Logistic ordinal regression for the calibration of oscillometric blood pressure monitors

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Abstract

Oscillometric Blood Pressure (BP) monitors are omnipresent and used on a daily basis for personalized healthcare. Nevertheless, physicians generally approach these devices cautiously since the mercury Korotkoff sphygmomanometer remains the golden standard. Various reasons explain the hesitating attitude of the medical world towards automated BP monitors: (i) its principle is based on the pressure pulsations arriving at the cuff by the cardiac cycle instead of an audio wave used by physicians triggered by the turbulences in the artery, (ii) the actual computation of the systolic and diastolic BP from the measured oscillometry is manufacturer dependent and not based on general scientific principles, (iii) the quality of the oscillometric monitors is labeled by a trial such that the devices correspond well to the Korotkoff method for the average healthy patient but deviates for patients suffering from hypo- or hypertension. In this paper, we develop a statistical learning technique to calibrate and correct an oscillometric monitor such that the device better corresponds to the Korotkoff method regardless of the health status of the patient. The technique is based on logistic regression which allows correcting and eliminating systematic errors caused by patients suffering from hyper- or hypotension. No user interaction is required since the technique is able to train and validate the calibration procedure in an unsupervised way. In our case study, the systematic error is reduced by nearly 50% corresponding to the performance specifications of the device.

Keywords: Blood pressure measurement, oscillometric signal, Pulse-Pressure Signals, Bio-signal Processing, Heart frequency estimation, Logistic regression, Calibration.

1. Introduction

Personalized healthcare and home health monitoring is a booming business. This is due to aging and the advanced technologies in telecommunication applications. From a technological perspective, one needs to ensure that we do not clear the path for a wild growth of medical monitors such that medical devices are based on scientific principles ensuring quality. The safeguard should consist of reducing the discrepancy between (low cost) home monitors and more expensive and specialized clinical devices. Nevertheless, the quality difference between the clinical device and the home monitoring system is apparent due to cost, space, and layman use.

In an effort to eliminate the gap between the Korotkoff method applied in the sphygmomanometer [1] and the oscillometric devices a calibration procedure needs to be established [2, 3]. The calibration method eliminates the systematic error by comparison with the Korotkoff results denoted as the golden standard. A few constraints to establish this procedure are: (i) the patient needs to be able to perform the calibration at home, (ii) additional hardware should be avoided, (iii) The patented algorithm should remain unchanged. The calibration problem is two-fold: The systematic error needs to be detected and the systematic error needs to be corrected. Since 1999, test simulators for the calibration of oscillometric devices have been introduced [4, 5, 6]. Such simulators apply test signals to the oscillometric device for testing the accuracy of the measurement. Either these simulators require additional hardware or they apply a database
of oscillometric signals to correct the algorithm computing the BP values [7]. Once a systematic error is detected, post-processing should account for this discrepancy without changing the patented algorithm. Most research on oscillometric blood pressure signals is focused on either understanding the fundamental signal properties [8, 9, 10], blood flow dynamics in the presence of deflating cuff [11, 12], filtering techniques [13] and signal feature detection with various methods [14].

The number of published results regarding the (post) correction of automatic blood pressure monitors is significantly less abundant. For instance a denoising approach to eliminate confounding and movement artifacts was studied in [15]. Temporal variability was modeled and suppressed in [16, 17]. To obtain more accurate readings the blood pressure variability was measured and taken into account in [18]. In [19, 20] we tried to correct the oscillometric monitors for patients with an extremely high or low blood pressure by taking into account a dynamic blood flow model.

In this paper, we want to study a statistical technique which can be used together with a test simulator for oscillometric devices. Test signals are fed to the oscillometric devices to quantify the discrepancy between the Korotkoff sphygmomanometer and the automatic oscillometric monitor. Based on the signal characteristics of the measured oscillometric signal and the oscillometric blood pressure readings a logistic regression is applied for the bank of test signals to identify a correction rule for the automatic blood pressure monitor. The actual relationship between the signal features and the Korotkoff blood pressure is highly nonlinear such that only a limited performance can be observed by using a linear regression to map the shape of the oscillometric signal to the blood pressure see [19]. To avoid a crusade for a very complex nonlinear model, we remain in the linear modeling framework. As a result, we can rely on the linear regression framework but not to estimate the correct blood pressure specifically but to estimate the correct range of the blood pressure. This type of linear regression is known as logistic regression.

2. Blood Pressure (BP) Measurement Campaign

2.1. Korotkoff sphygmomanometer against the oscillometric monitor

The golden standard for measuring the blood pressure remains the Korotkoff sphygmomanometer. The sphygmomanometer inflates a cuff wrapped around the patient’s upper arm until the blood circulation is stopped. A stagewise deflation of the cuff restores the blood flow while slowly opening the artery. Through the stethoscope, the physician listens at the turbulences caused by restoring the blood flow. The sound type of the turbulences undergo five phases known as the Korotkoff sounds [1]. The systolic blood pressure is determined at the start of the first Korotkoff sound whereas the diastolic pressure is defined by the final Korotkoff sound.

The oscillometric monitors also apply a cuff but instead of recording the turbulences in the arteries, the pressure pulsations of the heart arriving at the cuff are measured. The pulsations hold an oscillating nature due to the systolic and diastolic phases of the heart cycle. On top of that the deflation of the cuff acts as an amplitude modulation on the blood pulse oscillations. This amplitude modulated signal is known as the oscillometric signal. The determination of the systolic and diastolic BP from the oscillometric signal is obscure in the sense that a unique algorithm is not available. Indeed, there are two important theoretic schools which conjecture that the BP is a function of the relative height of the oscillometric signal w.r.t. the global maximum (height based school) whereas a second school is in favor of the BP as a function of the points of inflection of the oscillometric signal around the global maximum [21, 22]. On top of that, the exact implementation to determine the inflection points, the exact percentages to pin-point the relative heights w.r.t the global maximum differ from brand to brand due to possible patented software solutions. Once the algorithm computes the necessary points on the oscillometric signal, the time instant is traced back to the cuff pressure which determines the systolic and diastolic pressure (see Figure 1 as an illustration).

2.2. Measurement set-up

A measurement campaign following the protocols of the British Hypertension Society. In total, the campaign selected 75 patients. The patients were randomized in terms of age, sex, and socio-cultural
Figure 1: Oscillometric BP determination: oscillometric signal (top) - cuff pressure (bottom). BP values: systolic (left circle), MAP (cross marker), diastolic (right circle).

Table 1: Frequency table for the systolic and diastolic blood pressures

<table>
<thead>
<tr>
<th>Diastolic</th>
<th>Systolic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypertension</td>
<td>&lt;90</td>
</tr>
<tr>
<td>Desired</td>
<td>90-119</td>
</tr>
<tr>
<td>Pre-Hypertension</td>
<td>120-129</td>
</tr>
<tr>
<td>Hypertension stage 1</td>
<td>130-139</td>
</tr>
<tr>
<td>Hypertension stage 2</td>
<td>140-159</td>
</tr>
<tr>
<td>Crisis</td>
<td>&gt;160</td>
</tr>
</tbody>
</table>

Table 1 shows the Korotkoff readings performed by a certified physician for the different patients in the campaign. The table reveals that for the diastolic pressures slightly less than 53.3% of the patients reveal pressures beyond the desired range while for the systolic pressures this holds for 75.4% of the patients. Given that oscillometric devices are based on an algorithm computing the blood pressure. This suggests that these algorithms work reasonably for patients in the desired blood pressure range but these devices normalize the
estimated blood pressure for hypo- and hypertensive patients towards the desired ranges. As a result, the campaign shows quite some patients outside the desired range which poses a challenge for the oscillometric devices for which a correction of the oscillometric signal may be desired.

3. Oscillometric waveform: an amplitude modulated Hammerstein - Windkessel model

The calibration procedure to correct the readings of the oscillometric blood pressure monitor requires specific signal features of the oscillometric waveform. In order to extract these features, one needs to model the oscillometric waveform. On top of that, a good model serves to suppress disturbing noise corrupting the measured oscillometric signal resulting in a more accurate feature extraction.

3.1. Model building

One of the most popular and widely used cardiovascular models is the Windkessel model, [23]. Its popularity is based on the fact that it is a simple parallel RC-network driven by a current source and its results provide a good description of the central BP or the low-frequency behavior of the vascular system [24]. Hence, the corner stone of our model is the Windkessel model describing the linear dynamics relating the Cardiac Output (CO) to the BP. The CO is represented by a Fourier paradigm such that the signal is decomposed into harmonically related sinusoids. Hence, the CO is represented by a single sine wave corresponding to the fundamental heart frequency passing through a static nonlinearity inducing its higher harmonics. The fundamental heart frequency represents the beat-to-beat period while the static nonlinearity describes the effect of the different heart chambers. As a result, this representation of the Low Frequency behavior of the vascular system will be called throughout this paper the Hammerstein-Windkessel model. Finally the cuff-in- and deflation mode is modeled by an amplitude modulation acting on the pressure waveform given by the output of the Windkessel model.

In Figure 2 the model is represented as a feed-forward block design model. The signal u₀ denotes the beat-to-beat heart rate represented by a signal sinewave u₀ = A₀sin(2πf₀t + φ₀) with an unknown frequency f₀, phase φ₀ and amplitude A₀. The polynomial block generates higher harmonics such that q(t) = \sum_{k=0}^{R} A_k sin (2π(k+1)f₀t + φ_k) with R the highest degree of the polynomial. The parameters A_k and φ_k depend on the coefficients of the polynomial. The windkessel model represents the low frequency behavior of the cardiovascular system relating blood flow q(t) to blood pressure p(t) by means of the following differential equation

\[ Rq(t) = p(t) + CR\frac{\partial}{\partial t}p(t) \]

Besides the transient conditions of the pressure wave p(t) associated to each heart beat, the pressure signal is a cyclo-stationary signal. The steady-state response of a linear system to a multi-sine input is again a multi-sine such that we obtain

\[ p(t) = \sum_{k=0}^{R} B_k sin (2\pi(k+1)f₀t + ψ_k) \] (1)
The inflating and deflating cuff has an effect on the arterial compliance and resistance such that the parameters $R, C$ become time-varying which implies that the eventual oscillometric signal can be represented by

$$y(t) = \sum_{k=0}^{R} B_k(t) \sin \left(2\pi(k+1)f_0 t + \psi_k(t)\right)$$

Note that the heart rate in this model is assumed constant over time. This assumption is easily violated in practice although specific measurement protocols of for instance the British Hypertension Society. Perturbations in the heart frequency $f_0$ leads to leakage since the pressure signal $p(t)$ never reaches steady state. As a result, the presented approach treats this type of possible leakage as an additional noise disturbance.

### 3.2. Parameter Identification

The parameter identification is not a straightforward regression problem since we do not have access to an actual input. We have a rough estimate of the heart rate $f_0$, but the amplitude $A_0$ and phase $\phi_0$ of the fundamental heart signal $u_0$ are inaccessible. Hence, we need to solve a blind identification problem. This comes with the price that the individual substructures of the model in Figure 2 are not identifiable. Fortunately, the actual goal is to obtain the (time-varying) amplitude and phase in equation (2). The remainder of Section 3.2 describes the two step procedure to identify the oscillometric waveform (2).

#### Heart rate identification

To obtain the heart rate it is assumed that the dominant oscillations present in the oscillometric signal (see Figure 1) is driven by the heart frequency. Hence, the procedure tries to extract the frequency of the fundamental sine wave $u_0(t)$ which is maximally correlated to the oscillometric signal $y(t)$. Assume that both signals are sampled (possibly incoherently) at a sampling frequency $f_s$ then the Z-transform evaluated at angular frequency $\omega_k = \frac{2\pi k}{N f_s}$ or $z_k = \exp(j\omega_k)$ of the sampled signal $u_0(t_n)$ with $t_n = \frac{n}{f_s}$ reveals,

$$U_0(\omega_k) \triangleq \sum_{n=0}^{\infty} u_0(n \frac{f_s}{f_0}) z_k^{-n}$$

$$= A_0 \frac{\sin(\phi_0) + z_k^{-1} \sin \left(2\pi \frac{f_0}{f_s} - \phi_0\right)}{1 - z_k^{-1} 2 \cos \left(2\pi \frac{f_0}{f_s}\right) + z_k^{-2}}$$

where $\triangleq$ denotes equality by definition. The heart frequency $f_0$ can be determined by maximizing the correlation between the measured spectrum of the oscillometric signal $Y(\omega_k)$ computed by the Discrete Fourier Transform and the regressors

$$\begin{bmatrix} 1 \\ 1 - z_k^{-1} 2 \cos \left(2\pi \frac{f_0}{f_s}\right) + z_k^{-2} \end{bmatrix}$$

#### Least squares filter to determine the time varying amplitude and phase of the oscillometric signal

Now that the heart frequency $f_0$ is estimated, we can estimate the remaining unknown parameters in equation (2). The advantage of prior estimates of the heart frequency is that the remaining parameters are linear. Indeed, we can express equation (2) in the following form:

$$y(t) = \sum_{k=0}^{R} B_k(t) \sin \left(2\pi(k+1)f_0 t + \psi_k(t)\right)$$

$$= \sum_{k=0}^{R} B_k(t) \cos \left(\psi_k(t)\right) \sin \left(2\pi(k+1)f_0 t\right)$$
where $B_k(t) = B_k(t)\cos(\psi_k(t))$ and $B_k^*(t) = B_k(t)\sin(\psi_k(t))$. The estimation problem reduced to a linear regression problem with time-varying coefficients due to the amplitude modulation driven by the cuff deflation mode. We propose to track the time-varying coefficients by means of an adaptive Least Squares or a Kalman filter. An ordinary regression problem can be easily represented in matrix form, where the vector $y_N = [y(0), y(T_s), y(2T_s), \ldots, y((N-1)T_s)]^T$ with $T$ the transpose operator, $N$ the total number of measured samples and $T_s = \frac{1}{f_s}$ the sampling period. The $n$th row of the regression matrix $X$ is given by

$$x_n = \begin{bmatrix}
\sin(2\pi f_0 T_s n) & \cos(2\pi f_0 T_s n) & \ldots & \sin(2\pi f_0 (R+1) T_s n) & \cos(2\pi f_0 (R+1) T_s n) \\
\end{bmatrix}$$

The Weighted Least Squares (WLS) solution is given by the minimizer of

$$\arg \min_\theta \sum_{n=0}^{m-1} \lambda^{m-n-1} (y(n) - x_n \theta)^2$$

where $\lambda$ denotes the forgetting factor allowing to emphasize more to more recent samples and suppressing the information coming from past samples. The information of the past is exponentially decaying. A straightforward computation reveals the analytical solution of (5)

$$\hat{\theta}_{WLS}(m) = (X_m^T \Lambda_m X_m)^{-1} (X_m^T \Lambda_m y_m)$$

with $X_m = \begin{bmatrix} x_0 \\ \vdots \\ x_{m-1} \end{bmatrix}$ and $\Lambda_m = \begin{bmatrix} \lambda^{m-1} & 0 & \ldots & 0 \\ 0 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \lambda & 0 \\ 0 & \ldots & 0 & 1 \end{bmatrix}$. The estimator (6) can capture the time-varying features of the coefficients estimated in (4), it is computationally inefficient as it requires a matrix inversion for every computed estimator. Modifying the computation of (6) in a recursive manner, transforms the solution into a Kalman filter with gain $P_m^{-1} \mathbf{x}_{m}^T$ and residual $y(m) - x_m \hat{\theta}_{WLS}(m)$. The Kalman filter or recursive WLS estimator is given by

$$\hat{\theta}_{WLS}(m+1) = \hat{\theta}_{WLS}(m) + \frac{y(m) - x_m \hat{\theta}_{WLS}(m)}{\lambda + x_m P_m x_m^T} P_m x_m^T$$

$$P_m = \frac{1}{\lambda} \left( 1 - \frac{P_{m-1} x_{m-1}^T}{\lambda + x_{m-1} P_{m-1} x_{m-1}^T} \right) P_{m-1}$$

The proof of (7) is found in Appendix A. The choice of $\lambda$ is difficult to determine, we followed the rule given in [25]: $\lambda = 1 - \frac{f_s}{f_c}$ where $f_c$ is the sampling frequency of the oscillometric monitor. In Figure 3, we illustrate the Kalman filtering approach (7). The blue signal represents is a raw oscillometric signal as measured during the measurement campaign (see Section 2). The green signal is the modeled signal by means of the Kalman filtering approach and the forgetting factor chosen by the rule-of-thumb. To estimate the intermediate signal $p(t)$ (shown in magenta) which is stationary, we freeze the time-varying coefficients of (7) to the time instant $t_{MAP}$. This time instant $t_{MAP}$ is the time when the cuff pressure equals the Mean Arterial Pressure as determined by the blood pressure monitor (in this case PM-50). In the next section we show how the signal properties of the intermediate signal are used to calibrate the oscillometric device.
4. Calibrating the oscillometric signal

The philosophy of the calibration resides in the fact that an accurate estimation of the arterial resistance and compliance given by the Windkessel model allows correcting the oscillometric devices. Since the output signal of the Windkessel model is stationary, we retrieve the stationary signal \( p(t) \) from the time-varying model (4) estimated by the Kalman filter (7) by freezing the coefficients of (6) to the time instant \( t_{MAP} \). It is clear from (1) that the amplitudes and phases are a function of the Windkessel parameters \( R \) and \( C \). The oscillometric device provides systolic, diastolic and MAP readings which serve as initial estimates to classify the patient in one of the different categories (each category corresponds to a line) as defined in Figure 1. We want to update this initial category based on the features extracted from the oscillometric signal and compute the probability that this patient in fact belongs to a different category. The computation of the category probabilities is referred to as logistic regression and since the categories reveal an order from small to large we need to solve an ordinal logistic regression problem.

4.1. General introduction to ordinal logistic regression

Let \( Y_{bin} \) denote a nominal variable showing two possible categories \( \{0, 1\} \) such that \( Y_{bin} \) is binary. Assume that we have measured features \( F_1, F_2, \ldots, F_p \) which can be either continuous or discrete features. If one wants to model \( Y_{bin} \) as a linear regression of the measured features \( X_1, X_2, \ldots, X_p \) we need a link function \( G \) which maps the continuous linear combination to the binary variable \( Y_{bin} \)

\[
Y_{bin} = G \left( \sum_{k=1}^{p} \beta_k F_k + \beta_0 \right) + \epsilon_k
\]

where \( \epsilon_k \) denotes the zero-mean noise term. The expected value of this equation shows that the model predicts the probability \( p \) that \( Y_{bin} = 1 \) such that

\[
p = G \left( \sum_{k=1}^{p} \beta_k F_k + \beta_0 \right)
\]

Hence, the link function \( G \) maps the linear combination to the interval \([0, 1]\) such that \( G \) is a Cumulative Distribution Function (cdf). Standard choices are the probit link function which is the cdf of the standard normal distribution, whereas the more commonly used logit function is the cdf of the logistic distribution. As a result, the use of the logistic distribution provides the model

\[
p = \frac{1}{1 + \exp \left( - \sum_{k=1}^{p} \beta_k F_k - \beta_0 \right)}
\]

which is called binary logistic regression. Next, we need to generalize (8) to the case where \( Y_{bin} \) is a discrete random variable revealing different levels.
Let $Y_{\text{ord}}$ denote a variable showing different increasing values $\{0, 1, 2, 3, \ldots, M\}$. Since the link function $G$ is a cumulative distribution function, we do not want to predict $Y_{\text{ord}} = l$ but $Y_{\text{ord}} \leq l$ such that $Z_l = \{Y_{\text{ord}} \leq l\}$ is again a $\{0, 1\}$ variable since it is either true or false. Following the same philosophy as before we find that

$$Z_l = G \left( \sum_{k=1}^{p} \beta_k^{[l]} F_k + \beta_0^{[l]} \right) + \epsilon_k$$

and after taking the expected value we obtain

$$p_0 + p_1 + \ldots + p_l = G \left( \sum_{k=1}^{p} \beta_k^{[l]} F_k + \beta_0^{[l]} \right)$$

(9)

Equation (9) is not applied in practice as the model is overparametrized. Indeed, the difference between $Z_l$ and $Z_{l-1}$ is only one probability $p_l$ although $Z_l$ identifies $p + 1$ new parameters. As a result, the model's complexity of (9) is generally reduced to

$$p_0 + p_1 + \ldots + p_l = G \left( \sum_{k=1}^{p} \beta_k F_k + \beta_0^{[l]} \right)$$

(10)

where the parameters associated to the linear combination are the same regardless of the level $l$. The constants $\beta_0^{[l]}$ are called the ordinal thresholds. This type of ordinal logistic regression is referred to as the Walker and Duncan proportional odds ordinal logistic regression [26]. An overview article discussing the various approaches to deduct and perform an ordinal logistic regression analysis is found in [27]. Various software packages exist to estimate the parameters of (10). In matlab the “mnrfit” program is available while in SPSS the “plum” is available.

4.2. Calibration procedure

To calibrate an oscillometric device, we feed the database of oscillometric signals and their associated cuff inflation and deflation curves. The oscillometric devices can then compute the systolic, diastolic and MAP pressures given by $p_s^{[m]}$, $p_d^{[m]}$ and $p_{MAP}^{[m]}$. In Figure 1 we see the different blood pressure categories for the systolic and diastolic pressures. To compute the logistic regression, we select the following features per signal in the database corresponding to the signal characteristics of the estimated intermediate signal $p(t)$:

$$F_1 = B_0^s(t_{MAP}), F_2 = B_0^d(t_{MAP}), \ldots, F_{2d+1} = B_0^s(t_{MAP}), F_{2d+2} = B_0^d(t_{MAP}), F_{2d+3} = p_x^{[m]}$$

with $x = s, d$

The ordinal variable $Y_{\text{ord}}$ is a vector which specifies for each oscillometric signal in the database the systolic or diastolic blood pressure as measured by the Korotkoff technique. This allows estimating the logistic parameters of (10). For every oscillometric signal, we can predict the probability that $\{Y_{\text{ord}} \leq l\}$ which is computed as

$$\hat{p}_0 + \hat{p}_1 + \ldots + \hat{p}_l = G \left( \sum_{k=1}^{p} \beta_k F_k + \beta_0^{[l]} \right)$$

where the estimated logistic parameters $\hat{\beta}_k$ are used. From the predicted cumulative probabilities we can easily solve the probabilities $\hat{p}_k$ which predicts the probability that the oscillometric signal corresponds to a blood pressure in category $k$. Hence, we can assign new categories for every oscillometric signal corresponding to the category revealing the largest predicted probability.

If the category of the measured blood pressure $p_x^{[m]}, x = s, d$ remains the one with the largest predicted probability, then no calibration is required. However, if the predicted blood pressure category differs from the initial category as described by the measured blood pressure $p_x^{[m]}, x = s, d$ we need to calibrate and correct the measured blood pressure of the blood pressure monitor. Let the initial blood pressure category
Table 2: Statistics on the error between the Korotkoff readings and the output of the ABPM-06 [mmHg]

<table>
<thead>
<tr>
<th></th>
<th>Descriptive Statistics</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systolic Error</td>
<td>max 40.00 min 38.00</td>
<td>mean 3.36 std 12.38</td>
</tr>
<tr>
<td>Diastolic Error</td>
<td>max 19.00 min 22.00</td>
<td>mean 0.19 std 7.96</td>
</tr>
</tbody>
</table>

Table 2: Statistics on the error between the Korotkoff readings and the output of the ABPM-06 [mmHg]
determined by \( p^{[m]}_x, x = s, d \) be \( k^{[m]} \). We can define \( \delta^{[m]} \) the difference between the measured pressure \( p^{[m]}_x, x = s, d \) and the category’s mean. Let the predicted category by the logistic model be given by \( k^{[p]} \), then we can correct the blood pressure such that the difference between the calibrated pressure \( p^{[c]}_x, x = s, d \) and the mean pressure of the category \( k^{[p]} \) is \( \delta^{[m]} \).

5. Validation experiment

To test the procedure, we selected the oscillometric blood pressure monitor ABPM-06 of Contec. The device is a continuous-time oscillometric device taking up to 300 measurements during one day. The device has a systolic and diastolic range from 10 to 270 mmHg with a heart frequency range from 0.5 - 4 Hz. The blood pressure measurement resolution is 1 mmHg. The device’s accuracy is 6 mmHg and meets the ANSI/AAMI SP10-1992 standard. The oscillometric database as described in Section 2 is fed to the ABPM-06. The 75 signals are partitioned in a training set and a validation set. A cross-validation is performed where 1000 times the 75 signals are partitioned randomly in a training set of 60 oscillometric signals and a validation set of 15 signals. The logistic regression training is performed by the mnrt.m m-file in Matlab, the predicted probabilities per category is computed by the nnrval.m m-file in Matlab.

5.1. Results of the ABPM-06 to the database before calibration

We compute and study the difference between the Korotkoff readings associated to the oscillometric waveforms and the response of the ABPM-06 to the oscillometric waveform database. In Table 2 we see the descriptive statistics for the error of the systolic and diastolic blood pressures. The range of the errors is very large and the 95% confidence bounds reveal that the results show some heavy outliers. On top of that the Root Mean Square (RMS) error is 12.75 mmHg for the systolic and 7.91 mmHg for the diastolic. The error is significantly larger than the tabulated 6 mmHg as reported. Finally, the systolic error reveals a bias of 3.36 mmHg which should be corrected. As expected by medical literature, the diastolic errors are significantly lower than for the systolic case. Nevertheless, a large range of 41 mmHg is observed and the RMS error is larger than the expected 6 mmHg.

5.2. Results of the ABPM-06 after calibration

We report the results for both the training phase as well as the validation phase. The tables shows the average results over 100 validation runs. In Table 3 we see the results of the logistic regression model on the Training set of 60 oscillometric signal chosen randomly. The outliers have been significantly decreased by a factor 2 for the Systolic pressure. This is also reflected in the range which drops from 78 mmHg to 38 mmHg. The standard deviation and bias are reduced such that the RMS for the systolic pressure is approximately 6.02 mmHg and for the diastolic pressure only 4.71 mmHg. The specification of the error as reported by the ABPM-06 technical sheet is achieved.

In Table 4, we see the results of the validation phase performed on the remaining 15 oscillometric signals chosen randomly. Although, all statistics increase for the validation set, we still obtain significantly improved results with respect to the raw data in Table 2. On top of that, the increase of the error, bias and range from the Training phase to the Validation phase are, based on a proper statistical test, found to be insignificant.
Table 3: Statistics on the error between the Korotkoff readings and the calibrated output of the ABPM-06 during Training [mmHg]

<table>
<thead>
<tr>
<th>Training Phase</th>
<th>Descriptive Statistics</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>max</td>
<td>min</td>
</tr>
<tr>
<td>Systolic Error</td>
<td>20.00</td>
<td>-18.00</td>
</tr>
<tr>
<td>Diastolic Error</td>
<td>17.00</td>
<td>-15.00</td>
</tr>
</tbody>
</table>

Table 4: Statistics on the error between the Korotkoff readings and the calibrated output of the ABPM-06 during Validation [mmHg]

<table>
<thead>
<tr>
<th>Validation Phase</th>
<th>Descriptive Statistics</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>max</td>
<td>min</td>
</tr>
<tr>
<td>Systolic Error</td>
<td>22.00</td>
<td>-20.00</td>
</tr>
<tr>
<td>Diastolic Error</td>
<td>20.00</td>
<td>-23.00</td>
</tr>
</tbody>
</table>

Hence, the calibration procedure by means of the logistic regression approach successfully lowered the RMS error by almost a factor 2 regardless of the health condition of the patient.

To show the significance of the reduction between Table 2-3 and Table 2-4 we apply an F-test on the standard deviations of the errors. We do not perform a test on the mean since the reduction in bias or mean error is not significant. In Table 5, we analyze the significance of the reduction in standard deviation for the systolic pressure (row 1-2) and for the diastolic pressure (row 3-4). The test was performed at a confidence level of 95% with a null hypothesis $H_0: \sigma_i^2 = \sigma_j^2$ and alternative $H_1: \sigma_i^2 > \sigma_j^2$. The first two columns indicate which two standard deviations are being tested. The column 3 and 4 reveal the degrees of freedom for each variable which equals the number of observations used to compute the standard deviation minus 1. Column 5 computes the F-statistic which is given by the ratio of the 2 variances being tested. The critical value corresponding to a confidence level of 95% indicates that the alternative hypothesis $H_1$ holds for F-values exceeding this value. Finally, the last column shows the $p$-value which is the probability that a larger value for the F-statistic can be observed if the null-hypothesis holds. Table 5 reveals that the error reduction is significant when performing the calibration for the training set for both systolic and diastolic pressures. However, for the validation set we only observed a significant reduction for the systolic pressure but not for the diastolic pressure.

6. Conclusion

In this paper, we introduced a Hammerstein-Windkessel model to represent the central blood pressure system. It is a very simple representation capturing the low frequency dynamics of the cardiovascular system which was extended by an amplitude modulation representing the inflating and deflating cuff. A Kalman
filter estimated the time varying Fourier series representing the Windkessel model’s output signal. The Fourier coefficients of this output signal was then used in a next step to drive a logistic regression analysis. We showed that a logistic regression analysis based on the signal features of the oscillometric signal can detect hyper- and hypotension and correct the blood pressures accordingly.

Finally, we showed that the benchmark database of oscillometric test signals can be used together with the logistic regression analysis to calibrate the oscillometric device ABPM-06. We obtained an improvement in terms of error of a factor 2 due to calibrating the device. The calibration procedure can be performed unsupervised as it requires no skills or human interaction. As a result, the software, oscillometric test signals and calibration procedure can be potentially run from the USB-port of every common oscillometric device.

Appendix A. Proof of equation (7)

We compute \( \hat{\theta}_{WLS}(m+1) \) as a function of \( \hat{\theta}_{WLS}(m) \):

\[
\hat{\theta}_{WLS}(m+1) = \left( \begin{bmatrix} X_m^T & x_m^T \end{bmatrix} \begin{bmatrix} A_m \lambda & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_m \\ x_m \end{bmatrix} \right)^{-1}
\times \left( \begin{bmatrix} X_m^T & x_m^T \end{bmatrix} \begin{bmatrix} A_m \lambda & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_m \\ y(m) \end{bmatrix} \right)
\]

\[
= \left( \lambda X_m^T A_m X_m + x_m^T x_m \right)^{-1} \left( \lambda X_m^T A_m y_m + x_m^T y(m) \right)
\]

where \( \tilde{X}, \tilde{\Lambda} \) is the design matrix and weighting matrix respectively corresponding to \( m \) observations. Next, we can apply the Sherman-Morrison lemma which computes the inverse of a rank-one perturbation of a matrix (see [28]):

\[
(A + x^T x)^{-1} = A^{-1} - \frac{1}{1 + x A^{-1} x^T} A^{-1} x^T A^{-1} x
\]

(A.1)

Application of the Sherman-Morrison formula (A.1) and a straightforward simplification reveals,

\[
\hat{\theta}_{WLS}(m+1) = \hat{\theta}_{WLS}(m) + \frac{y(m) - x_m \hat{\theta}_{WLS}(m)}{\lambda + x_m (X_m^T A_m X_m)^{-1} x_m} \times (X_m^T A_m X_m)^{-1} x_m
\]

Let \( P_m = (X_m^T A_m X_m)^{-1} \) then we can compute the following recursive formula by applying (A.1) once more

\[
P_m = \left( \lambda X_{m-1}^T A_{m-1} X_{m-1} + x_{m-1}^T x_{m-1} \right)^{-1}
\]

\[
= \frac{1}{\lambda} \left( 1 - \frac{P_{m-1} x_{m-1}^T x_{m-1}}{\lambda + x_{m-1} P_{m-1} x_{m-1}} \right) P_{m-1}
\]

This establishes the result.

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References