Optimal Integrated Call Admission Control and Dynamic Pricing with Handoffs and Price-Affected Arrivals

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Abstract—Call admission control and dynamic pricing have been proposed as separate measures to reduce congestion in a network. In this paper, we integrate the call admission control and dynamic pricing problems, formulating them as a Markov Decision Problem (MDP), operating in a multiservice, resource-sharing cellular system. Our model incorporates price-affected behaviour of network users, considering effects on arrivals, retrials and substitutions among services and time. The network exercises admission control, to reject or accept a new connection requests, and price control to set a state-dependent price per bandwidth time with the objective to maximise the long-term revenue. The results show that the proposed method improves revenue and reduces congestion compared to other conventional sub-optimal policies.

I. INTRODUCTION

Call admission control (CAC) and dynamic pricing have been proposed as arbitration mechanisms to regulate traffic and reduce congestion in a network. CAC is such a provisioning strategy to limit the number of call connections. It means that users are not automatically admitted even though there are resources available. Dynamic pricing makes adjustments often, according to the demand pattern and congestion level in a network in order to influence the way users utilise network resources, especially to allocate limited resources to those who value them most. Dynamic pricing also enhances network operators’ ability to recover costs and make profits to finance capacity expansions. By influencing the demand patterns, operators could avoid the costly need to provision a network so that it can always meet its peak demand.

Many pricing schemes have been put forward. The concept of auction-based, smart-market pricing is proposed in [1]. A charge that is based on effective bandwidth is constructed in [2]. Charges depend upon both static parameters, being traffic contracts, and dynamic measures, being the current usage of network resources. Pricing models in multiservice networks have been considered in [3] for the single link case and in [4] for the network case, where near-optimal static pricing policies have been established for a network with many small sources of traffic. We refer our readers to [5] for a comprehensive survey on the vast number of pricing schemes proposed. These schemes are proposed for broadband networks like the Internet where resource reservation for handoff users is a non-issue.

In this paper, we provide a generic framework to formulate an integrated call admission and dynamic pricing problem for multiservice, resource-sharing cellular systems as a Markov Decision Problem (MDP) and solve it using dynamic programming methods [6]. The objective is to maximise the long-term expected revenue. Given a particular configuration of network users, the objective is to determine both whether or not to accept a new connection and the optimal price per bandwidth time to shape demand. Since premature termination of ongoing calls is more undesirable than rejection of new call requests, it has been widely accepted that a system should allocate a higher priority to handoff call requests. Our model allows for bandwidth reservation for handoff calls by associating a Satisfaction Revenue (SR) with the admission of handoff calls. SR is not a real income to the service operator.

A major and inaccurate assumption in all of the papers mentioned is that users arrive independently according to a Poisson process and will be automatically lost if blocked. This queuing model creates an underestimation of the actual arrival rate in the system, especially during congestion. As depicted in Fig. 1, we consider an advanced arrival model that incorporates retrials and substitution effects among services and through time. Assuming that some users remain in the vicinity of the system when blocked, they have a choice to defer their call requests or to use another service as a substitute. These deferred users, together with new arrivals, provide a more realistic estimate of the actual arrival rate to the system than do conventional arrival models. However, handoff connection requests are not price-affected since they are pre-admitted at other prices and are assumed to leave the system if blocked.

A similar work has been considered in [7]. In their work, the dynamic pricing component of the integrated system is only active when the arrival rate of the system exceeds a pre-determined optimal level. Instead, we will use prices as incentive to encourage arrivals during low-traffic period, in addition of congestion control during high-traffic period. The CAC component of [7] only assumes a certain number of guard

![Fig. 1. Integrated Call Admission and Dynamic Pricing Model.](image-url)
channels and, unlike our work, does not optimally reserve resources for future-arriving handoff and new users that will maximise the long-term revenue. Besides, our problem is different because we consider the integrated problem in a decision-theoretic framework and solve it using different tools.

The rest of the paper is organised as follows. In Section II, we describe our network model. We then formulate our integrated problem in section III and present computational results in Section IV. Conclusions are summarised in Section V.

II. NETWORK MODEL

We consider a network with \( B \) units of bandwidth and \( J \) classes of service. The price per unit time for using one unit of bandwidth of service class \( j \) is denoted as \( p_{ij} \), \( i = 1, 2, \ldots, J \). Each service class \( j \), which uses \( b_j \) units of bandwidth, is characterised by its Poisson-distributed new and handoff call arrival rates \( \lambda^a_j \) and \( \lambda^b_j \), exponentially-distributed call holding time \( 1/\mu_j \) and Weibull-distributed willingness to pay (WTP) \( \Psi_j \) with mean \( \psi_j \) and shape \( \beta_j \). WTP quantifies the satisfaction gained from a call.

We use the Weibull distribution to model users’ WTP because it is versatile and can take up the characteristics of other types of distributions, based on the value of the shape parameter. Within the telecommunications framework, it has been used to model the traffic characteristics of packet audio streams in [8] and to simulate data traffic in [9]. Different values of the shape parameter have different effects on the behaviour of the distribution. Exponential demand functions \( (\beta = 1) \), as used in [7], are special instances of the WTP distribution considered in this work.

CAC is triggered when a user makes a call connection request. At the time a new or handoff call is accepted, the system must have at least \( b_j \) units of bandwidth available. A handoff connection request will always be accepted if the system has sufficient bandwidth. A CAC policy determines the state-dependent admission policy, \( u_j(s) = (u_{c1}(s), \ldots, u_{cj}(s)) \), for new users, for every state \( s \). A new connection can either be accepted with \( u_{cj} = 1 \) or rejected with \( u_{cj} = 0 \). A new user will decide to either make a connection request if his or her budget is sufficient to cover the expected call cost, i.e. \( \Psi_j \geq u_{cj}b_j/\mu_j \) or defer the request otherwise.

When blocked, handoff users will leave the system immediately. However, new users of service \( j \) who are rejected by control or have insufficient WTP will either retry later with probability \( \alpha_{Rj} \), substitute out to another service \( k \neq j \) with probability \( \alpha_{SOj} \) or abandon the system with probability \( \alpha_{Aj} \). New users of service \( j \) who retry later and users of service \( k \neq j \) who substitute in to service \( j \) are said to be in orbit and will independently generate requests for service at exponentially-distributed time intervals with mean \( 1/\sigma_j \) until they obtain service, abandon or substitute out.

Price control determines the state-dependent pricing vector \( u_p(s) = (u_{p1}(s), \ldots, u_{pj}(s)) \), with \( u_{pj} \in U_{pj} \), to regulate new call arrivals to the network. \( U_{pj} \) is the set of possible values of \( u_{pj} \). Therefore, the demand for service of new users depends on \( u_{pj} \), the desired service length and their WTP. We denote the probability of having the sufficient WTP to make a call as the access probability:

\[
\alpha_{pj} = 1 - F_W(u_{pj}, b_j/\mu_j),
\]

where \( F_W(y) = P[\Psi_j \leq y | \psi_j, \beta_j] \) is the cumulative distribution function of the Weibull-distributed \( \Psi_j \). \( \lambda_j^b \) is the maximum new arrival rate, limited by the access probability \( \alpha_{pj} \). The total price-affected arrival rate to a service is therefore \( \lambda_j = \alpha_{pj}(\lambda^a_j + x_j\sigma_j) \), where \( x_j \) is the number of users in orbit and \( \sigma_j \) is the retrial rate of these users. Access probability can be seen as an arrival gate that controls the flow of price-affected arrivals to the system. By varying \( u_{pj} \), the arrival rate of a particular service can be encouraged or discouraged to give way for arriving handoff or higher revenue-generating users. The objective is to exercise call admission and pricing to maximise the long-term expected revenue.

As mentioned in [7], the parameters that define the WTP distribution function must be identified by adequate market research. In reality, the information on users’ call budget can be extracted from an operator’s historical data on users’ spending patterns and expenditure. Users can also willingly inform network operator of their WTP and this procedure can be automated by including a simple call-budgeting program in users’ mobile device. Although it is expected that most users would like to spend as little as possible and only indicate their minimum WTP, higher-end users would place a higher value on a call during congestion when their initial WTP is not sufficient.

III. MARKOV DECISION PROBLEM FORMULATION

The system can be described by a Markov Chain on the state space \( S = \{(x, n) : 0 \leq x_j \leq X_j, n^j b \leq B, \forall j \leq J \} \). Vectors \( n = (n_1, \ldots, n_J) \), \( x = (x_1, \ldots, x_J) \) and \( b = (b_1, \ldots, b_J) \) denote the number of active connections, the number of users in orbit and the amount of bandwidth required for all services respectively. To ensure that the state space remains finite, we limit the number of users in orbit to \( X \). When the number of users in the orbit of service \( j \) reaches \( X_j \), the blocked calls will be lost and no users from other services can substitute in. This truncation method is often used in the analysis of retrial system to reduce the complexity involved and methods of choosing appropriate level of truncation are discussed in [10].

We will now derive the transition rates from a state \( s = (x, n) \). The number of active connections in the system increases due to arriving handoff users at a rate of:

\[
q((x, n), (x, n + e_j)) = \lambda^a_j I(n^j b + b_j \leq B) = \lambda_{0j}.
\]

The value of an indicator function \( I(a) = 1 \) if condition \( a \) is true and 0 otherwise. New and retrying users in orbit increase the number of connections at the following rates respectively

\[
q((x, n), (x, n + e_j)) = \alpha_{pj} \lambda^b_j u_{ej} = \lambda_{1j}
\]

\[
q((x, n), (x - e_j, n + e_j)) = \alpha_{pj} x_j \sigma_j u_{ej} = \lambda_{2j}
\]
where \(e_j\) is a unit vector with 1 in its \(j^{th}\) position. The number of connections will decrease at a rate of:

\[
q((x, n), (x, n - e_j)) = n_j \mu_j I(n_j > 0). \tag{5}
\]

The number of users in orbit \(j\) will only decrease if users abandon service at a rate of

\[
q((x, n), (x - e_j, n)) = (1 - \alpha_{pj} u_{cj}) \alpha_{Rj} x_j \sigma_j = \lambda_{3j} \tag{6}
\]

or substitute out to another service \(k, k \neq j\), at

\[
q((x, n), (x - e_j + e_k, n)) = (1 - \alpha_{pj} u_{cj}) \alpha_{SOkj} x_j \sigma_k = \lambda_{4kj}. \tag{7}
\]

The number of users in orbit \(j\) will increase if new users retry at a rate of

\[
q((x, n), (x + e_j, n)) = (1 - \alpha_{pj} u_{cj}) \alpha_{Rj} x_j \sigma_j = \lambda_{5j} \tag{8}
\]

\[
\sum_{k \neq j} (1 - \alpha_{pk} u_{ck}) \alpha_{SOkj} x_k \sigma_k = \lambda_{5j}
\]

or users of another service, say \(k\) where \(k \neq j\), substitute in at a rate of

\[
q((x, n), (x + e_j - e_k, n)) = (1 - \alpha_{pj} u_{cj}) \alpha_{SOkj} x_k \sigma_k = \lambda_{4kj}. \tag{9}
\]

Note that (7) and (9) are actually the same. In order to avoid calculating the same rates twice, we only need to consider the rates associated with a user substituting out for each service. We also note that transitions \(q((x, n), (x - e_k, n + e_j))\) are disallowed and set to zero. This means that users substituting in from service \(k\) need to enter the orbit of service \(j\) before reattempting. Users who reattempt unsuccessfully and decide to reattempt again, and new users who abandon do not change the state of the system.

Even though the process evolves in continuous time, we only have to consider the state of the network when certain events take place. We say that an event happens at a certain time if any of the transitions derived in the previous section occurs. Let \(\Omega\) denote the set of possible events, i.e. \(\Omega = \{\omega | \omega \in \{0, 1, 2, 3, 4, 5, 6, 7\}^E\}\). The list of possible events corresponds to the possible transitions outlined previously, i.e. event 0 indicates an handoff arrival, 1 indicates a new arrival, 2 indicates an arrival from orbit and so on. Event 7 indicates no event occurring. For each state \(s \in S\) and event \(\omega \in \Omega\), \(U(s, \omega)\) is the set of available decisions:

\[
U(s, \omega) = \begin{cases} \{U_c \times U_p\} & \text{if } \omega \in \Omega_a \\ \{U_p\} & \text{if } \omega \notin \Omega_a \end{cases} \tag{10}
\]

where \(\Omega_a\) is the set of all events consisting of arrival events 1 or 2, \(U_c\) and \(U_p\) denote the set of all possible call admission and price control decisions and are defined as \(U_c = \{u_c | u_{cj} \in \{0, 1\}, \forall j\}\) and \(U_p = \{u_p | u_{pj} \in U_p, \forall j\}\) respectively.

Given that the system is in state \(s \in S\) with control actions \(u \in U\) available and an event \(\omega \in \Omega\) occurred, the next state, \(s' \in S\), is given by a function \(f : S \times \Omega \times U\) such that

\[
f(s, \omega, u) = \begin{cases} (x, n + e_j) & \text{if } \omega_j = 0, n'b + b_j \leq B \\ (x, n + e_j) & \text{if } \omega_j = 1, u_{cj} = 1 \\ (x - e_j, n + e_j) & \text{if } \omega_j = 2, u_{cj} = 1 \\ (x - e_j, n) & \text{if } \omega_j = 3 \\ (x - e_j + e_k, n) & \text{if } \omega_j = 4 \\ (x + e_j, n) & \text{if } \omega_j = 5 \\ (x, n - e_j) & \text{if } \omega_j = 6 \\ (x, n) & \text{otherwise.} \end{cases} \tag{11}
\]

Using uniformisation [6], the continuous-time MDP can be transformed into its discrete-time equivalence with the so-called uniform transition rate, where the total transition rate out of any state is bounded by \(\nu\). The transition probabilities \(p(s, \omega, u)\) for state \(s\) are then given by

\[
p(s, \omega, u) = \begin{cases} \lambda_{0j} \nu & \text{if } \omega_j = 0 \\ \lambda_{1j} \nu & \text{if } \omega_j = 1 \\ \lambda_{2j} \nu & \text{if } \omega_j = 2 \\ \lambda_{3j} \nu & \text{if } \omega_j = 3 \\ \lambda_{4kj} \nu & \text{if } \omega_j = 4 \\ \lambda_{5j} \nu & \text{if } \omega_j = 5 \\ n_{j}\mu_j \nu & \text{if } \omega_j = 6 \\ 1 - \nu(s) & \text{and if } \omega_j = 7, \end{cases} \tag{12}
\]

where the total transition rate out of state \(s \in S\) is given by

\[
\nu(s) = \sum_{j=1, k \neq j} \lambda_{0j} + \lambda_{1j} + \lambda_{2j} + \lambda_{3j} + \lambda_{4kj} + \lambda_{5j} + n_j \mu_j.
\]

The revenue rate collected by the system at a particular state \(s = (x, n)\) is be given by the reward function

\[
g(s, \omega, u) = \begin{cases} SR & \text{if } \omega_j = 0 \text{ and } n'b + b_j \leq B \\ r_j = u_{pj} b_j / \mu_j & \text{if } \omega_j = 1, 2 \text{ and } u_{cj} = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{13}
\]

Reward \(r_j\) is the revenue collected when a user of class \(j\) is admitted. In order to reflect the higher importance of accepting handoff calls, SR should be greater than the actual revenue provided by the admission of new call requests. A user is admitted based on a single admission price that will not change for the entire duration of the call. The average reward-to-go function is given by the Bellman’s equation:

\[
J^* + h(s) = \max_{u \in U(s, \omega)} \left[ \sum_{\omega' \in \Omega} p(s, \omega, u)[g(s, \omega, u) + h(s')] \right] \tag{14}
\]

where \(s' = f(s, \omega, u)\) is given in (11) while \(J^*\) and \(h(s)\) denote the optimal expected revenue per stage and the relative or differential revenue rate of state \(s \in S\), respectively. A stage here means a transition in the uniformised chain. The optimal expected revenue per stage is independent of the initial state. It has been argued that the standard infinite-horizon average reward dynamic programming theory applies and there exists a stationary policy which is optimal [6].
IV. Numerical Results

We compare our Optimal Call Admission and Dynamic Pricing (OCADP) policy numerically with three other policies: Always Accept and Static Pricing (AASP), Optimal Call Admission and Static Pricing (OCASP) and Always Accept and Dynamic Pricing (AADP). The Always Accept component in AASP and AADP always admits a new user if sufficient bandwidth is available, which is equivalent to having no CAC at all. For both AASP and OCASP, we set the price per bandwidth time to the average price that gives access probability \( \alpha_{P_j} = 0.5 \), \( j = 1, 2 \). For AADP and OCADP, we allow price \( u_{P_j} \), and therefore \( \alpha_{P_j} \), to vary. We purposely set up service 2 to generate higher expected revenue than service 1. The simulation parameters used are: \( B = 8 \), \( X = (3, 4) \), \( \lambda^n = (0.5, 1) \), \( \lambda^h = 0.25 \lambda^n \), \( 1/\mu = (1, 2) \), \( \sigma = (0.5, 0.5) \), \( b = (1, 2) \), \( \alpha_R = (0, 6, 0.6) \), \( \alpha_{SO} = (0, 2, 0.2) \), \( \alpha_A = (0, 2, 0.2) \) and \( SR_j = 3r_j \). We will analyse the results for two WTP distributions (see Fig. 2), i.e. with \( \beta = 3.5 \) and 1.0. Both have the same \( \psi = (1, 1) \) per bandwidth time.

As shown in Fig. 3, integrated policy OCADP generates the highest optimal expected revenue per stage, \( J^* \), followed by AADP, OCASP and AASP. The reward advantage of OCADP over other policies increases as the arrival rate of service 2 increases. The optimal policy maximises revenue by exercising call admission and pricing control to:

- reserve bandwidth for handoff users,
- block (new) users of the lower revenue-generating service 1 to reserve bandwidth for (new) users of service 2,
- set higher price per bandwidth time (and therefore lower access probability) for service 1 to discourage the arrival of its users, and
- adjust prices according to the system state to encourage and discourage arrivals of all services.

New users are blocked when the expected revenue gained from future-arriving handoff users exceeds that of an immediate admission of service 1 and 2 users. Similarly, service 1 users are blocked more often than of service 2 because the latter provide more expected revenue per call with higher arrival rate and call holding time. As the arrival rate of service 2 to the system increases (see Fig. 4), more service 1 users are blocked so that scarce system resources will only be allocated to the service that provides more revenue. However, the number of states where a service 1 user is blocked under OCADP is less than of OCASP as the latter has no control over the setting of price per bandwidth time to further influence the arrival rate of both services. In cases where network is lightly loaded, e.g. \( \lambda = (0.5, 1) \), the expected average revenue generated by both services are almost the same.

A typical dynamic pricing policy is illustrated in Fig. 5. The network charges higher prices per bandwidth time as the network becomes congested. Additional revenue is generated by offering below average prices per bandwidth time to users when the network is under-utilised. This result obtained is in line with the laws of demand and supply from economics which require that prices rise when demand is large relative to available supply and fall in the contrary case. A static pricing scheme that offers average, state-independent price per bandwidth time generates lower expected average revenue as it fails to offer the same incentives to users.

The stationary probabilities of using AASP and OCADP are shown Fig. 6(a) and 6(b) respectively. The spikes in both graphs are due to the variation of the number of users in orbit. There is an evident displacement of probabilities from congested to less congested states when OCADP is used. The explanation for this phenomenon is simple. OCADP blocks all or some new call requests that generate lower revenue, compared to SR, and sets high prices to subsequent call admissions when the network is congested. Persistent users
The higher revenue is due to the higher WTP of users at \( \alpha \beta \) average optimal access probabilities for OCADP when state of service 1, Fig. 5. Optimal Dynamic Pricing Policy. The x and y axes indicate the distribution by setting \( s \) and \( 3 \). OCADP and AADP achieve higher distribution function of the WTP reduces to an exponential network to always cater for the peak demand.

Similar results (Fig. 7) are obtained when the probability distribution function of the WTP reduces to an exponential distribution when \( s = 3 \) and \( \beta = 1 \). However, OCAS and AASP provide lower revenue but OCADP and AADP achieve higher revenue, compared to when \( \beta = 3.5 \) is used. Please refer to Fig. 2 for the following explanations. The previous is due to the higher WTP placed by users with \( \beta = 1 \) for the same access probability \( \alpha P = 0.5 \). In the dynamic pricing case, the average optimal access probabilities for OCADP when \( \beta = 1 \) and \( 3.5 \) are \( \alpha P = (0.11, 0.25) \) and \( (0.13, 0.45) \) respectively. The higher revenue is due to the higher WTP of users at \( \alpha P = 0.25 \) when \( \beta = 1 \) compared to at \( \alpha P = 0.45 \) when \( \beta = 3.5 \).

V. CONCLUSIONS

We have analysed a model for optimal integrated call admission and dynamic pricing in a cellular system with handoffs and price-affected new arrivals. Integrated policy OCADP has the flexibility to reject new connection requests to reserve resources for handoff users or new users who value the connection more. This strategy also provides monetary incentive to low-WTP users to access the network when the load is relatively light and allocate resources to high-WTP users when the network is relatively congested.

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