Asymmetric Network Coding for Two-Way OFDM Relay System with Power Allocation

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Abstract—Asymmetric data transmission commonly exists in two-way orthogonal frequency division multiplexing (OFDM) relay system. In this paper, we propose a novel asymmetric network coding scheme for two-way OFDM relay system with power allocation to improve the throughput and bit error rate (BER) performance. In the multiple access channel (MAC) phase, the relay receives asymmetric data from two terminals, respectively. The Max-Min power allocation is used to improve the system BER performance during the MAC phase. In the broadcast (BC) phase, a novel asymmetric network coding scheme based on the thought of adaptive modulation (AM) is proposed. The stronger link can reliably transmit more data by using higher-order modulation. The weaker link can adaptively reduce the modulation order with the help of bit-inserting zero approach and well-designed signal constellation and bit labeling, hence, improve the transmission reliability. Simulation results validate that the proposed scheme outperforms the symbol-inserting zeros scheme in BER and throughput.

Keywords—asymmetric data transmission; network coding; two-way OFDM relay system; power allocation

I. INTRODUCTION

Recently, the two-way relaying has obtained research interests [1], wherein the terminals A and B need to exchange data by two-hop connections at the relay node R. A number of two-way relay protocols have been proposed known as amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) and so on.

Several Network coding (NC) schemes [2-6] were proposed to improve the throughput performance of two way relay system. In order to enhance the transmission reliability, optimal power allocation for two way relay system employing analog network coding is proposed in [7]. In the two-way relay system based on NC, both terminals decode the combined data bits from the same transmit symbols in the broadcast (BC) phase. In most cases, the channel qualities to the two terminals are asymmetric and the data rates from the relay are limited by the weaker link. [8] shows that it is possible for the relay to transmit data rates equal to the individual link capacities simultaneously to the two receivers by using random coding according to information theory.

Adaptive modulation (AM) can take full use of channel information, adjust sending parameter in time and optimize the overall performance of system, thus it has also widely used in modern wireless communication systems. We propose a novel asymmetric network coding scheme based on the thought of AM for two-way DF OFDM relaying system. According to their individual link qualities, the relay can adaptively select the modulation order of the two links to meet both bit error rate (BER) constraints in the BC phase. The stronger link can reliably transmit more data by using higher-order modulation. The weaker link can adaptively reduce the modulation order with the help of bit-inserting zero approach, well-designed signal constellation and bit labeling, hence, improve the transmission reliability. At the same time, we can also see that the weaker link terminals have to transmit more available data in the MAC phase, and the system throughput performance is limited by the weaker link. In this paper, the Max-Min power allocation approach is proposed to improve the BER performance. The proposed asymmetric network coding two-way OFDM relaying scheme outperforms traditional scheme [9], which was called symbol-inserting zero, in BER and throughput performance.

The rest of the paper is organized as follows: system model using the NC protocol is briefly described and Max-Min power allocation is proposed in Section II. In Section III, the proposed transmission scheme are discussed. Numerical results and conclusion are presented in Section IV and Section V, respectively.

II. SYSTEM MODEL AND POWER ALLOCATION

We consider terminal A and B exchange data via a half-duplex relay R, as shown in Fig. 1. It is assumed that there is no direct path between the two terminals. In the MAC phase, the channel in frequency domain from A and B to R are denoted by $G_A$ and $G_B$, respectively. In the BC phase, the channel in frequency domain from R to A and B are denoted by $H_A$ and $H_B$, respectively. The bit sequences that A and B want to send to each other are $\{b_A(k)\}$ and $\{b_B(k)\}$, respectively.

In the MAC phase, terminal A and B transmit their signal to the relay R during their own time slot, respectively. In other words, there are two time slots in the MAC phase. In the first slot, the bit sequence $\{b_A(k)\}$ is firstly mapped to data symbols $s_A(k)$ and N data symbols $s_A(k)$ consist of a data block $S_A = [s_A(1), s_A(2), \ldots, s_A(N)]^T$ in terminal A. Afterwards, the IFFT of the data block yields the time domain
sequence $s_A = [s_A(1), s_A(2), \ldots, s_A(N)]^T$ and cyclic prefix (CP) larger than the maximum path delay of the channel between terminal A to the relay R is appended. Finally, the OFDM signals are sent to channel with transmission power $P_A$.

At the relay R, the CP will be removed from the received signals. Thus, the ISI is eliminated. After passing through an N-point DFT, the receive signal $y_{AR}(n)$ can be expressed as

$$y_{AR}(n) = \sqrt{P_A} s_A(n) + w_{R}(n)$$

where $L$ is the number of multi-paths, $s_A(n,l)$ is the complex gain of the $l$th path at time $n$, and $w_{R}(n)$ is the complex additive white Gaussian noise (AWGN) at time $n$. Then the received signal $y_{AR}(n)$ is transformed into frequency domain using FFT and

$$Y_{AR} = \sqrt{P_A} S_A \odot G_A + W_R$$

where $\odot$ denotes the Hadamard product, i.e., component-wise product. $Y_{AR}$ and $W_R$ are received signal and AWGN in frequency domain. Then the relay R demodulates and decodes the received signal $Y_{AR}$ into $\hat{b}_A(k)$. In the second slot, the same as the first slot, the relay R decodes the received signals into $\{\hat{b}_A(k)\}$.

In our scheme, the stronger link (assuming terminal A) will transmit more data than the weaker link (namely, terminal B) in the BC phase. It brings about a problem that terminal B must transmit more data to the relay in the MAC phase. Hence, we must allocate more resources and use lower modulation order to terminal B to ensure the reliable reception of the weaker link’s signal. In this paper, the goal of power allocation is to minimize BER. To minimize the average sum BER, the two terminals is dominated by the worse one, a suboptimal power allocation is proposed to minimize the higher BER between the two terminals. We call this scheme as Max-Min power allocation. The formulation of this optimal problem is represented as:

$$\min_{P_A, P_B} \max(p_A(A | G_A, H_B), p_B(B | G_B, H_A))$$

subject to $P_A + P_B \leq P_{\text{total}}$

where $p_b$ denotes BER. The formulation can be expressed in detail as:

$$p_b(A | G_A, H_B) = 1 - [1 - p_b(A | G_A)][1 - p_{\text{BICM-ID}}(m)]$$

$$p_b(B | G_B, H_A) = 1 - [1 - p_b(B | G_B)][1 - p_{\text{BICM-ID}}(n)]$$

$$p_{\text{BICM-ID}}(m) = \frac{M - 1}{M} \sin^2 \left( \frac{\pi}{M} \right)$$

where $M$ denotes modulation order, $m = \log_2 M$, $P_b$ denotes the SNR per bit, $A(w_1, \cdots, w_m)$ is the enumerating function of the information weight corresponding to error events with coded output weight $(w_1, \cdots, w_m)$, $d_i$ is the Euclidean distance of bit $i$.

In the BC phase, relay R combines the decoded bit sequences $\{\hat{b}_A(k)\}$ and $\{\hat{b}_B(k)\}$ by using the bit level XOR operation. Then the combined bit sequence $\{\hat{b}_R(k)\}$ is remodulated to OFDM symbol $S_R = [S_R(1), S_R(2), \cdots, S_R(N)]^T$, i.e.,

$$\{\hat{b}_A \oplus \hat{b}_B\} = \{\hat{b}_R\} \mapsto S_R$$

where $\oplus$ denotes XOR operation. The received signal at A and B in frequency domain can be expressed as

$$Y_A = H_A S_B + W_A$$

$$Y_B = H_B S_R + W_B$$

where $W_A \sim \mathcal{CN}(0, \sigma_A^2 I_N)$ and $W_B \sim \mathcal{CN}(0, \sigma_B^2 I_N)$ are the AWGN at terminal A and B, respectively. The two terminals demodulate the received signals and recover the unknown data bits by XOR-ing the decoded data $\{\hat{b}_A\}$ with their own data on the bit level, i.e.,

$$\{\hat{b}_B\} = \{\hat{b}_R \oplus \hat{b}_A\} \quad \text{at terminal A}$$

$$\{\hat{b}_A\} = \{\hat{b}_R \oplus \hat{b}_B\} \quad \text{at terminal B}$$

Considering both terminals can decode the data transmitted from the relay R. The relay has to transmit data in a rate supported by both terminals. This results in the consequence that stronger link can't be made full use of and should be avoided in practice. In the next section, the scheme we propose is to achieve asymmetric data rate from the relay to the two terminals according to their own link qualities.
III. ASYMMETRIC DATA RATE TRANSMISSION IN DETAILS

A. Transmission Strategy at the Relay

Our purpose is to design a scheme that utilize the stronger link to transmit more data, at the same time the weaker link transmits at a data rate that can guarantee BER of the weaker link according to AM.

Bit-interleaved coded modulation (BICM) introduces a bit interleaver instead of a symbol interleaver at the output of channel encoder and before the modulator, thus provides improved performance over fading channels.

We propose a novel asymmetric network coding scheme named as bit-inserting zero instead of traditional symbol called symbol-inserting zero [9] of the relay to combine and forward the two received sequences with unequal length. The proposed asymmetric network coding is based on the thought of AM and outperforms traditional symbol-inserting zero scheme on the BER performance with the help of well-designed signal constellation and bit labeling. The proposed scheme can change the modulation order adaptively according to channel quality.

As is shown in Fig. 2, the information bit sequence \( \{b_A\} \) is shorter than \( \{b_B\} \), the relay must insert zeros into the sequence \( \{b_A\} \) to guarantee that both sequences have the same length in order to achieve XOR-ing NC. There are two schemes we can choose. At first, we can distribute the information bits to the first bit of each symbol in turn. Then the second bit and so forth like in (a). We can also distribute the information bits to each bit of the first symbol, then the second and so on like in (b). Our idea is that we can exploit constellation design to turn 8PSK symbol inserted dummy zeros into QPSK in (a). How to design the constellation to improve the BER performance will be discussed in section B.

Fig.2. Comparison between bit-inserting zero and symbol-inserting zero (8PSK as an example)

Now we focus on the transmitter structure at Relay R. After decoded in the MAC phase, the information bit sequences \( \{b_A\} \) and \( \{b_B\} \) are encoded individually by a encoder with coding rate \( r \). We assume the bit sequences are encoded by the same encoder at the relay. The relay sends \( m \) bits to terminal A and \( n \) bits to terminal B, where \( n < m \). \( n \) and \( m \) can change over time according to their individual channel quality. Without loss of generality, we consider \( n \) and \( m \) are constant at the relay for ease of implementation.

The transmitter structure is depicted in detail: after encoded by a encoder simultaneously, the information bit sequences \( \{b_A\} \) and \( \{b_B\} \) are bitwise interleaved to form the code sequence \( \{c_A\} \) and \( \{c_B\} \), where \( c_A, c_B \in \{0, 1\} \). Then the code sequences are partitioned every \( n \) and \( m \) bits, respectively. \( c_A = [c_A^n, ..., c_A^1] \) and \( c_B = [c_B^m, ..., c_B^1] \) are each pair of corresponding bit group. For each \( c_A \), we insert \((m-n)\) zeros for bit level XOR operation, i.e.,

\[ \tilde{c}_A = [0, ..., 0, c_A^n, ..., c_A^1]^{T} \]

After inserting zeros, network coding with XOR operation is applied to combine \( \tilde{c}_A \) and \( c_B \) into \( c_R = [c_R^m, ..., c_R^1]^{T} \), i.e.,

\[ c_R = c_B \oplus \tilde{c}_A = [c_B^n, ..., c_B^1, c_B^n, c_B^n, ..., c_B^n, c_B^n, c_B^n, c_B^n, c_B^n, c_B^n, ..., c_B^n, c_B^n, c_B^n, c_B^n, c_B^n]^{T} \]

From the formula, we can see that \([c_B^n, ..., c_B^n] \) are kept unchanged after the XOR operation, and those are known to terminal B (weaker link).

Each combined bit group \( c_R \) is sent to modulator to transfer to complex symbol \( s_R = \mu(c_R) \), where \( \mu() \) denotes the mapping function. Each modulated symbol \( s_R \) is in the M-ary PSK or QAM symbol alphabets \( \mathcal{A} = \{a_1, ..., a_M\} \), where \( M = 2^m \). We assume the encoding scheme and the mapping...
scheme at the relay are open to terminal A and B. Then $s_R$ is sent to fading channels.

B. Iterative Decoding Strategies at the Receivers

After receiving $Y_A$ and $Y_B$, terminal A and B demodulate the received signals, and reveal the unknown data according to the bits contained in $s_R$ and their own data bits. At the receivers, iterative decoding algorithm is exploited to improve the performance. Specific structures are shown in Fig.3.

We know the bits in $c_A$ have different value for A and B. For terminal A, every bit in $c_A^i, \forall i \in \{1, \ldots, m\}$ is useful and must be decoded, whereas for terminal B, the useful bits are $c_B^i, \forall i \in \{1, \ldots, n\} (n < m)$. The $(m-n)$ bits are a priori known to its receiver. Therefore, terminal B doesn’t need to decode every bit in $c_B$. Terminal B can only demap on the subset of the transmit signal constellation with $[c_B^m, \ldots, c_B^{n+1}]$ contained at the corresponding positions.

Take 8PSK ($m = 3$) constellation for example, we assume $n = 2$ and $c_B^3 = 0$. Terminal B knows $c_B^3 = 0$ and it only needs to consider the symbols whose third bit is 0 when demodulating. That is to say, the equivalent symbol alphabets are $S(c_B^m, \ldots, c_B^{n+1}) \subset A_i$, i.e.,

$$S(c_B^m, \ldots, c_B^{n+1}) = \{ s | c_m^i(s) = c_B^m, \ldots, c_B^{n+1}(s) = c_B^{n+1}, s \in A_i \}$$

where $c_j^i(s)$ is the $j$th bit associated with the label of symbol $s$. Assuming $c_B^3 = 0$, the equivalent symbol alphabets to be demapped at terminal B can be denoted as $S(0)$. Different labeling schemes result in different subsets for the given a priori bits which influence the decoding performance at the receivers. Fig.4. shows two labeling schemes: the Gray labeling and the set partitioning (SP) labeling. For Gray labeling, the minimum Euclidean distance (MED) between symbols in $S(0)$ is the same as MED in 8PSK. On the contrary, for SP labeling, the components in $S(0)$ become QPSK, which brings about MED between symbols in $S(0)$ increasing and needs lower SNR to decode compared to that of 8PSK under the same BER.

Generally speaking, modulation is decided by channel information. When modulation switching, the symbol constellation will change to maximum the MED between adjacent signal phase. We can call this AM on OFDM for NC.

In the following, we will discuss iterative decoding at the receivers in detail. At terminal A, the a posteriori Log-

Likelihood Ratio (LLR) for each of the coded bits $c_R^i, i \in \{1, \ldots, m\}$ can be calculated as

$$L(c_R^i) = \ln \frac{p(c_R^i = 1 | y_A)}{p(c_R^i = 0 | y_A)}$$

Assuming each of the constellations has an equal possibility to choose, i.e., a priori possibility $p(c_R^i = 1) = p(c_R^i = 0)$. We can derive

$$L_A(c_R^i) = \ln \frac{p(y_A | c_R^i = 1)p(c_R^i = 1)}{p(y_A | c_R^i = 0)p(c_R^i = 0)} = \ln \frac{p(y_A | c_R^i = 1)}{p(y_A | c_R^i = 0)}$$

$$= \frac{\sum s_i \in A_i \exp(-\frac{1}{2\sigma_A^2}(y_A - H_s^i s_R^i)^2)}{\sum s_i \in A_i^0 \exp(-\frac{1}{2\sigma_A^2}(y_A - H_s^i s_R^i)^2)}$$

where $A_i^1$ and $A_i^0$ represent the sets of transmit symbols whose $i$th bit labeling is 1 and 0, respectively. To lower computation, we use Log-Max approximately to simplify the formula (11),

$$L_A(c_R^i) = \ln \frac{p(c_R^i = 1 | y_A)}{p(c_R^i = 0 | y_A)}$$

$$\approx \max_{s_i \in A^1_i} \left( \frac{|y_A - H_s^i s_R^i|^2}{2\sigma_A^2} \right) - \max_{s_i \in A^0_i} \left( \frac{|y_A - H_s^i s_R^i|^2}{2\sigma_A^2} \right)$$

Similarly, the a posteriori LLR value $L_B(c_B^i)$ for the coded bits $c_B^i, i \in \{1, \ldots, n\}$ can be calculated as follows:

$$L_B(c_B^i) = \ln \frac{p(c_B^i = 1 | y_B)}{p(c_B^i = 0 | y_B)}$$

$$\approx \max_{s_i \in S^1} \left( \frac{|y_B - H_s^i s_R^i|^2}{2\sigma_B^2} \right) - \max_{s_i \in S^0} \left( \frac{|y_B - H_s^i s_R^i|^2}{2\sigma_B^2} \right)$$

where $S^1$ and $S^0$ represent the sets of transmit symbols whose $i$th bit labeling is 1 and 0 in the constellation subset $S(c_B^m, \ldots, c_B^{n+1})$, respectively.

For iterative decoding, a priori possibility $p(c_R^i = 1)$ and $p(c_R^i = 0)$ should not be seem as equal possibility. The a priori LLR values from the feedback of the channel are subtracted from $L_A(c_R^i)$ and $L_B(c_R^i)$ to generate the extrinsic LLRs $\lambda_A(c_R^i)$ and $\lambda_B(c_R^i)$

$$\lambda_A(c_R^i) = \max_{s_i \in A^1_i} \left( \frac{|y_A - H_s^i s_R^i|^2}{2\sigma_A^2} \right) + \sum_{j=1, j \neq i}^m \lambda_A(c_R^j) c_R^j$$

$$- \max_{s_i \in A^0_i} \left( \frac{|y_A - H_s^i s_R^i|^2}{2\sigma_A^2} \right) + \sum_{j=1, j \neq i}^m \lambda_A(c_R^j) c_R^j$$

(14)
\[ \lambda_B (c_R^i) \approx \max_{s_R \in S_R} \left( \frac{|y_B - H_B s_R|^2}{2\sigma_B^2} + \sum_{j=1,j\neq i}^n \Lambda_B (c_R^j) c_R^i \right) - \max_{s_R \in S_R} \left( \frac{|y_B - H_B s_R|^2}{2\sigma_B^2} + \sum_{j=1,j\neq i}^n \Lambda_B (c_R^j) c_R^i \right) \]  

We must consider the output of the demappers are the LLRs for \( \{c_R^1\} \) and they must be converted to the LLRs for \( \{c_A^1\} \) and \( \{c_B^1\} \). We get LLRs for \( \{c_A^1\} \) and \( \{c_B^1\} \) using extended XOR operation. Given the LLR \( \lambda \) of \( c_R^1 \) and the known bit \( c \) (i.e. \( c_R^1 \) or \( c_A^1 \)), the LLR of the unknown bit (i.e. \( c_B^1 \) or \( c_A^1 \)) can be calculated as

\[ \lambda_{\Theta c} = \begin{cases} \lambda, & c = 0 \\ \sim \lambda, & c = 1 \end{cases} \]  

Then the LLRs for \( \{c_A^1\} \) and \( \{c_B^1\} \) are used as the input of the convolutional decoder, where the MAP algorithm is applied. Similarly, extrinsic LLRs and the bit sequences \( \{\hat{c}_A\} \) and \( \{\hat{c}_B\} \) are given to the input of the extended XOR operation to get LLRs for \( \{c_R\} \). At last, the decoder outputs the hard decisions on the information bits.

### IV. Simulation Result

In this section, we give the performance about the transmission scheme. The number of information bits from the relay to terminal A and terminal B are 18000 and 12000, respectively. The convolutional encoder is a rate of 1/2 with generator (4, 7). The interleaver length is 12000 bits. 8PSK is used to modulate the information bits. The OFDM symbol includes 1024 carriers with 256 CP. Each subcarrier corresponds to a Rayleigh fading channel. The simulated BER performance is shown in Fig. 5.

We consider different labelings: Gray labeling, SP labeling and Optimal labeling [10]. It is obvious that SP labeling outperforms Gray labeling. Moreover, the optimal labeling outperforms SP labeling by achieving lower BER at the high SNR regime. However, it may lead to worse BER in low SNR.

In Fig. 6, we give the comparison between bit-inserting zeros (BIZ) scheme and symbol-inserting zeros (SIZ) [9] in BER and throughput. Our scheme has lower BER than traditional one. The reason is that the weaker link has adaptively reduced the modulation order from 8PSK to QPSK.

Fig. 7 gives the BER under different power allocation when \( \text{SNR}=4\text{dB} \). Taking BER and data rate into consideration, terminal B need higher data rate using QPSK and terminal A need lower BER using BPSK.

### V. Conclusion

In this paper, we consider two-way OFDM relay system with asymmetric network coding. In the MAC phase, we propose a Max-Min power allocation scheme. In the BC phase, we design symbol constellation to achieve asymmetric data transmission scheme by bit-inserting zeros according to AM. We make use of a priori information of the weaker link to achieve asymmetric data rate. Numerical results validate our scheme outperforms the symbol-inserting zeros scheme in BER and throughput. We will consider symbol superposition instead of bitwise XOR in our following papers.
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