\gamma$ ($\gamma \geq 0$), is denoted as $F_\gamma(r) = \Pr\{\gamma < r\}$; this can be calculated as

$$F_\gamma(r) = \Pr\{Z/(2Y) < r\} = \Pr\{Y > Z/(2r)\}$$

$$= \int_{0}^{+\infty} f_Z(z) \int_{z/(2r)}^{+\infty} f_Y(y) dy dz$$

(16)

To calculate the integral in (16), we first realize that, for a general (MTx, NRx) antenna configuration, the moment generating function of $Z$ can be expressed as

$$\Phi_Z(s) = E[e^{sz}] = \left(1 - s \times P_{\mu k}\right)^{MN}$$

(17)

For the (2Tx, 2Rx) case, $M = N = 2$. Further, we denote $\prod_{j=1,j \neq k}^{K} P_{\mu k,d}^{P_k}$ as $\xi_k$; by utilizing the fact that

$$\int_{0}^{+\infty} f_Y(y) dy = 1$$

we can easily obtain $\sum_{k=1}^{K} \xi_k = 1$. Then, using (15), (16) becomes

$$F_\gamma(r) = \int_{0}^{+\infty} f_Z(z) \sum_{k=1}^{K} \xi_k \exp\left(-\frac{z}{P_k\mu_k}\right) dz$$

$$= \int_{0}^{+\infty} f_Z(z) \sum_{k=1}^{K} \exp\left(-\frac{z}{2rP_k\mu_k}\right) dz$$

$$= \sum_{k=1}^{K} \xi_k \int_{0}^{+\infty} \exp\left(-\frac{1}{2rP_k\mu_k}\right) dz$$

$$= \sum_{k=1}^{K} \xi_k \cdot E[\exp\left(-\frac{1}{2rP_k\mu_k}\right)]$$

(18)

By replacing (17) with $s = -1/(2rP_k\mu_k)$ and using $M = N = 2$ into (18), the closed-form CDF expression of the post-processing SIR is given as

$$F_\gamma(r) = \sum_{k=1}^{K} \frac{\xi_k}{1 + \left(\frac{2rP_k\mu_k}{P_k\mu_k}\right)^{2\times2}}$$

(19)

**Remarks:** For a general (MTx, NRx) antenna configuration, using the same mathematical techniques, the closed-form pdf expression of $Y$ and the closed-form CDF expression of the post-processing SIR can be obtained. When the $K$ interfering BSs use MRT [16], because the precoding matrix for MRT has the same functional form as that for MBF, the approximate expression for the post-processing SIR has the same form as using MBF at the interfering BSs [11]. Thus, under realistic propagation conditions and with the existence of unequal-power interferers, the closed-form pdf for the denominator of the post-processing SIR and the closed-form CDF for the post-processing SIR can be easily obtained.

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**IV. SIMULATION RESULTS**

In this section, the validity of the theoretical analyses is verified through comparing the CDFs obtained with Monte-Carlo simulations. In particular, as an example, a (2Tx, 2Rx) antenna configuration is considered. Here, as in Section III, STBC and MBF are used to represent the first and second category of interfering MIMO schemes, respectively.

For the general scenarios where $K > 1$ and $P_i\mu_i \neq P_j\mu_j$ for $i \neq j$ with $i, j \in \{1, ..., K\}$, the comparisons are shown in Fig. 1 for both categories of MIMO interferers. The notation “$K = 6 \{1, 0.9, 0.8, 0.7, 0.6, 0.5\}$” means there are $K = 6$ interfering BSs; the $k$-th number within the braces stands for the value of $P_k\mu_k$ for the $k$-th interfering BS. Moreover, for the desired link, $P_0\mu_0$ is always set as 1. It can be observed from Fig. 1 that, for the first category of interferers, the analytical results match the Monte-Carlo results very well. For the second category of interferers, the analytical results perform very well in the low-SIR region; in the high-SIR region, a small gap exists between the analytical and simulation results and the gap decreases with an increase in the number of interfering BSs. This is because, for the second category of interferers, the expression for the post-processing SIR which we use to analyze the statistical distribution is derived with some approximation.

When comparing the two categories of interfering MIMO schemes, it can be seen from Fig. 1 that, under the same condition, the second category of interferers has less impact on the performance of STBC transmission in the desired link, especially in the high-SIR region. In addition, the performance gap between the two categories decreases with an increase in the number of interfering BSs. These observations are the same as that in [11] for equal-power MIMO interferers with i.i.d. channels.

Fig. 2 is for the special cases in which (i) $K = 1$ or (ii) $K > 1$ with $P_1\mu_1 = ... = P_K\mu_K$. Again, the results verify the validity of the theoretical analyses. All the other related observations are similar as what we observe in Fig. 1 for the general scenarios.
Fig. 2. Comparison of the analytical results and Monte-Carlo simulation when $P_1 \mu_1 = \ldots = P_K \mu_K$.

V. CONCLUSIONS

In this paper, using a realistic channel model including the effects of path-loss and Rayleigh fading and considering the existence of unequal-power interferers, we analyzed the statistical distributions of the post-processing SIRs for downlink STBC transmission with different categories of co-channel MIMO interferers. Simulation results verify the validity of the analytical derivations.

REFERENCES