NEW METHODS FOR BLIND FINE ESTIMATION OF CARRIER FREQUENCY OFFSET IN OFDM/OQAM SYSTEMS

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ABSTRACT

Carrier frequency offset (CFO) in OFDM/OQAM systems can be estimated based on the conjugate correlation function of the received signals. Traditionally this is done before demodulation. We present new methods based on the subchannel signals. The proposed estimators are robust to multipath effects due to the narrow-band property of the subchannels. The performance of the estimators is evaluated by asymptotic analysis and simulation results. Our results show that methods based on estimation in subchannels have better performance than estimators operating before demodulation.

1. INTRODUCTION

It is well known that multicarrier systems are much more sensitive to carrier frequency offset (CFO) than single carrier systems. For OFDM/OQAM systems, it is reported that CFO should be less than 2% of the subchannel spacing to guarantee a signal to interference ratio higher than 30 dB [1]. OFDM/OQAM systems using single carrier QAM transmission systems [5]. Bolcskei’s estimator transmitted one QAM symbol \( a_m[n] = a_m^R[n] + j a_m^I[n] \) per \( T \) seconds. QAM symbols are formed by shifting the imaginary part of the QAM symbols by \( T/2 \). By summing up all the subchannels, the modulator generates a \( T/N \) spaced output sequence

\[
x[l] = \sum_{k=0}^{N-1} w_k \sum_{n=-\infty}^{\infty} (a_k^R[n] g[l - nN] + j a_k^I[n] g[l - nN - N/2]) e^{j (\frac{2\pi f_c}{T} + \frac{\pi}{2}) k}. \tag{1}
\]

The transmitter filter \( g[l] \) and receiver filter \( f[l] \) operate with the same sampling interval \( T/N \) and are bandlimited to \( [-1/T, 1/T] \). Here we assume a stationary channel which can be modelled as a discrete linear time-invariant system. The number of subchannels \( N \) is large enough, we may approximate the equivalent channel response of subchannel \( k \) as flat-fading with an attenuation factor \( \mu_k \). The channel model also includes an additive circular white Gaussian noise source \( \nu[l] \) with variance \( \sigma_n^2 \). The carrier frequency offset is normalized with respect to \( 1/T \) and to be denoted \( f_c \).

Then we can write the received sequence from the channel as

\[
r[l] = e^{j \frac{2\pi f_c}{T} l} \sum_{k=0}^{N-1} w_k \mu_k \sum_{n=-\infty}^{\infty} (a_k^R[n] g[l - nN] + j a_k^I[n] g[l - nN - N/2]) e^{j (\frac{2\pi f_c}{T} + \frac{\pi}{2}) k} + \nu[l].
\]

In subchannel \( k \) of the receiver, the received sequence is first down-converted by multiplying with \( e^{-j (\frac{2\pi f_c}{T} + \frac{\pi}{2}) k} \), then filtered by the receiver filter \( f[l] \) and \( N/2 \) times down-sampled to generate a \( T/2 \) spaced sequence

\[
b_k[s] = r[l] e^{-j (\frac{2\pi f_c}{T} + \frac{\pi}{2}) k} * f[l] \big|_{l = s + (s-2n)}
\]

\[
= e^{j \pi f_c s} \sum_{n=0}^{N-1} w_m \mu_m \sum_{n=-\infty}^{\infty} (a_m^R[n] p_m,k[s - 2n])
\]

\[
+ j (-1)^{(m-k)} a_m^I[n] p_m,k[s - 2n - 1] + \nu_k[s]. \tag{2}
\]

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where \( p_{m,k}[s] = \text{def} p^{(o)}_{m,k}[sN] \) and \( \nu_{b}[s] = \text{def} v^{(o)}_{k}[sN] \) are respectively the \( N/2 \) times down-sampled versions of \( p^{(o)}_{m,k}[l] \) and \( v^{(o)}_{k}[l] \) which are defined as

\[
\begin{align*}
p^{(o)}_{m,k}[l] &= \text{def} \, g[l] e^{j\frac{2\pi}{N} (m-k) f_0 l} e^{-j\frac{2\pi}{N} m f_0 l} , \\
v^{(o)}_{k}[l] &= \text{def} \, v[l] e^{j\frac{2\pi}{N} k f_0 l} . \tag{3}
\end{align*}
\]

Note that although the sequence immediately before the decimator (or immediately after the receiver filter), \( v[l] e^{-j\frac{2\pi}{N} k f_0 l} \), contains more information than the \( N/2 \) down-sampled sequence \( b_k[s] \), this signal is not directly available in a receiver based on FFT and polyphase filters [9, 10]. Therefore we will base our methods on \( b_k[s] \).

### 2.2. Conjugate correlation function

The conjugate correlation function is defined as

\[
\hat{c}_k[s, \tau] = r_k(\tau, f_c) e^{j2\pi (f_0 + 1/2)s}, \tag{4}
\]

where

\[
r_k(\tau, f_c) = \frac{1}{2} e^{j\pi f_c \tau} \sum_{m=0}^{N-1} \left( \sum_{n=-\infty}^{\infty} \left( p_{m,k}[2n + \tau] p_{m,k}[2n] - p_{m,k}[2n + \tau + 1] p_{m,k}[2n + 1] \right) \right) , \tag{5}
\]

and

\[
A_{m,k}(\tau, f_c) = \sum_{n=-\infty}^{\infty} \left( p_{m,k}[2n + \tau] p_{m,k}[2n] - p_{m,k}[2n + \tau + 1] p_{m,k}[2n + 1] \right) . \tag{6}
\]

We see clearly that \( \hat{c}_k[s, \tau] \) is cyclostationary with a period \( f_0 + 1/2 \). To be useful for CFO estimation, we must in addition check that \( \hat{c}_k[s, \tau] \neq 0 \). In that case, the spectrum (with respect to \( s \)) of \( \hat{c}_k[s, \tau] \) will have a sharp peak at \( f_c + 1/2 \), which can be used to estimate \( f_c \). By assuming that the transmitter \( f_0[l] \) and receiver \( g[l] \) are identical real-valued and symmetric pulses, it is proved in appendix A that \( \hat{c}_k[s, \tau] \equiv 0 \) in the case of unweighted systems and AWGN channel (i.e. \( w_k = 1, \mu_k = 1 \)). This implies that no information about \( f_c \) is present in \( \hat{c}_k[s, \tau] \).

### 3. ESTIMATION ALGORITHMS

In the previous section, we saw that if the case of \( w_k = 1 \) and \( \mu_k = 1 \), the conjugate correlation function is zero. Here we consider two weighting methods:

**Method 1:**

Set \( w_k = 0 \) for all selected subchannels \( k_1, k_2, \ldots, k_L \), which can be referred as null-subchannels, while the other factors set to 1. If the null-subchannels are sparsely distributed and subchannel \( k \) is a null subchannel, it can be easily verified that \( \hat{c}_{k-1}[s, \tau], \hat{c}_k[s, \tau] \) and \( \hat{c}_{k+1}[s, \tau] \) contain information of \( f_c \). The amplitude of the conjugate correlation function of subchannels \( k - 1 \) and \( k + 1 \) is lower than subchannel \( k \), and simulation shows that the benefit from including contributions from subchannel \( k \pm 1 \) is quite marginal. For low SNR, it even causes a higher threshold. Therefore we use only the output sequence of subchannel \( k \) to estimate CFO. Another practical problem is to choose \( \tau \). Numerical results show that \( \left| A_{k,k}(\tau, f_c) \right| \) approaches zero, and is negligible for \( \tau > \tau_{\text{max}} \), where \( \tau_{\text{max}} \) depends on the shaping filters \( g[l] \) and \( f[l] \). Note that \( \hat{c}_k[s, \tau] \) with negative \( \tau \) is just a shifted version of that with positive \( \tau \) and thus contains no extra information.

In practice only one finite-length data record \( b_k[s] \) is available. This means that we must use a sample single estimate for \( \hat{c}_k[s, \tau] \):

\[
y_k[s, \tau] = b_k[s + \tau] b_k[s] . \tag{7}
\]

By defining \( y_k[s] = [y_k[s, 0] y_k[s, 1] \cdots y_k[s, \tau_{\text{max}}]]^T \), we can express the estimation problem as:

\[
\hat{f}_{e,M} = \text{arg} \max_{f_c(0,1)} J_M(f_c) - \frac{1}{2} . \tag{8}
\]

\[
J_M(f_c) = \sum_{k=1}^{L} \left\| \frac{1}{M} \sum_{m=0}^{M-1} y_{k}[s] e^{-j2\pi f_c s} \right\|^2 . \tag{9}
\]

We see that \( J_M(f_c) \) contains the contribution from different \( \tau \) and different null-subchannels.

**Method 2:**

Alternatively we may use an interleaved weighting pattern, i.e. \( w = \{ w_1, w_2, w_3, \ldots \} \), where \( w_1 \not= w_2 \). For this case, we have \( \hat{c}_k[s, \tau] \simeq \frac{1}{2} \left| \mu_k^2 \left( w_1^2 - w_2^2 \right) A_{k,k}(\tau, f_c) \right| \). Therefore the CFO can be estimated over all subchannels, and we can express the estimation problem as:

\[
\hat{f}_{e,M} = \text{arg} \max_{f_c(0,1)} J_M(f_c) - \frac{1}{2} . \tag{10}
\]

\[
J_M(f_c) = \sum_{k=1}^{L} \left\| \frac{1}{M} \sum_{m=0}^{M-1} y_{k}[s] e^{-j2\pi f_c s} \right\|^2 . \tag{11}
\]

We see that estimator (10) and (9) have the same acquisition range \((-0.5, 0.5)\). While estimator (8) is more useful since null-subchannels are always present for practical OFDM systems. The estimator (9) is included in order to show the benefit of using subchannel signals for the estimation.

### 4. ASYMPTOTIC ANALYSIS FOR METHOD 1

In this section, we will present an asymptotic analysis for weighting method 1. Using a reasoning similar to that in [11], we find that (see [12] for more details) \( f_{e,M} \) is an asymptotically unbiased estimation for \( f_c \), and the asymptotic variance is given by

\[
\gamma = \frac{3}{2\pi^2 M^2} \sum_{\nu=1}^{L} \text{Re} \left\{ \Psi_{\nu}(f_c) - \Psi_{\nu}(f_c) \right\}^2 , \tag{12}
\]

where

\[
\Psi_{\nu}(f_c) = r_{\nu}(f_c) P_{\nu} r_{\nu}(f_c) , \quad \tilde{\Psi}_{\nu}(f_c) = r_{\nu}^H(f_c) \tilde{P}_{\nu} r_{\nu}^H(f_c) , \quad \Phi_{\nu}(f_c) = r_{\nu}^H(f_c) \Phi_{\nu} r_{\nu}(f_c) .
\]
Here $r_k(f_e) = [r_k(0, f_e), r_k(1, f_e), \ldots, r_k(\tau_{\text{max}}, f_e)]^T$ and $P_k, \hat{P}_k$ are matrices with entries given by

$$
\begin{align*}
[\hat{P}_k]_{\tau_1, \tau_2} &= S_{\tau_k}(f_{e} + 1/2, \tau_1, \tau_2) \\
[\hat{P}_k]_{\tau_1, \tau_2} &= \bar{S}_{\tau_k}(f_{e} + 1/2, \tau_1, \tau_2)
\end{align*}
$$

respectively.

Explicit expressions for $S_{\tau_k}(f_{e} + 1/2, \tau_1, \tau_2)$ and $\bar{S}_{\tau_k}(f_{e} + 1/2, \tau_1, \tau_2)$ are found in [12]. Based on these expressions, we can rewrite the theoretical MSE as

$$
\gamma = \left[ A(f_e) + B(f_e)/\text{SNR} + C(f_e)/\text{SNR}^2 \right] / M^3, \quad (11)
$$

where $\text{SNR} \eqdef 1/\sigma^2_n$, and $A(f_e), B(f_e), C(f_e)$ are related to $f_e$ but independent of data record length $M$.

We see that the MSE decreases as $O(M^{-3})$. We also note that even in the case of a noise-free channel, a non-zero $A(f_e)$ will cause a certain MSE floor.

### 5. Simulation Results

The following conditions apply to all simulations except where specially indicated:

- The number of subchannels $N$ is set to 16;
- 16QAM modulation is used in all subchannels, and the input symbols are uniformly distributed;
- $g[l]$ and $p[l]$ are square root raised cosine pulses with a roll off factor $\alpha = 1.0$, giving $\tau_{\text{max}} = 2$;
- Each result is obtained by averaging over 1000 Monte Carlo trials.

In addition to an expected peak at $f_e + 0.5$, the objective function $J_M(f)$ will have local maxima caused by noise. If the desired peak around $f_e + 0.5$ is lower than other peak(s) caused by noise, false detection occurs. The peak is found in two steps. First a coarse search is made using FFT with four times oversampling accomplished by zero padding (Simulations show that only marginal improvement is attained by using larger oversampling rate). Then the simplex method is used to find the precise maximum point.

**Simulation 1: performance of method 1 (null-subchannels) versus SNR over an AWGN channel**

In this simulation, we choose a data record length $M = 256$ (corresponding to 128 OFDM symbols since the OQAM symbols are two times over-sampled), and the number of null-subchannels $L = 1$. The curves of MSE versus SNR for different CFO are shown in Fig. 1. The simulation results match well with the theoretical predictions (based on (10)), except for SNR below a certain threshold. The derivation below this threshold is caused by false peak detection of $J_M(f)$. We also note that for $f_e = 0$, the simulated results match well with theoretical predictions, and no MSE floor is present (due to the fact that $A(0) = 0$ in (11)). For $f_e = 0.2$ and 0.4, the simulated values deviate from theoretical predictions for high SNR. This is because the theoretical prediction is an asymptotic result. It will be shown in simulation 2 that such a deviation disappears asymptotically with increasing $M$.

**Simulation 2: performance of method 1 (null-subchannels) versus data record length $M$ over an AWGN channel**

In this simulation, we set the number of null-subchannels to $L = 1$. The curves for MSE versus the data record length $M$ for different SNR are shown in Fig. 2. We see that MSE decreases with increasing $M$. For SNR = 0 dB, the large deviation between theoretical and simulated results, which is due to the threshold effects, disappears for $M > 1800$. This suggests that the deviation caused by threshold effects can be eliminated by increasing $M$. For SNR = 20, 40 and 60 dB, no threshold effect is present. There is a small gap between theoretical and simulated results. This is due to the asymptotic approach of theoretical analysis. This small gap disappears asymptotically with increasing $M$.

**Simulation 3: performance of method 1 (null-subchannels) over a stationary multipath fading channel**

In this simulation, we set $M = 256$, $f_e = 0.2$. We assume a time-invariant five-path channel with impulse response $h[l] = \sum_{d=0}^{4} \lambda_d \delta[l-d]$, where $\lambda_d$ is the complex-valued path attenuation factor.
We change the coefficients $\lambda$ for each trial. For the purpose of simulation, we set the path attenuation factors $\lambda_k$ to be circular Gaussian and independently distributed, and with a variance $1/5$ so that the average received power is identical to the AWGN case. We simulate two cases: the number of null-subchannels $L = 1$ and $L = 4$. The simulation results are shown in Fig. 3. The simulation results for $L = 1$ over multipath channel are obtained by averaging over 10000 trials to get a sufficiently smooth curve. We see that the unknown multipath channel will degrade the estimator performance in two ways: (1) higher SNR threshold; (2) larger MSE in the high SNR region. Both phenomena are due to the low equivalent SNR when the null-subchannels suffer a deep fading. Then the estimate (7) is dominated by noise and no prominent peak in $J_M(f)$ is present. For $L = 1$ this kind of degradation is most obvious since only a single null-subchannel is used. The performance can be improved by properly selecting the position of null-subchannel(s) if the channel is known. The degradation is less obvious for larger $L$ since the probability of all null-subchannels suffering deep fading is smaller. This implies that we can improve the robustness to multipath fading by using more null-subchannels.

Such improvement is obtained by sacrificing spectrum efficiency.

In this simulation, we set $f_c = 0.2$ and $M = 512$. For weighting method 1, the number of null-subchannels $L = 1$. For method 2, two weighting cases are simulated: $w_1 = \sqrt{3}/2$, $w_2 = \sqrt{5}/2$ and $w_1 = \sqrt{3}/2$, $w_2 = \sqrt{6}/2$. This corresponds to 1.25 and 3.0 dB attenuation of the weakest subchannels, respectively. The simulation results are shown in Fig. 4. For the modified C/S estimator with $w_1 = \sqrt{3}/2$, $w_2 = \sqrt{5}/2$, the self-noise will cause a false detection ratio about 4%, then a high MSE floor. For the case of $w_1 = \sqrt{3}/2$, $w_2 = \sqrt{5}/2$, both method 2 and the modified C/S estimator can work properly, while Method 2 has lower SNR threshold and MSE floor than the modified C/S estimator.

Thus better performance can be achieved based on the estimation of subchannel signals. We also note that method 1 outperforms method 2 and the modified C/S estimator for SNR $> 12$ dB. This can be partly explained by noting that the spectral loss of weighting method 2 decreases with increasing SNR, while the spectral loss of weighting method 1 is fixed to $L/N$, which is independent of SNR.

**Simulation 4: performance comparison between method 1 (null-subchannels), method 2 (interleaved weighting) and the modified Ciblat/Serpedin estimator over an AWGN channel**

In [6], Ciblat and Serpedin present a blind CFO estimation algorithm based on conjugate cyclostationarity of the received sequence before demodulation. However, they use a different modulation system that corresponds to multiplication of subchannel $k$ by a weighting factor $j^k$. Application of Ciblat and Serpedin’s algorithm to a standard OFDM/OQAM system is denoted modified C/S estimator. The modified C/S estimator will not work in an unweighted system because the conjugate correlation function of the signal before demodulation is zero. For weighting method 1, the conjugate correlation function is quite weak, so that the modified C/S estimator doesn’t work properly (also confirmed by simulation). Therefore we simulate the modified C/S estimator only for weighting method 2.

**6. Conclusion**

We have shown how to estimate CFO based on the conjugate correlation function of subchannel signals, and that this method requires non-uniform power distribution (weighting). Two different weighting patterns are studied. One of these, method 1, based on null-subchannels is shown to be asymptotically unbiased. The other one, method 2, uses an interleaved weighting. Simulation results show that method 1 is robust to multipath fading, and it outperforms method 2 at SNR above 12 dB for AWGN channel. The simulation results also show that methods based on the estimation of subchannel signals have lower MSE than estimators operating before demodulation for AWGN channel.
A. PROOF OF CONJUGATE CORRELATION FUNCTION TO BE ZERO FOR UNWEIGHTED OFDM/OQAM SYSTEMS OVER AN AWGN CHANNEL

Proof: First by defining \( P_{m,k}(f) = \sum_{n=-\infty}^{\infty} p_{m,k}[n] e^{-j2\pi nf} \) and using the decimator formula (see formula 4.1.13 in [13]), we get

\[
P_{1m,k}(f) \equiv \sum_{n=-\infty}^{\infty} p_{m,k}[2n] e^{-j2\pi nf} = \frac{1}{2} ( P_{m,k}(f) + P_{m,k}(f-1) )
\]

\[
P_{2m,k}(f) \equiv \sum_{n=-\infty}^{\infty} p_{m,k}[2n+1] e^{-j2\pi nf} = \frac{1}{2} e^{j\pi f} ( P_{m,k}(f) - P_{m,k}(f-1) ).
\]

Then by using (12) and Parseval’s relation, we can rewrite (6) in frequency domain as

\[
A_{m,k}(\tau, f_s) = \frac{1}{2} \int_{-1}^{1} T_{m,k}(f) e^{j\pi f \tau} df,
\]

where \( T_{m,k}(f) \equiv P_{m,k}(f) P_{m,k}(1-f) \).

From the definition of \( p_{m,k}^{(o)}[n] \) in (3) and the relationship of \( p_{m,k}[n] = P_{m,k}^{(o)}[n/2] \), we can write \( P_{m,k}(f) \) as

\[
P_{m,k}(f) = j^{m-k} \sum_{n=-\infty}^{\infty} G(f - m + k - 2n) G(f + f_s - 2n),
\]

where \( G(f) \) is the prototype filter and band limited to \([-1, 1]\).

We see that \( P_{m,k}(f) \) is a periodic extension of \( G(f - (m - k)) G(f + f_s) \). Then by substituting \( P_{m,k}(f) \) into the definition of \( T_{m,k}(f) \), we have

\[
T_{m,k}(f) = (-1)^{m-k} \sum_{n=-\infty}^{\infty} \sum_{1}^{\infty} G(f - m + k - 2n_1)
\times G(f + f_s - 2n_1) G(f + m - k + 2n_2 + 1)
\times G(f + 2n_2 + 1 - f_s).
\]

(14)

Without the loss of generality (we will show later that the acquisition range is \([f_s, 0.5]\)), we may assume \( 0 \leq f_s < 0.5 \). Then \( P_{m,k}(f) \) is nonzero only if \( m \in \{k-2, k-1, k, k+1\} \). Based on the facts that \( G(f) \) is bandlimited to \([-1, 1]\) and \( 0 \leq f_s < 0.5 \), we have

\[
T_{k-2,k}(f) = 0
\]

\[
T_{k-1,k}(f) = - \sum_{n=-\infty}^{\infty} G(f - 2n) G(f + f_s - 2n)
\times G(f - 2n + 1) G(f - 2n + 1 - f_s)
\times G(f - 2n - 1) G(f - 2n - 1 - f_s)
\times G(f - 2n) G(f + f_s - 2n)
\times G(f - 2n + 1) G(f - 2n + 2 - f_s)
\times G(f - 2n - 1) G(f - 2n - 2 - f_s)
\times G(f - 2n) G(f + f_s - 2n)
\times G(f - 2n - 1) G(f - 2n - 2 - f_s)
\times G(f - 2n) G(f + f_s - 2n)
\times G(f - 2n - 1) G(f - 2n - 2 - f_s)
\times G(f - 2n) G(f + f_s - 2n)
\times G(f - 2n - 1) G(f - 2n - 2 - f_s)
\]

Then we can conclude that \( \sum_{m=0}^{N-1} A_{m,k}(\tau, f_s) = 0 \), which implies that \( \sum_{m=0}^{N-1} A_{m,k}(\tau, f_s) = 0 \) based on (13). Thus for the case of unweighted systems and AWGN channel, i.e. \( w_k = 1, \mu_k = 1 \), we get \( e_k[s, \tau] \equiv 0 \) based on (4). \( \square \)

B. REFERENCES


