Outage Probability Analysis of Asymmetrical Bi-directional Multi-Relay System with Network Coding

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Abstract: In this paper, we construct a practical asymmetrical bi-directional multi-relay system model with considering the effect of distance between source nodes and destination nodes. Then an outage probability expression of the system based on network coding is given. The theoretical analysis is proved to be correct by Monte Carlo simulations. Also we analyze the system average outage probability with different situations of power allocation coefficient, locations of the relay nodes, SNR and the number of relay nodes, indicating the inherent relationship between power allocation coefficient and total system power as well as the number of relay nodes. Simulation results indicate that the optimal power allocation coefficient of the system based on network coding varies with respect to the distance between source nodes and destination nodes. Meanwhile, simulation results show that the system outage probability performance improves with the number of relay nodes increasing.

Keywords: Asymmetrical Bi-directional Multi-relay System; Outage Probability; Network Coding; Physical-layer Network Coding; Power Allocation Coefficient

I. INTRODUCTION

Network coding (NC) has been shown its power for improving the network throughput of bi-directional relay system significantly, which was first proposed in [1]. Physical-layer network coding (PNC) proposed in [2] can markedly improve the wireless network throughput.

Outage probability is one of the most important performance measures for wireless communication systems. In [3], Yingda Chen et al. analyzed the outage probability of multiple access channel based on NC. The authors in [4] has analyzed analog network coding (ANC) protocol in terms of single-user outage probability instead of considering the overall system outage. The system outage performance of practical PNC schemes for two-way relay channels is given in [5]. [6] studied the capability of bi-directional relay system based on DF, and the relevant arrival rate region was given. The authors in [7] investigated the strategies that can maximize the overall two-way rate for several 2 and 3 step PNC schemes. Despite the abundance of literature on one-way relay with multiple relay nodes [8], research on the two-way multi-relay channel remains limited to date.

In practical wireless communication systems, the distance between two nodes is a very important factor when we analyze the outage probability performance. In [9], the authors evaluated the outage probability performance of ANC protocol for a two-way half-duplex relaying system with asymmetric traffic requirements at the end terminals. The authors in [10] studied the outage probability performance of cooperative communication system based on asymmetrical channels. An outage probability expression of cooperative communication system which considers the location of relay nodes is given in [11].

With the above motivation, we analyze the outage probability of asymmetrical bi-directional multi-relay system with considering the effect of distance between source nodes and relay nodes with NC. In this paper, an outage probability expression of the system with NC is given and theoretical analysis is proved correct by Monte Carlo simulations. Numerical and simulation results reveal that the optimal power allocation coefficient of asymmetrical bi-directional multi-relay system with NC varies with respect to the distance between source nodes and relay nodes. Meanwhile, we show that the system outage probability performance improves with the number of relay nodes increasing.

The remainder of the paper is organized as follows. In Section II, a practical model of asymmetrical bi-directional multi-relay system is introduced. We then analyze the outage probability of the system based on NC in Section III. Simulation results are given in Section IV, and the conclusion is made in Section V.

II. SYSTEM MODEL

We consider an asymmetrical bi-directional multi-relay systems as shown in Fig.1., where two source nodes $S_1$ and $S_2$ exchange information with the help of a set of $N$ relay nodes. All the relay nodes are denoted by $r_1, r_2, \ldots, r_N$, and
In the second phase, we analyze the probability of the third phase based on the probability of set $R^{NC}$. Secondly, we analyze the probability of the third phase based on the probability of set $R^{NC}$.

A. NC relay nodes set $R^{NC}$

According to the definition of outage probability and [5], $R^{NC1}$ and $R^{NC2}$ can be expressed as

$$R^{NC1} = \left\{ r_{k}^{NC} \in R^{NC1} \mid \frac{1}{3} \log_2(1 + \frac{\|h_{S_{1}r_{k}^{NC}}\|^2}{d_{S_{1}r_{k}^{NC}}N_0}) > R \right\}$$

and

$$R^{NC2} = \left\{ r_{k}^{NC} \in R^{NC2} \mid \frac{1}{3} \log_2(1 + \frac{\|h_{S_{2}r_{k}^{NC}}\|^2}{d_{S_{2}r_{k}^{NC}}N_0}) > R \right\}$$

From (1) and (2), then we can define $R^{NC}$ as

$$R^{NC} = \left\{ r_{k}^{NC} \in R^{NC} \mid \frac{1}{3} \log_2(1 + \frac{\|h_{S_{1}r_{k}^{NC}}\|^2}{d_{S_{1}r_{k}^{NC}}N_0}) > R, \frac{1}{3} \log_2(1 + \frac{\|h_{S_{2}r_{k}^{NC}}\|^2}{d_{S_{2}r_{k}^{NC}}N_0}) > R \right\}$$

where $P_1$ and $P_2$ denote the power of $S_1$ and $S_2$, respectively. $d_{S_{1}r_{k}^{NC}}$ denotes the distance between $S_1$, $S_2$, and relay nodes, respectively. From (3), we can get the probability of discretionary relay $r_{k}^{NC} \in R^{NC}$ as

$$P_{NC}(r_{k}^{NC}) = P_{NC}(\frac{1}{3} \log_2(1 + \frac{\|h_{S_{1}r_{k}^{NC}}\|^2}{d_{S_{1}r_{k}^{NC}}N_0}) > R)P_{NC}(\frac{1}{3} \log_2(1 + \frac{\|h_{S_{2}r_{k}^{NC}}\|^2}{d_{S_{2}r_{k}^{NC}}N_0}) > R)$$

$$= \exp\left(-\frac{d_{S_{1}r_{k}^{NC}}^2(2^R-1)}{P_1N_0}\right)\exp\left(-\frac{d_{S_{2}r_{k}^{NC}}^2(2^R-1)}{P_2N_0}\right)$$

(4)

Let $P_1 = P_2 = \frac{\alpha P}{2}$, $\frac{P}{N_0} = SNR$, (4) can be transformed as

$$P_{NC}(r_{k}^{NC}) = \exp\left(-\frac{2(\frac{\alpha P}{2} d_{S_{1}r_{k}^{NC}}^2 + \frac{\alpha P}{2} d_{S_{2}r_{k}^{NC}}^2)(2^R-1)}{\alpha SNR}\right)$$

(5)

Let $d_{S_{1}r_{k}^{NC}} = d_{S_{2}r_{k}^{NC}} = d_{S_{i}r_{k}^{NC}}, b_{NC} = (\frac{2^R-1}{\alpha SNR})$, then (5) can be expressed as

$$P_{NC}(r_{k}^{NC}) = e^{-2\lambda^{d_{S_{i}r_{k}^{NC}}}}$$

(6)

Whether the relay nodes can decode the received signals rightly is mutual independent, so the probability of $R^{NC}$ is given by

$$P_{NC}(R^{NC}) = \prod_{r_{k}^{NC} \in R^{NC}} P_{NC}(r_{k}^{NC})\prod_{r_{k}^{NC} \not\in R^{NC}} P_{NC}(r_{k}^{NC})$$

(7)

B. The conditional outage probability based on $R^{NC}$

During the third phase, all the relays in $R^{NC}$ broadcast their packets to source nodes. In this paper, we assume that the system is interruptive when one of $S_1$ and $S_2$ cannot decode rightly, so we only analyze the conditional outage probability
of $S_i$. The system is sure to be interruptive when $|R^{NC}| = 0$, so we only analyze the condition that $|R^{NC}| > 0$.

Maximal ratio combining (MRC) is considered at source $S_1$ and $S_2$, then the mutual information at $S_i$ can be expressed as

$$I = \frac{1}{3} \log_2 \left( 1 + \sum_{k=1}^{M} \frac{|h_{k,S_i}|^2 P_{k,S_i}}{d_{k,S_i}^\alpha N_0} \right)$$

The conditional outage probability based on $R^{NC}$ is calculated as follow

$$P_{outage}^{NC}(I < R | R^{NC}) = P \left( \frac{1}{3} \log_2 \left( 1 + \sum_{k=1}^{M} \frac{|h_{k,S_i}|^2 P_{k,S_i}}{d_{k,S_i}^\alpha N_0} \right) < R \right)$$

Let $P_{k,S_i} = (P-P_k-P_i) = (1-\alpha)P$, $\gamma_{S_i,S_k} = \frac{|h_{k,S_i}|^2}{d_{k,S_i}^\alpha}$, then

$$\gamma^{NC} = \frac{(2^{\gamma-1} - 1) |R^{NC}|}{(1-\alpha)SNR}$$

$$\gamma^{NC} = \sum_{k=1}^{M} \gamma_{S_i,S_k}$$

(9) can be transformed as

$$P_{outage}^{NC}(I < R | R^{NC}) = P \left( \sum_{k=1}^{M} \gamma_{S_i,S_k} < \gamma^{NC} \right) = \int_0^{\gamma^{NC}} f_{\gamma}(\gamma^{NC}) d\gamma^{NC}$$

(10)

Let $\lambda_{k}$ denote the expectation of $\gamma_{S_i,S_k}$, and we assume that $\phi_{\gamma}(s)$ and $\phi_{\gamma_{S_i,S_k}}(s)$ denote the Characteristic Function (CF) of $\gamma^{NC}$ and $\gamma_{S_i,S_k}$, respectively. According to the property of CF, we can get

$$\phi_{\gamma_{S_i,S_k}}(s) = \prod_{k=1}^{M} \phi_{\gamma_k}(s) = \prod_{k=1}^{M} \frac{1}{1-s/\lambda_{k}}$$

(11)

We assume that $K_1$ expectations are equal to $\lambda_{k_1}$, $K_2$ expectations are equal to $\lambda_{k_2}$, $K_m$ expectations are equal to $\lambda_{k_m}$ and the rest of $M$ expectations are different of each other, i.e., $\lambda_{k_1}, \lambda_{k_2}, ..., \lambda_{k_m}, \lambda_{k_{m+1}}, ..., \lambda_{k_M}$ are different from each other and $K_1 + K_2 + ... + K_m + M = |R^{NC}|$, so (11) can be transformed and expanded:

$$\phi_{\gamma_{S_i,S_k}}(s) = \left( 1 - \frac{1}{1-s/\lambda_{k_1}} \right)^{K_1} \left( 1 - \frac{1}{1-s/\lambda_{k_2}} \right)^{K_2}$$

$$\cdots \left( 1 - \frac{1}{1-s/\lambda_{k_m}} \right)^{K_m} \prod_{k=1}^{M} \left( \frac{1}{1-s/\lambda_{k}} \right)$$

$$= \sum_{k=1}^{K_1} \frac{A_k}{(1-s/\lambda_{k_1})^k} + \sum_{k=1}^{K_2} \frac{B_k}{(1-s/\lambda_{k_2})^k}$$

$$\cdots + \sum_{k=1}^{K_m} \frac{M_k}{(1-s/\lambda_{k_m})^k} + \sum_{k=1}^{M} \frac{E_k}{1-s/\lambda_{k}}$$

(12)

where $A_k, B_k, ..., E_k$ can be got by dynamic residues method, then the probability disturbed function of $\gamma^{NC}$ can be expressed as follow

$$f_{\gamma^{NC}}(\gamma^{NC}) = \sum_{k=1}^{N_c} \frac{E_k}{\lambda_{k}} e^{-\frac{\lambda_{k}}{\gamma^{NC}}} + \sum_{k=1}^{K_1} \frac{A_k}{\lambda_{k_1}} e^{-\frac{\lambda_{k_1}}{\gamma^{NC}}}$$

$$\cdots + \sum_{k=1}^{K_m} \frac{M_k}{\lambda_{k_m}} e^{-\frac{\lambda_{k_m}}{\gamma^{NC}}}$$

$$U(\gamma^{NC})$$

where $\Gamma(k)$ and $U(\gamma^{NC})$ denote gamma function and unit step function, respectively.

By Substituting (13) in (10), we can get the conditional outage probability expression as

$$P_{outage}^{NC}(I < R | R^{NC}) = \sum_{k=1}^{K_1} \frac{A_k}{\lambda_{k_1}} e^{-\frac{\lambda_{k_1}}{\gamma^{NC}}} + \sum_{k=1}^{K_2} \frac{B_k}{\lambda_{k_2}} e^{-\frac{\lambda_{k_2}}{\gamma^{NC}}}$$

$$\cdots + \sum_{k=1}^{K_m} \frac{M_k}{\lambda_{k_m}} e^{-\frac{\lambda_{k_m}}{\gamma^{NC}}}$$

(14)

C. The average outage probability

For a bi-directional multi-relay system with $N$ relay nodes, there are 2$^N$ conditions for $R^{NC}$, and all of $R^{NC}$ are considered, the average outage probability of our system can be expressed as

$$P_{outage}^{NC}(I < R) = \sum_{k=1}^{K_1} \frac{A_k}{\lambda_{k_1}} e^{-\frac{\lambda_{k_1}}{\gamma^{NC}}} + \sum_{k=1}^{K_2} \frac{B_k}{\lambda_{k_2}} e^{-\frac{\lambda_{k_2}}{\gamma^{NC}}}$$

$$\cdots + \sum_{k=1}^{K_m} \frac{M_k}{\lambda_{k_m}} e^{-\frac{\lambda_{k_m}}{\gamma^{NC}}}$$

(15)

By Substituting (7) and (14) in (15), we can get the average outage probability of the system.

IV. SIMULATION RESULTS

In this section, we examine the system outage probability performance through numerical investigations and simulations. Let $S_i$ and $S_j$ be located at $(0, 0)$ and $(1, 0)$, respectively. $r_i$ is distributed equally in the rectangular region, and the four vertexes of the region are $(0, 0.5)$, $(0.2, -0.5)$, $(0.8, 0.5)$ and $(0.8, -0.5)$. We consider $R = 1$ bps/Hz, $\beta = 3$, $h_{k_1, i} \sim CN(0,1)$ is the channel coefficient between source $S_i$ and $r_k$, $r_k \in (0, 1)$ and $SNR = P/N_0$ ($P$ denotes the total power, $N_0$ denotes the noise power).

![Fig2. The range of relay nodes in reference frame](image-url)
Table 1. The different locations of the 8 relay nodes.

<table>
<thead>
<tr>
<th></th>
<th>r_1</th>
<th>r_2</th>
<th>r_3</th>
<th>r_4</th>
<th>r_5</th>
<th>r_6</th>
<th>r_7</th>
<th>r_8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.23, 0.25)</td>
<td>(0.4, -0.3)</td>
<td>(0.25, 0.39)</td>
<td>(0.35, -0.42)</td>
<td>(0.31, 0.45)</td>
<td>(0.2, 0.3)</td>
<td>(0.22, 0.39)</td>
<td>(0.21, 0.28)</td>
</tr>
</tbody>
</table>

Table 2. Three cases of 5 different locations of relay nodes.

<table>
<thead>
<tr>
<th>Case</th>
<th>r_1</th>
<th>r_2</th>
<th>r_3</th>
<th>r_4</th>
<th>r_5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>(0.23, 0.25)</td>
<td>(0.4, -0.3)</td>
<td>(0.25, 0.39)</td>
<td>(0.35, -0.42)</td>
<td>(0.31, 0.45)</td>
</tr>
<tr>
<td>Case 2</td>
<td>(0.53, 0.4)</td>
<td>(0.7, -0.3)</td>
<td>(0.55, 0.49)</td>
<td>(0.65, 0.41)</td>
<td>(0.25, 0.45)</td>
</tr>
<tr>
<td>Case 3</td>
<td>(0.5, 0.2)</td>
<td>(0.53, -0.15)</td>
<td>(0.25, 0.5)</td>
<td>(0.48, 0.41)</td>
<td>(0.45, 0.45)</td>
</tr>
</tbody>
</table>

Fig. 3. Comparison of theoretical and simulated results with different power allocation coefficient based on NC.

Fig. 4. Comparison of theoretical and simulated results with different relay number based on NC.

Fig. 5. Comparison of the system outage probability at three cases of five different locations of relays.

Fig. 6. Relative curves of outage probability and power allocation coefficient with N and SNR at the three cases based on NC.
outage probability has marked difference when the locations which are given in Table2. It can be seen that the system probability at three cases of five different locations of relays see the system outage probability is decreasing with N and compared to Monte Carlo simulations. From Fig.4, we can see that the system outage probability is markedly different when the power allocation coefficient is changed.

Fig.4. shows the theoretical and simulated results with different relay number based on NC when $\alpha = 0.8$, and there are $4(r_1 - r_2)$ and $6(r_1 - r_2)$ relay nodes which are chosen from Table1 in the system. Theoretical results are proved correct by Monte Carlo simulation in Fig.3. From Fig.3, we can see that the system outage probability is decreasing with N and SNR increasing.

Fig.5. shows the comparison of the system outage probability at three cases of five different locations of relays which are given in Table2. It can be seen that the system outage probability has marked difference when the locations of relays are changed, so the locations of the relays must be considered when we analyze the outage probability of any wireless communication system.

Fig.6. shows the relation of outage probability and power allocation coefficient with the same N and SNR at the three cases based on NC. From Fig.7, we can see that the optimal power allocation coefficient is relative with the locations of relay nodes, such as the optimal power allocation coefficient of Case1 and Case3 is 0.6, and Case2 is in $(0.7 \sim 0.8)$.

Fig.7. shows the relative of outage probability and power allocation coefficient with the same locations of relays based on NC, and the locations of relays are given in Table1. Fig.7. tells us that the optimal power allocation coefficient is fixed when the locations of relay nodes are fixed, and the optimal power allocation coefficient is fixed although SNR is changed. In this case, the optimal power allocation coefficient is in $(0.7 \sim 0.8)$.

V. CONCLUSION

In this paper, we analyze the outage probability of asymmetrical bi-directional multi-relay system based on NC, and we give an accurate outage probability expression which is proved correct by Monte Carlo simulations. In the analysis process, we consider the influence of distance between source nodes and relay nodes, simulation results indicate that the distance is a very important factor when we analyze the outage probability of asymmetrical bi-directional multi-relay system. Simulation results tell us that the optimal power allocation coefficient of asymmetrical bi-directional multi-relay system based on NC is relative with the distance between source nodes and relay nodes, and we know that the system outage probability performance improves with the number of relay nodes increasing.

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