Assessing the Potential of Predictive Control for Hybrid Vehicle Powertrains using Stochastic Dynamic Programming

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Abstract—The potential for reduced fuel consumption of Hybrid Electric Vehicles by the use of predictive powertrain control was assessed on measured drive data from an urban route with varying topography. The assessment was done by evaluating the fuel consumption using three optimal controllers, each with a different level of information access to the driven route. The study showed that good performance, 1-3% from the lowest theoretical fuel consumption, can be achieved with a time invariant controller that is optimized to the environment. If the predictive control was based on information supplied by the vehicle navigation system the attainable fuel consumption is almost indistinguishable from the theoretical lowest.

I. INTRODUCTION

Over the last years many new telematic systems have been introduced in road vehicles. For instance, global positioning systems (GPS) and mobile phones have become a de facto standard in premium cars. With the introduction of these systems, the amount of information about the traffic environment available in the vehicle has increased.

Another trend in automotive research and development is the introduction of hybrid electric vehicles (HEVs). This trend is driven by the need for reduced environmental impact of road transport. It is well known that the energy management strategy (EMS) of an HEV largely influences its competitiveness.

With the availability of traffic information, predictions of the vehicle propulsion load can be made. This enables predictive control of the hybrid powertrain, potentially increasing the overall efficiency. Previous work within this field includes [1] and [2]. In contrast to earlier publications, the main purpose of this study is not to develop a predictive control scheme that can meet the demands of real-time implementation. Instead, the focus is on investigating what can be achieved if the available information is used optimally. This is done in order to assess the potential of predictive control for HEV powertrains as well as to determine what type of traffic environment information that should be provided to the powertrain controller.

The approach in this study is to model a specific transport mission, i.e. a specific vehicle traveling a specific route, as a stochastic process based on collected data. The route is divided into a number of intervals, and the vehicle state at the end of each interval is modeled by a discrete time Markov chain. The transition probabilities of the Markov chain are identified from collected driving data.

Three types of optimal controllers are studied. Each controller represents a different level of access to information about the transport mission. The controller with the most detailed information access simulates the unrealistic situation where the future power demand is completely known to the control system, hence this controller can be regarded as an ideal controller.

The second highest information level represents the case where the controller has feedback of the position along the specific route. In addition to this, the controller has access to statistical data about the traffic flow along the route. This represents a vehicle equipped with a GPS combined with a traffic flow information system. The information access is modeled with a position dependent Markov chain with the Markov state consisting of the velocity and the power demand. The optimal control law is derived by solving a finite horizon stochastic dynamic programming problem. The resulting optimal control law is position dependent and optimal in an average sense when evaluated against the position dependent Markov chain.

The lowest information level represents what can be achieved if the controller knows the transport mission, but has no feedback of the exact position. This information level results in a time invariant control law that is optimized against a homogeneous, i.e. position invariant, Markov chain model of the collected driving data, [3],[4],[5]. Hence, there is no position dependency and the Markov state consists of the velocity and the power demand that the powertrain must deliver to fulfill the driver’s request. This information level represents a vehicle that is being driven in a certain environment, e.g. city driving, and that the controller has been optimized for that type of environment.

The potential fuel savings with predictive control is assessed by comparing the fuel consumption resulting from the application of each of the three controllers. Results for three variants of the powertrain with different degrees of hybridization are presented.

II. STOCHASTIC MODELING OF A TRANSPORT MISSION

Two different stochastic processes that model the transport mission were identified from the data, collected during actual driving. The processes are of Markov type. This means that the future evolution of the processes, conditioned on their present state, is independent of the past.

A. Data collection

Drive data was collected in Gothenburg under similar conditions: relatively low traffic density and no traffic con-
suggestions. Measurements where done with a Toyota Prius 2. The data was collected on the same route by driving it 37 times. The studied route stretches along 16 km of mixed city driving. The altitude varies between 2 to 95 meters and the speed limits are 30, 50 and 70 km/h.

The measurements were done with a GPS which logged velocity and position at every second. The measured velocity trajectories and the altitude profile are shown in Fig. 1.

B. Homogeneous Markov chain model

The objective of the first model constructed is to describe the “average” (stationary; time and position invariant) distribution of velocity and power demand along the route, without taking into account the specific position. In [6] a stochastic drive cycle generator was identified from collected drive data of mixed city driving. This model will be used to smooth the data before creating the Markov matrix in the Markov chain model. The drive behavior is in [6] modeled as a discrete time, continuous state space, time invariant Markov process with acceleration and velocity as the Markov states. The acceleration \( a \) is a stochastic variable modeled as beta distributed with the beta parameters dependent on previous acceleration \( a \) and velocity \( v \).

\[
A_{k+1} \sim \text{Beta}(a_k, v_k) \tag{1}
\]

The model is verified by comparing simulated velocities and accelerations with the measurements. A velocity-acceleration diagram for both measured and simulated data is shown in Fig. 2. The figure shows that the model is able to reproduce the behavior exhibited in the measurements.

By extensive simulations with the drive cycle generator the data set is expanded from \( \approx 50000 \) s in the measurements to 2500000 s. The next step is to simulate a chassis on the generated drive data. The chassis model has three inputs \( a, v \) and the road slope \( \theta \). \( \theta \) comes from the topography shown in Fig. 1. The output from the chassis model is the power demand \( P_{\text{demand}} \) that the powertrain need to deliver in order to follow the generated drive cycle. By quantizing \( P_{\text{demand}} \) and \( V \) a Markov chain model is now identified from the simulated data. The state vector in the Markov chain is defined as:

\[
X_k^h = (V_k, P_{\text{demand}}), \tag{2}
\]

The probability distribution for a combination of \( P_{\text{demand}} \) and \( v \) at the next time step is given by the transition probabilities (conditional probabilities)

\[
P(P_{\text{demand}, k+1} = p_{\text{demand}, k+1}, V_{k+1} = v_{k+1} \mid P_{\text{demand}, k}, V_k), \tag{3}
\]

which are defined for all possible combinations of \( P_{\text{demand}} \) and \( v \). All these transition probabilities are determined from the simulated data by counting the occurrence of each transition and dividing for each state with the times the state is visited.

Note that the Markov chain model is time invariant, which implies that the transition probabilities are independent of \( k \).

C. Position dependent Markov chain model

The second Markov chain model is aimed at describing the ensemble of velocity trajectories as a function of position along the route. First the raw data needs to be condensed. The route was divided into a number of discrete intervals measured in distance. The velocity and the slope at the end of each interval was approximated by one of a finite number of values. The result of this quantization is a representation of the data, which is discrete in both time/position and in the state (i.e. velocity and slope). Details for the two different models are given below. At a number of positions along the route the vehicle may come to a complete stop. Naturally, it will then remain at zero velocity for some period of time. To handle this, a model describing the length of the time interval is needed. Using the exponential distribution, a good approximation of the time interval lengths of the collected data is achieved. However, it is required that the mean value of the interval length is individually set for each possible stop position.

With the route divided into discrete steps, the Markov model is defined by the transition probabilities, linking the state at one position to the state at the next position. In this case, the state vector is defined as:

\[
X_k^p = (V_k, A_k), \tag{4}
\]

Since the slope is given as a deterministic function of the position, \( P_{\text{demand}} \) is a function of the state \( X_k^p \).

To be more specific, given the velocity \( v_k \) and the acceleration \( a_k \) at a certain position \( k \) along the route, the probability distribution for the velocity at the position \( k+1 \) is given by the transition probabilities

\[
P_k(V_{k+1} = v_{k+1} \mid v_k, a_k) \tag{5}
\]
of a nonhomogeneous Markov chain. Note that the transition probabilities now depend on the position \( k \), and therefore implicitly on the slope.

The transition probabilities are determined from the collected data. The velocity is assumed to change linearly between every position. Since the velocity is a stochastic process, the time needed to go between two fixed positions will vary. Therefore, the distances between the points on the route are adapted to the possible velocities at each position, keeping the time step as close to one second as possible. With the velocity resolution set to 1 m/s the longest sample time that can occur at any position is approximately 5 seconds and the shortest is approximately 0.2 seconds. However, the vast majority of the time steps lie around 1 second. The reason for choosing a low velocity resolution of 1 m/s is the relative lack of data. Remember that each engine (ICE) can add or absorb torque. The resulting electric power demand and the shaft speed \( \omega_{\text{demand}} \) is transferred to the driving wheels via a 5 stepped gearbox and a differential gear. At the ICE flywheel, an Electric Machine (EM) can add or absorb torque. The resulting electric power to or from the EM is transferred to the energy buffer via an electric power converter.

### III. VEHICLE MODEL

#### A. Vehicle description

The studied HEV is a city taxi with a parallel configuration for the hybrid powertrain. Three variants of the powertrain are studied. The vehicle specifications are given in Tab. I. Note that total vehicle mass includes the driver and passengers. The HEV is powered by an Internal Combustion Engine (ICE) with a Nickel Metal Hydride (NiMH) battery as a buffer. The mechanical power produced by the PPU is transferred to the driving wheels via a 5 stepped gearbox and a differential gear. At the ICE flywheel, an Electric Machine (EM) can add or absorb torque. The resulting electric power to or from the EM is transferred to the energy buffer via an electric power converter.

#### B. Chassis model

The chassis model outputs at the differential gear the power demand \( p_{\text{demand}} \) and the shaft speed \( \omega_{\text{demand}} \) that is required to propel the vehicle at a certain velocity, acceleration and road slope. Longitudinal driving dynamics need to be considered in order to assure vehicle stability during regenerative braking. Therefore, a longitudinal bicycle model is used to represent the chassis. During braking, the vehicle is assumed to absorb as much energy as possible through the driving wheels, but without ever locking them.

#### C. Powertrain model

Three input signals are used in the powertrain model; \( p_{\text{demand}}, \omega_{\text{demand}} \) and the PPU power \( p_{\text{PPU}} \). \( p_{\text{demand}} \) and \( \omega_{\text{demand}} \) comes from the chassis model and arises, from the driver’s intention while \( p_{\text{PPU}} \) is determined by the EMS. The gear is chosen so that \( p_{\text{PPU}} \) is delivered at the engine speed \( \omega_{\text{ICE}} \) that results in the highest possible engine efficiency \( \eta_{\text{ICE}}(p_{\text{PPU}}, \omega_{\text{ICE}}) \). \( \eta_{\text{ICE}}(p_{\text{PPU}}, \omega_{\text{ICE}}) \) is given by linear interpolation in an ICE efficiency map. There are no restrictions on gear changes and inertia of the engine is not modeled. With \( \omega_{\text{ICE}} \) and \( p_{\text{PPU}} \) determined, the buffer power \( p_{\text{EB}} \) is for \( p_{\text{demand}} > 0 \) given by

\[
p_{\text{EB}} = p_{\text{PPU}} - p_{\text{demand}}/\eta_{\text{EB}} - \omega_{\text{ICE}} * T_{\text{ICE,friction}},
\]

and for \( p_{\text{demand}} \leq 0 \) by

\[
p_{\text{EB}} = p_{\text{PPU}} - \eta_{\text{EB}} p_{\text{demand}} - \omega_{\text{ICE}} * T_{\text{ICE,friction}}.
\]

The gearbox efficiency \( \eta_{\text{g}} \) is assumed to be the same for all gears. If \( p_{\text{PPU}} > 0 \) the engine friction is included in the ICE efficiency map and \( T_{\text{ICE,friction}} = 0 \). The electric machine efficiency \( \eta_{\text{EM}}(p_{\text{EB}}, \omega_{\text{ICE}}) \) is given by linear interpolation in an efficiency map and the electric converter efficiency \( \eta_{\text{EC}} \) is modeled as constant. The battery efficiency \( \eta_{\text{BAT}}(p_{\text{BAT}}) \) is modeled by a simple resistive circuit where the open circuit voltage is independent of the SoC. The SoC at the next time point is thus determined by

\[
\text{SoC}_{k+1} = \text{SoC}_k + t * \eta_{\text{BUF}}(p_{\text{EB}}, \omega_{\text{ICE}}) * p_{\text{EB}} / Q,
\]

where \( \eta_{\text{BUF}}(p_{\text{EB}}, \omega_{\text{ICE}}) \) denotes the total efficiency of electric energy flow, \( t \) the time step and \( Q \) the effective capacity. A number of restrictions must be considered when simulating and optimizing the powertrain energy flow. The buffer power, \( p_{\text{EB}} \) and the SoC is limited according to

\[
p_{\text{EB}} \in [p_{\text{EB,min}}, p_{\text{EB,max}}],
\]

\[
\text{SoC} \in [0, 1].
\]

It should be noted that SoC refers to effective buffer capacity. To avoid excessive wear of the EB, the effective capacity is only 15% of the total battery capacity. The PPU power, \( p_{\text{PPU}} \), is limited according to

\[
p_{\text{PPU}} \leq p_{\text{PPU,max}}.
\]

The PPU and EM torques, \( T_{\text{PPU}} \) and \( T_{\text{EM}} \), are further limited by rotational speed dependent constraints

\[
T_{\text{PPU}}(\omega_{\text{ICE}}) \leq T_{\text{PPU,max}}(\omega_{\text{ICE}}),
\]

\[
T_{\text{EM}}(\omega_{\text{ICE}}) \in [T_{\text{EM,min}}(\omega_{\text{ICE}}), T_{\text{EM,max}}(\omega_{\text{ICE}})].
\]

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**TABLE I**

**MAIN PARAMETER VALUES FOR FULL-SIZE TAXI CAR**

<table>
<thead>
<tr>
<th>Vehicle concepts T1-T3</th>
<th>Code</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-15</td>
<td></td>
<td>Vehicle configuration</td>
<td>Parallel, FWD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Vehicle Mass</td>
<td>1800 kg</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gearbox</td>
<td>5 stepped automatic</td>
</tr>
<tr>
<td>T1-12</td>
<td></td>
<td>PPU type and max power</td>
<td>ICE, 125 kW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Buffer type and max power</td>
<td>NiMH Battery, 12 kW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Effective buffer capacity</td>
<td>300 kW</td>
</tr>
<tr>
<td>T1</td>
<td></td>
<td>Buffer type and max power</td>
<td>NiMH Battery, 16 kW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Effective buffer capacity</td>
<td>400 kW</td>
</tr>
<tr>
<td>T2</td>
<td></td>
<td>Buffer type and max power</td>
<td>ICE, Atkinson, 43 kW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Buffer type and max power</td>
<td>NiMH Battery, 32 kW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Effective buffer capacity</td>
<td>800 kW</td>
</tr>
</tbody>
</table>

---

IV. DYNAMIC PROGRAMMING

Dynamic programming is used to find the optimal control, minimizing the expected fuel consumption, for each vehicle concept and each information level. The control signal $u$ is: $u = p_{PPU}$. When quantizing $u$, care should be taken so that at all states where it is possible, one control action should result in a $p_{PPU}$ exactly equal to zero.

A. Position invariant controller

An infinite horizon problem is formulated on the homogeneous Markov chain. The optimization finds an optimal policy, $u = \pi(x)$ that minimizes the expected fuel consumption over an infinite horizon [3], [4], [5]:

$$J^*_N(x_0) = \lim_{N \to \infty} E\{\sum_{k=0}^{N-1} \lambda^k c(x_k, \pi(x_k))\},$$

(14)

where $c$ is the cost for one time step, $\lambda$ is the discount factor, $x_k$ is the dynamic state vector at the $k$th time point and $E\{\ldots\}$ denotes the expectation with respect to the considered prediction model defined by the time invariant Markov chain. The dynamic state vector is defined as

$$x_k = [\text{SoC}_k, p_{\text{demand},k}, v_k].$$

(15)

The discount factor $\lambda < 1$ assures convergence of the infinite sum. The optimization is subjected to constraints (9)-(13). The cost for one time step $c(p_{\text{demand},k}, v_k, u_k)$ is simply the instantaneous fuel consumption which is precalculated in order to reduce computational time.

The optimal policy $u = \pi(x)$ is found by using a modified policy iteration algorithm [7].

Note that since $\text{SoC}_{k+1}$ is not constrained to be on the state grid, calculating $J^*_N(x_{k+1})$ involves interpolation. Linear interpolation is used here.

The optimal policy $\pi(x)$ is a nonlinear time invariant control law given as a lookup table on the discrete grid $x$. When the policy is used as an EMS in a real vehicle, interpolation is needed.

B. Position dependent controller

Dynamic programming applied to the position dependent Markov chain leads to a finite horizon problem. The expected fuel consumption if policy $\pi_k$ is used for the finite horizon and the system is in state $x_0$ at the initial position is defined by

$$J^*_N(x_0) = E\{\sum_{k=1}^{N-1} c_k(x_k, \pi_k(x_k)) + r_k(\text{SOC}_N)\},$$

(16)

where $c_k$ is the cost for one step, $x_k$ is the dynamic state vector at the $k$th position and $r_k(\text{SOC}_N)$ is a final cost. The final cost $r_k(\text{SOC}_N)$ is used to control the final SoC. The dynamic state vector is defined as

$$x_k = [\text{SoC}_k, v_k, a_k].$$

(17)

The optimization is subjected to constraints (9), (10), (11), (12) and (13).

The optimal policy $u_k = \pi_k(x_k)$ is found by backwards iterations. Standstill is modeled by one extra decision epoch, where the PPU power is constant during the stop. Note that $\text{SoC}_{k+1}$ is here constrained to be on the state grid, interpolation is thus not used. This means that the discretization of the SoC must be done with a high resolution to keep the discretization noise on the control signal $u = p_{PPU}$ at an acceptable level.

C. Ideal controller

The drivecycle is here completely known to the controller. The optimal policy $u_k = \pi_k(x_k)$ where $k$ is either a position or time index is found by backwards iterations from the final position or time sample. Note that since the drivecycle is perfectly known the dynamic state is simply $x = \text{SoC}$.

V. RESULTS AND DISCUSSION

Three different variants of a parallel hybrid taxi are included in this study, representing different degrees of hybridization. The sparse quantization and the varying time step for the position dependent Markov chain means that the controllers cannot be evaluated in a straightforward and fair manner using the same drivecycle. The evaluation is instead done in two steps. First, the position dependent controller is compared with the ideal controller on quantized versions of the measured drivecycles. The ideal controller represents the unrealistic situation where the future power demand is completely known to the control system. The quantization of the measured drivecycle is the same as the one used when constructing the position dependent Markov chain. Simulating with this quantization of the measured drivecycles removes the need for interpolation in the policies and reduces the effect of the sparse gridding of the velocity. The sparse quantization of the drivecycle can be considered as a modeling error which results in more aggressive driving than what was measured.

The second step in the evaluation is to compare the position invariant controller with the ideal controller. Here the comparison is done by simulating the vehicle variants with the two controllers both on the measured drivecycles and on drivecycles generated from (1). The drivecycles are not quantized and the time step is 1 second. Interpolation techniques must be used to find $u(x)$ from the policies. This means that in this second comparison there is an interpolation error.

A. Comparison between the position dependent controller and the ideal controller

The vehicle concepts are simulated on 10 quantized versions of the measured drivecycles. The study shows practically no difference in consumption between the ideal and the position dependent controller on all simulated drivecycles and for all studied concepts. The difference in consumption is less than 0.2% for all simulated drivecycles with no difference in final SoC. This result means that the planning of the use of the buffer is insensitive to the fine
structure of the driving that lies ahead, only the "average" information about the route is significant. Fig. 3 shows the SoC-trajectory for the T1 concept and it is seen that the planning is done on a long horizon, where energy is saved for the high speed section and the buffer is depleted at the two altitude peaks.

B. Comparison between the position invariant controller and the ideal controller

The vehicle concepts are simulated both on the measured drivecycles and on random drivecycles generated from (1). The sample time is 1 second. The quantization grid used for calculating the position invariant controller is refined until no improvement of the fuel consumption is noted in the simulations.

The simulated fuel consumptions are presented in Tab. II. The difference between the ideal and the position invariant controller is interpreted as the savings that can only be achieved if position dependent predictions of the route are used. When simulated on the measured drivecycles these savings are 0.8% for the T1 concept, 0.9% for the T2 concept and 1.6% for the T3 concept. When simulated on the randomly generated drivecycles the savings are 2.1% for the T1 concept, 0.9% for the T2 concept and 2.5% for the T3 concept. The T3 concept with a higher degree of hybridization benefits more from using position dependent predictive control than the two concepts with low hybridization.

Fig. 4 shows the SoC-trajectories for the three concepts when simulated on one of the measured drivecycles. In the three upper graphs the solid lines show the results from the ideal controller and the dotted lines are the position invariant controller. The bottom graph shows the topography of the route.

Fig. 5 shows the SoC-trajectories for the three concepts when simulated on a randomly generated drivecycle. Studying the SoC-trajectory for the T3 concept it is seen that energy is saved for the uphill sections and the buffer is depleted at the two altitude peaks. This shows that the topography plays an essential part in the planning. Again this behavior is similar but not as clear for the T1 and T2 concept. Fig. 6 show that the ideal controller moves the operating points for the PPU slightly to higher efficiencies compared to the position invariant controller.
VI. CONCLUSIONS

The performance of the ideal and the position dependent controller is almost identical. The conclusion is that the planning is done on a long horizon where the topography plays a crucial part. Furthermore, this also means that if position dependent information is used in the EMS, the planning is insensitive to the particular fine structure of the drivecycle, only the "average" information about the route is significant. The comparison between the fuel consumption between the position invariant and the ideal controller shows that if the controller uses position dependent information the potential fuel savings is between 1-2.5% for the studied route and concepts. The study therefore shows that good performance is attainable without position dependent information, conditioned that the EMS is well tuned to the route. If position dependent information is used, the fuel consumption is reduced further. An integration of the GPS navigator with the EMS is therefore very promising.

REFERENCES


