Abstract—In this paper, we propose an unequal error protection (UEP) framework for progressive image transmission over wireless networks with both random errors and packet loss. By sophisticatedly interleaving each coded frame, the packet loss is converted into randomly punctured bits in a Turbo code. Therefore, error control of channels with both types of distortions is equivalent to dealing with random bit errors only through turbo codes with reduced rates. To optimally allocate the overall transmission rate between the source and channel codes, a genetic algorithm (GA) based method is proposed which not only largely reduces the optimization complexity but also gives results close to the brute force search in tested scenarios. Then, we extend the UEP to a product code scheme by adding Reed-Solomon (RS) code across the frames. An efficient rate allocation method for this product code with the use of interleaving is further presented. The effectiveness of the proposed schemes is demonstrated with extensive simulation results.

I. INTRODUCTION

An embedded image coder, such as SPIHT [1], provides the progressive mode of transmission but is very sensitive to channel noise. The data after the first bit error may impair the decoding and have to be discarded completely. Recently, joint source-channel coding (JSCC) is regarded as an effective tool to improve the image transmission performance. JSCC has two main design aspects. One is to design the appropriate channel protection methods, and the other is to optimally allocate the given bandwidth resource between the source coding and the channel coding so as to achieve the best possible end-to-end performance in the noisy environment.

Because of the different priority in segments of the progressive data, unequal error protection (UEP) [2]–[4] was often used than equal error protection (EEP) [5]. Many error-control solutions [2]–[14] have been proposed to deal with different kinds of channel noise and preserve the source coding efficiency. For binary symmetric channels (BSCs), in [5], [7], [8] rate-compatible punctured convolutional (RCPC) codes were used as inner code to protect progressive image transmitted over noisy channels and cyclic redundancy check (CRC) codes were adopted as outer coder for error detection. Fading channels or channels with packet loss were discussed in [7]–[11]. Since Reed-Solomon (RS) codes and convolutional codes are effective in handling the burst and random errors respectively, product codes, consisting of RCPC/CRC along the row direction and RS codes along the column direction, were used for channels with both random errors and packet loss (or burst errors) in [9], [10]. Recently, rate-compatible punctured Turbo codes (RCPT) [15], [16] with high coding gain were also considered in [2] for BSCs and in [11] for fading channels to achieve better error protection. Regarding the frame organization format, fixed length of information bits in a frame (but with various channel codeword length) was used in [5]. However, in practice it is more convenient to use the fixed channel codeword length but with variable length of information bits [2], [10].

A primary difficulty in the JSCC is the distortion based optimization process which is very complex in general. This is particularly prominent for product codes where two different types of codes are considered simultaneously [10]. In fact, though Sherwood and Zeger gave the basic product code scheme in [9], they did not give any method to find the optimal rate allocation for their system.

In this paper, both design aspects of JSCC are considered for the image transmission over channels with both random errors and packet loss. These two types of distortion often coexist in a wireless network where packet loss is due to wired network and the bit errors are more due to the wireless channels. First, we propose a new protection scheme by applying an interleaver to each coded frame so that the packet loss will be equivalent to random bit erasures throughout the frame. The effect can be further regarded as randomly puncturing a turbo code with reduced rate. As a result, when both packet loss and random errors occur, the error control can be considered as the design of optimal punctured turbo code rate for each frame at a specific bit error rate. The source and channel code rates will be jointly optimized for the fixed channel frame size, under the distortion criterion. Second, to reduce the optimization complexity, we propose a genetic algorithm (GA) based method which obtains the performance almost the same as the results of brute-force search in the tested scenarios. Finally, we further extend the consideration to a product code with interleaving, and present an algorithm to obtain more performance improvement.

The rest of paper is organized as follows. In section II, we describe the forward error correction technique used in this research and formulate the problem. In section III, we present the GA-based optimization algorithm in details. Analysis of the product code scheme with UEP is discussed in Section IV. Simulation results are presented in section V. Finally, conclusion is drawn in section VI.

Lei Yao and Lei Cao
Department of Electrical Engineering
The University of Mississippi, University, MS 38677, USA
Email: {lyao,lcao}@olemiss.edu

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2007 proceedings.
In a higher BER. The new BER errors can be seen as a channel with only random bit errors the receiver, the channel with both packet loss and random bit errors respectively. When the lost packet indices are not known to PLR and BER to be 

\[ P \] to random bit erasures throughout the frame. We set the packet loss and random bit errors, the packet loss is equivalent along the column. Therefore, for noisy channels with both packet loss and random bit errors, we can first optimize the RCPT to handle the random bit errors and get a rate allocation \( \{ r_{k_1}, r_{k_2}, \ldots, r_{k_M} \} \) to protect frames \( \{1, 2, \ldots, N \} \) with information length \( \{ s_1, s_2, \ldots, s_N \} \). Then, we reduce these code rates to handle the additional bit erasures converted from packet loss. For PLR \( P_t \), we have

\[
[(1 - P_t)/r_{k_i}^*] = 1/r_{k_i} \quad 1 \leq k_i \leq M
\] (2)

where \( r_{k_i}^* \) is the new RCPT code rate. Since packet loss is random, punctured positions caused by packet loss are random as well. The performance of random puncturing is inferior to the conventionally regular puncturing. Therefore, we need to add additional protection by increasing denominator value of \( r_{k_i}^* \) by 1. For example, if we get code rate \( r_{k_i}^* = 4/6 \) for equation (2), we further set it as 4/7. Simulation results show that this scheme gives more performance improvement.

Therefore, the essential idea is that by using an interleaver, both packet loss and random channel errors can be considered within one scenario that only uses RCPT/CRC coding and decoding, with an increased bit error rates based on equation (1) and (2). In packet-based network, it is more convenient to fix the size of the channel codeword and allow the number of information bits to be different [8], [10]. In this paper, we design UEP using RCPT for the fixed channel frame size. Important frames will be allocated more channel parity bits, i.e., with lower Turbo code rate, than less important frames.

Our objective is to find a channel code rate allocation \( \{ r_{k_1}, r_{k_2}, \ldots, r_{k_M} \}, r_{k_i} \in R \), corresponding to the N frames that maximizes the expected quality of the received image, i.e., minimizes the expected distortion \( \bar{D} \) of the image, subject to the given transmission rate. It needs to note that, an optimization process has been proposed for the case of fixed information length in [14], but still needs in-depth investigation for the case of fixed coded frame length.

Let \( p(i) \) be the probability of decoding error in the \( i \)th frame protected by channel code rate \( r_{k_i} \). The probability of no decoding error in all decoded \( N \) frame is \( P_N = \prod_{j=1}^{N} (1 - p(j)) \). When the first \( i \) frames are turbo decoded without errors but the \( (i+1) \)th frame has errors, we denote the corresponding probability and distortion as \( P_i \) and \( D_i \). Then,

\[
P_i = p(i + 1) \prod_{j=1}^{i} (1 - p(j)) \quad 0 < i < N,
\] (3)

\[
\bar{D} = \sum_{i=0}^{N} P_i D_i
\] (4)

Here, \( D_0 \) denotes the distortion when no correct frame is received. The probability of decoding error in the first frame is \( P_0 = p(1) \).

Brute force search may be used to solve the problem when \( M \) and \( N \) are small. However, it is not practical when \( M \) and \( N \) values are large. This is because for a \( M \) tuple channel code rate set and \( N \) frames of source bits, the number of candidate solutions is \( M^N \). It is also known that when UEP in channel
encoding is used, since the most important source bits are placed in the first frame and the source rate-distortion function is convex, the Turbo code rate can be assumed nondecreasing with \( n, 1 < n \leq N \). Therefore, we have

\[
R_1 \leq R_2 \leq \cdots \leq R_N, \ R_i \in \mathbf{R}, \ 1 \leq i \leq N
\]

\[
s_1 \leq s_2 \leq \cdots \leq s_N, \ s_i \in \mathbf{S}, \ 1 \leq i \leq N
\]

Define \( n_j \) as the number of consecutive frames that use the same channel code rate \( r_j, 1 \leq j \leq M \), then we have

\[
n_1 + n_2 + \cdots + n_M = \sum_{j=1}^{M} n_j = N.
\]

Since \( n_j \geq 0 \), this is the typical integer solution problem and the number of candidate allocations is reduced from \( M^N \) to \( (M-1)^N \). However, this value is still very large for large \( M \) and \( N \) values. For example, when \( M = 10 \) and \( N = 32 \), this value is \( 1.4714 \times 10^{10} \). As a result, the exhaustive search complexity is prohibited for any practical system. In Section III, we propose a GA based method which closely approaches the optimal solution with very low complexity.

III. GA BASED TURBO CODE OPTIMIZATION

One key point in design of a joint source-channel coding scheme is to find an allocation of channel code rates with low complexity that optimizes the performance measurement. For a large number of frames, the complexity of searching algorithm is large. In this paper, we propose a genetic algorithm (GA) [17] based method. Although this method is theoretically suboptimal, the complexity could be great reduced which is desired in any real time transmission system.

GA is a popular searching technique used in many fields to find approximate solutions to various optimization problems, with low complexity. In our scheme, first, we randomly generated \( Q \) individual solutions to form an initial population. Here, one solution represents one possible allocation of channel code rates \( \mathbf{R} \). We calculate the fitness of each solution. The distortion function defined in equation (4) is used as the fitness function and each individual solution corresponds to an image distortion value. According to these fitness values, we choose half of the generated population that have the lower distortion than anyone in the other half as a pool. Then, two “parents” are selected from the pool to breed new generation through crossover and mutation operations. Since only “parents” with high fitness values are used, the created new offsprings typically share many of the better characteristics of their “parents”. With this process, the average fitness value will increase in general, i.e., corresponding distortion will be decreased since only good individuals from the each generation are selected for breeding. In each generation, a half of population that have low fitness values will be discarded. This process is repeated until a termination condition has been reached.

To summarize, the GA based method is as follows:

1) Randomly generate initial population. We define population size as 100.

2) Repeat following steps until a predefined number of generations have processed. In this paper, GA will end when maximum fitness values of consecutive five generations are same or the process run up to 100 generations.
   a) Evaluate the individual fitnesses through the distortion criterion in each generation.
   b) Pick 50 percent of the population with high fitness values as parents pool.
   c) Select pairs of individuals to generate new offsprings through crossover and mutation.

Let \( R_1 = (r_{k_1}, \ldots, r_{k_n}, \ldots, r_{k_N}) \) and \( R_2 = (r_{j_1}, \ldots, r_{j_n}, \ldots, r_{j_N}) \) be two parents. Suppose a position \( n \) is selected randomly, the crossover generates children \( C_1 = (r_{k_1}, \ldots, r_{k_n}, \ldots, r_{r_n+1}, \ldots, r_{j_N}) \) and \( C_2 = (r_{j_1}, \ldots, r_{j_n}, \ldots, r_{k_n+1}, \ldots, r_{k_N}) \). Only children with the non-decreasing code rates are kept. The mutation is then done for each child. One position is selected first and then the code rate at this point will be randomly either increased or decreased to the next code rate. All its neighbor code rates will be either increased or decreased accordingly if necessary to satisfy the non-decreasing code rate requirement.

In our simulation, the results of GA-based method are almost the same as the exhaustive search in all tested scenarios. The total complexity for the GA-based method is roughly determined by population size and the number of generations. In this research, it is at most \( 100 \times 10^6 \approx 5000 \), which greatly reduces the complexity of the optimization. For example, as shown in [2], when \( N = 128 \) and \( M = 7 \), there are more than \( 3.6 \times 10^6 \) candidates needs to be considered. With the GA-based method, only 5000 candidates were checked.

IV. PRODUCT CODE UEP

Sherwood [9] illustrated that even for a BSC, using product codes consisting of RCPC/CRC and RS code still gives a better performance than using the RCPC/CRC codes only. This is because the error floor in convolutional codes could be broken through the RS code in the other direction. Therefore, for noisy channels with random bit errors converted from packet loss by interleaving, we also consider to use additional Reed-Solomon (RS) code protection across the frames based on previous RCPT/CRC UEP.

The product code used in present paper is described as a two-dimensional code consisting of RCPT/CRC along the row direction and RS codes along the column direction. Because it is a difficult combinatorial optimization problem for all possible RCPT and RS code rates [10], the problem can be eased out by using EEP in one direction. In this paper, we use equal RS protection in column direction as shown in Fig.2 where all data in one frame are still interleaved. As in Section II, for overall transmission rate \( R \), we have totally \( N \) frames and each has length \( L \). We set the number of information frames and RS protection frames to be \( N_I \) and \( N_R, N_I + N_R = N \). Each information frame is protected by RCPT with code rate \( r_k \) and RS code \( (N_I, N) \). At the receiver side, the product code first tries to correct transmission errors for every frame by turbo decoding. Some frames will be erased.
if CRC code in those frames detects error. Then, RS decoder is used to recover the erased frames if the number of erased frames does not go beyond the error correction capability.

Let \( p_i^{(r,n)} \) be the probability that the \( i^{th} \) frame cannot be correctly decoded with the protection \( N_{R} \), here, \( N_{R} \) is the RCPT code rate for each frame, \( n = N_{R} \) is the number of RS protection frames of RS code \( (N, N_{R}) \). When the first \( i \) information frames are decoded without errors but the \((i+1)^{th}\) information frame has errors, we denote the corresponding probability and distortion as \( P_i^{(r,n)} \) and \( D_i^{(r,n)} \). Therefore, the probability of no decoding error in all decoded \( N_{I} \) information frames is \( P_{N_{I}}^{(r,n)} = \prod_{i=1}^{N_{I}} (1 - p_i^{(r,n)}) \). \( P_i^{(r,n)} \) and the expected distortion \( D_i^{(r,n)} \), \( 0 < i < N_{I} \), can be calculated by

\[
P_i^{(r,n)} = p_i^{(r,n)} \prod_{j=1}^{i-1} (1 - p_j^{(r,n)}) \quad (8)
\]

\[
D_i^{(r,n)} = \sum_{i=0}^{N_{I}} P_i^{(r,n)} D_i^{(r,n)} \quad (9)
\]

Here, \( D_i^{(r,n)} \) is the distortion of the case that no correct frame is received. The probability of a decoding error in the first frame is \( P_0^{(r,n)} = p_0^{(r,n)} \).

For a \( (N, N_{I}) \) RS code, it can recover \( N - N_{I} \) symbols when the decoder knows erasure position. Therefore, RS decoder recovers all erased frames if the number of them in all \( N \) frames less than or equal to \( N - N_{I} \). Otherwise, the decoder discards the first erased frame and all following frames. Suppose \( p_i^{(r)} \) is the probability that the \( i^{th} \) information frame cannot be correctly decoded by RCPT. For \( W \) frames, there are \( \binom{W}{N} \) possible combinations that \( j \) frames are erased after RCPT decoding, \( 0 \leq j \leq W \). We define \( P_m(W, j) \) as the probability that the \( m^{th} \) possible combination is occurred, \( 1 \leq m \leq \binom{W}{N} \). For example, if we have three frames \( \{1, 2, 3\} \), \( W = 3, j = 2 \), then \( \binom{W}{N} = 3 \) and \( \{1, 2\}, \{1, 3\} \) and \( \{2, 3\} \) are three possible combinations. Thus we have \( P_1(3, 2) = p_1^{(r)} p_2^{(r)} (p_3^{(r)} + 1 - p_3^{(r)}) \), \( P_2(3, 2) = p_1^{(r)} p_3^{(r)} (1 - p_2^{(r)}) \), \( P_3(3, 2) = p_2^{(r)} p_3^{(r)} (1 - p_1^{(r)}) \). \( P_0^{(r,n)} \) is the probability that no frame is correctly decoded by product code, which is equal to the probability that the 1st frame is the first one that cannot be correctly decoded by product code, i.e., \( P \) (the first frame cannot be correctly decoded by RCPT) \( \times P \) (more than \( N - N_{I} - 1 \) frames cannot be correctly decoded by RCPT from the 2nd frame to the last frame), can be calculated by

\[
P_0^{(r,n)} = p_1^{(r)} \sum_{N-I=1}^{N} \sum_{m=1}^{N-I-1} P_m(N-I, j) \quad (10)
\]

The probability of correctly decoding all information frames is \( P_N^{(r,n)} = 1 - \sum_{j=0}^{N_I-1} P_j^{(r,n)} \). In general, \( P_i^{(r,n)} = P_i^{C} \times P_{i+1}^{E} \). \( P_i^{C} \) is the probability that the first \( i \) frames are correctly decoded by product code and \( P_{i+1}^{E} \) is the probability that the \( (i+1)^{th} \) frame cannot be correctly decoded by product code. Based on previous equations, we have

\[
P_i^{C} = 1 - \sum_{j=0}^{i-1} P_j^{(r,n)} \quad (11)
\]

\[
P_{i+1}^{E} = P_r^{(r)} \sum_{j=N-N_I}^{N-I} \sum_{m=1}^{N-I-1} P_m(N-1, j), \quad 0 < i < N_{I} \quad (12)
\]

Therefore, equation (8) and (9) can be expressed as

\[
P_i^{(r,n)} = P_i^{C} P_{i+1}^{E} = (1 - \sum_{j=0}^{i-1} P_j^{(r,n)}) P_{i+1}^{C} \cdot \sum_{j=N-N_I}^{N-I} \sum_{m=1}^{N-I-1} P_m(N-1, j), \quad 0 < i < N_{I} \quad (13)
\]

\[
D_i^{(r,n)} = \sum_{i=0}^{N_{I}} P_i^{(r,n)} D_i^{(r,n)} = P_0^{(r,n)} D_0^{(r,n)} + \sum_{i=1}^{N_{I}-1} (1 - \sum_{j=0}^{i-1} P_j^{(r,n)}) P_{i+1}^{C} \sum_{j=N-N_I}^{N-I-1} \sum_{m=1}^{N-I-1} P_m(N-1, j) + \sum_{i=1}^{N_{I}-1} \sum_{j=0}^{N-I-1} P_j^{(r,n)} P_{i+1}^{E} = \sum_{i=0}^{N_{I}} P_i^{(r,n)} D_i^{(r,n)} \quad (14)
\]

\( P_i^{(r,n)} \) is image independent and can be obtained through Monte Carlo simulation. Based on equation (13), (14), the minimization of the expected distortion can be achieved using proposed GA method. We should notice that the above analysis is only valid with the proposed scheme where an interleaving is used after the coding of each frame. Otherwise, the packet loss cannot be regarded as the random bit errors or randomly punctured bits in the decoding of RCPT in the row direction.

V. PROPOSED ALGORITHMS AND EXPERIMENTAL RESULTS

In this paper, we propose and test the following three algorithms based on previous analysis for image transmission over channels with both packet loss and random errors.

**Algorithm 1:**
Simulation results show that GA-based optimization actually improves the PSNR compared to brute force search. Table I presents the expected PSNR of 0.25 bpp and 0.5 bpp. The optimization was done for channel with only random bit errors.

<table>
<thead>
<tr>
<th>Code Rate (bpp)</th>
<th>Channel BER</th>
<th>0.01</th>
<th>0.05</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSNR</td>
<td>Rate</td>
<td>PSNR</td>
<td>Rate</td>
</tr>
<tr>
<td>0.25</td>
<td>GA</td>
<td>32.25</td>
<td>0.69</td>
<td>33.78</td>
</tr>
<tr>
<td></td>
<td>BF</td>
<td>32.25</td>
<td>0.69</td>
<td>33.78</td>
</tr>
<tr>
<td>0.5</td>
<td>GA</td>
<td>35.11</td>
<td>0.68</td>
<td>33.78</td>
</tr>
<tr>
<td></td>
<td>BF</td>
<td>35.12</td>
<td>0.68</td>
<td>33.79</td>
</tr>
</tbody>
</table>

1) Transform PLR to BER based on equation (1), and get the new BER for the channel.
2) Use GA-based optimization with the new BER to minimize equation (4).

Algorithm 2:
1) Use GA-based optimization with original channel BER to minimize equation (4).
2) Based on equation (2), reduce the RCPT code rate for each frame.
3) Add additional protection by increasing 1 for the denominator of each code rate.

Algorithm 3:
1) Transform PLR to BER based on equation (1), get the new BER for the channel.
2) Use GA-based optimization with new BER to minimize equation (14).

Algorithm 1 and algorithm 2 use RCPT/CRC UEP while algorithm 3 uses product code UEP scheme.

The test image is 8 bits per pixel (bpp) grayscale 512 × 512 Lena image. Optimal allocations were designed for channels with PLR 0.1 and BER from 0.01 to 0.1. The coded frame length is set as 4096 bits. We use a 16-bit CRC with generator polynomial 0x8005. The turbo code consists of two identical recursive systematic convolutional codes with generator polynomial (21, 37) octal and the mother code rate is 1/3. The memory length of turbo encoder is four and the RCPT code rate set is (4/12, 4/11, 4/10, 4/9, 4/8, 4/7, 4/6, 4/5). The embedded source coder is based on SPIHT algorithm [1]. All the RCPT codes used in test were selected from [16]. Log-MAP decoding is adopted for turbo decoding and the decoding ends after 20 iterations. All experiment results were obtained with 5000 independent simulations.

First we test the proposed GA optimization method in channels only with random bit errors. We use RCPT/CRC as the channel codes. Table I presents the expected PSNR of two algorithms: GA-based optimization and brute force search. Simulation results show that GA-based optimization actually obtains the same result with the exhaustive search results in 0.25 bpp case and very close performance in 0.5 bpp case, with much lower complexity.

Then we consider channels with both packet loss and random bit errors and test different UEP algorithms. We compare our results with those based on the structure of [9] but with RCPT/CRC and RS codes. Table II and Table III list the expected PSNR from the four algorithms with 0.1 PER and different BER. All three algorithms we proposed outperform the first one. This is due to the use of the interleaver in each frame before the transmission which converts the packet loss into random bit error and produces a more efficient source-channel rate allocation. In Algorithm 1, we transform PLR to BER by equation (1). However, when we know the positions of erased bits but do not use this information in RCPT decoding, the new BER based on equation (1) is overrated and some frames are overprotected. Algorithm 2 considers the benefit from the position information. Therefore, it has better performance than Algorithm 1. Algorithm 3 improves the performance even more by using product code, which also verifies the result of Sherwood [9] that product code could have better performance for noisy channel which has only random bit errors, compared with the RCPT/CRC scheme.

The advantage of the proposed methods is more clear when comparing the cumulative quality distribution which is reported in Fig. 3 and Fig. 4. The optimization were done for 0.1 PLE and 0.08 BER. We can find that in low decoded PSRN, Algorithm 1, 2, 3 all have less probability than the method in [9]. Both Algorithms 1 and 3 have lower maximal PSNR than Algorithm 2. However, they all have lower probability in low PSNR. Overall, Algorithm 1 has the weakest performance among these three method. It is due to that some frames are overprotected and product code has stronger error correction ability.

VI. CONCLUSION

This paper considers the joint source-channel coding for the transmission of progressive images over wireless networks with both packet loss and random bit errors. Before each
frame is transmitted over noisy channels, an interleaver is applied to convert packet loss into random bit erasures in the punctured turbo codes. Therefore, a noisy channel with both packet loss and random bit errors is equivalent to a channel with only random bit errors so that a single RCPT/CRC code can be designed for channel protection. With a GA-based low complexity optimization algorithm, two RCPT/CRC UEP algorithms are presented. Furthermore, we extend RCPT/CRC UEP to product code by adding a RS code across the frames and present an efficient rate allocation algorithm. Performance improvement of the proposed methods is demonstrated with experimental results.

Fig. 3. Cumulative distribution of decoded PSNR for Lena image at transmission rate 0.25 bpp with 0.1 PLE and 0.08 BER. The optimization were done for 0.1 PLE and 0.08 BER.

ACKNOWLEDGEMENTS

This work was supported by the Mississippi NASA EPSCoR under grant 300222505A.

REFERENCES