EXACT CORRECTION OF SHARPLY VARYING OFF-RESONANCE EFFECTS IN SPIRAL MRI

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ABSTRACT

Magnetic Resonance Imaging with non-Cartesian acquisition schemes suffer from blurring artifacts induced by off-resonance. Conventional algorithms for off-resonance correction are approximate in nature and assume that the off-resonance map is varying smoothly across the object. This assumption is violated in the case of susceptibility related off-resonance which is sharply varying near certain tissue interfaces. Iterative methods have been proposed for sharply varying off-resonance correction but are either computationally intense in the exact form or need approximations like time segmentation to reduce computational cost. Here a new method for accurate and computationally efficient off resonance correction using spatial domain deconvolution is presented.

1. INTRODUCTION

Non-Cartesian acquisition schemes have an important role in MR imaging. For example, time-efficient spiral k-space trajectories are used to acquire data in just a few excitations [1], reducing total scan time, while being highly robust to motion and flow artifacts [2]. Their primary drawback is a trajectory dependent object domain blurring artifact induced by off-resonance, which stems from chemical shift, B0 field inhomogeneity and susceptibility. Blurring caused by chemical shift can be avoided by using spectral spatial excitation or fat saturation pre-pulses, while main field inhomogeneity is smooth in nature and can be corrected by conventional techniques. One major remaining concern is susceptibility related off-resonance which is experienced near the boundaries between regions of different magnetic susceptibility, for example near air-tissue interfaces.

Conventional approaches for dealing with off-resonance include conjugate phase reconstruction (CPR) [3] and its derivatives : multifrequency interpolation (MFI) [4], block regional off-resonance correction (BRORC) [5] and spatial domain deconvolution [6] among others. In CPR, separate k-space data sets are computed for each pixel by demodulating the entire raw data at that pixel’s resonance offset, which is obtained from the field map. Each pixel is reconstructed independently by evaluating the 2D inverse Fourier transform of the corresponding demodulated k-space data set at that pixel’s location. In reconstructing each pixel independently, CPR assumes that neighboring pixels have the same or similar frequency offsets i.e. the field map is smooth. MFI is an approximation to CPR in which several images are reconstructed at a reduced number of demodulation frequencies and on-resonant regions are selected under the guidance of a field map. Regions requiring other resonance offsets are obtained by interpolation. Ahunbay et al. [6] propose a spatial domain deconvolution scheme which involves estimating the spiral time evolution function as separable along x and y directions and perform a fast separable deconvolution in the object domain. In BRORC [5] the uncorrected image is divided into several blocks and the effect of the average frequency offset for each block is demodulated out in the frequency domain using time-maps. All of these methods are approximate in nature and are limited by the primary assumption that the field map is smoothly varying.

Man et al. [7] address this problem and propose an iterative scheme for correcting the residual blur after a full CPR. Munger et al. [8] also bypass this assumption and offer an iterative solution for EPI trajectories without any CPR. In [9] also an iterative scheme for single shot spirals is presented. Recently Sutton et al. [10] proposed an iterative method for general non-Cartesian trajectories which is computationally intense in the exact form or requires time segmentation. Time segmentation basically involves splitting time into constant segments over small intervals in order to make the effect of off-resonance more manageable. Here we present a computationally efficient and accurate off-resonance correction method for arbitrary field map variations with flexibility to deblur over specific regions of interest.

2. THEORY

The signal equation in MRI, incorporating the effect of off-resonance can be expressed as a function of time or k-space position:
\( s(t) = \int m(\vec{x})e^{-j\omega(\vec{x})t}e^{-j2\pi\vec{k}(t)\cdot\vec{x}}d\vec{x} \) (1)

\[ M_{\text{off}}(\vec{k}) = \int m(\vec{x})e^{-j\omega(\vec{x})t}e^{-j2\pi\vec{k}\cdot\vec{x}}d\vec{x} \] (2)

where \( \vec{x} \) is location in the object domain, \( \vec{k} \) is location in k-space, \( t \) is time, \( s(t) \) is the received signal, \( M_{\text{off}}(\vec{k}) \) represents the corrupted k-space data, \( m(\vec{x}) \) is the actual signal distribution of the object being measured, \( \omega(\vec{x}) \) is the off-resonance map and \( t(\vec{k}) \) is the time map. After incorporating Fourier reconstruction the effect of off-resonance can be reduced to:

\[ m_{\text{off}}(\vec{x}') = \int M_{\text{off}}(\vec{k})e^{j2\pi\vec{k}\cdot\vec{x}'}d\vec{k} = \int \int m(\vec{x})e^{-j\omega(\vec{x})t(\vec{k})}e^{-j2\pi\vec{k}\cdot\vec{x}'}d\vec{x}e^{j2\pi\vec{k}\cdot\vec{x}'}d\vec{k} = \int m(\vec{x}) \begin{array}{l}
\int e^{-j\omega(\vec{x}')t(\vec{k})}e^{j2\pi\vec{k}\cdot(\vec{x}'-\vec{x})}d\vec{k}d\vec{x}'
\end{array} \]

\[ = \int m(\vec{x}) \mathcal{F}^{-1}\{e^{-j\omega(\vec{x}')t(\vec{k})}\}_{\vec{x}'-\vec{x}} d\vec{x}' \] (3)

where \( m_{\text{off}}(\vec{x}') \) is the blurred image, \( e^{-j\omega(\vec{x})t(\vec{k})} \) is the trajectory’s phase evolution function for a given resonance offset \( \omega(\vec{x}) \), \( \mathcal{F}^{-1} \) denotes the inverse Fourier transform with respect to \( \vec{k} \) and \( h(\vec{x}, \vec{x}') \) is the spatial domain blurring function. With this new representation the effect of off-resonance can be modeled directly as a spatially varying convolution in the object domain. Since only finitely many measurements of the blurred image are available, the integral equation in (3) is not only ill-posed but is also extremely sensitive to noise [11]. If the objective is to deblur a finite discrete image then the integral can be reduced to a summation and the blurring kernel discretized. Using the matrix treatment of a discrete convolution, where actual and corrupted data are viewed as column vectors and the blurring kernel is embedded in the columns of a large matrix we can re-write (3) in the discrete form as:

\[ \tilde{m}_{\text{off}} = H\tilde{m} \] (4)

For a \( N \times N \) image, the \( H \) matrix is of size \( N^2 \times N^2 \) and the column of \( H \) corresponding to the pixel at location \((p, q)\) is obtained by column ordering of \( \mathcal{F}^{-1}\{e^{-j\omega(\vec{x}')t(\vec{k})}m(\vec{x}')\}_{\vec{x}'-\vec{x}} \) shifted by \((p, q)\).

As mentioned initially, conjugate phase and its derivatives require the field map to be smoothly varying, which mathematically translates into assuming that the columns of the blurring matrix are orthogonal. Under this assumption each on-resonant pixel can be obtained by projecting the off-resonant data onto the corresponding column from \( H^* \). Conjugate phase carries out these projections in the frequency domain while [6] carries out these projections in the spatial domain.

For exact correction, this system of equations needs to be solved. Direct inversion of \( H \) is practically infeasible, since matrices arising from discretisation of integral equations like the one in (3) have a cluster of singular values at zero [12] and have large dimensionality. There are several approaches for extracting the actual data from the blurred image. Approximate solutions using Kronecker products and singular value decomposition are described by Nagy et al. [12, 13]. Because the region of support of the blurring kernel is very localized an accurate solution can be obtained by using the conjugate gradient (CG) technique in a computationally efficient way [14]. CG should be applied to the normal equations associated with (4) as \( H \) may not be symmetric. Appropriate regularization is necessary to obtain a stable solution quickly as the condition number of \( H^*H \) is square of that of \( H \), which is already ill-conditioned.

3. METHODS

Each column of the blurring matrix \( H \), is a shifted version of the inverse Fourier transform of the trajectory’s phase evolution function at some resonance offset. Since there is a practical limit to the maximum range of off-resonance, these inverse FFT’s can be pre-computed in small discrete steps. Each blurring kernel (column of \( H \)) is of the same size as the image, which is typically \( 256 \times 256 \).

Pre-computing such large vectors will lead to memory constraints. But if correction is performed on blocks smaller than the actual image then the required size of the blurring kernel will be the same as the image block. The block-sized blurring kernels can be obtained by simply gridding the original phase evolution function onto a grid of the same size as the image block. Also flexibility to deblur small image blocks independently enables region specific correction which can be implemented in real time. For example, if the range of off-resonance is -600 to 600 hertz and a step size of 2 Hz is used, then for a \( 32 \times 32 \) image block, 600 blurring kernels of the same size are needed. Only half of these blurring kernels need to be pre-computed since the columns with negative frequencies can be obtained from the columns with corresponding positive frequencies using the complex conjugation property of the Fourier transform.

The spiral trajectory has a circular footprint which means that the frequency spectrum of the entire image and the trajectory’s phase evolution function will be zero beyond a certain radius in k-space. But the frequency spectrum of an image block may have some signal beyond that k-space radius. Hence care must be to taken to fill the four corners of the trajectory’s time map with the maximal value of time [5].

The entire image is divided into overlapping blocks of size \( N \times N \) such that only the central \( M \times M \) sub-block doesn’t overlap with the neighboring blocks [5]. Correction is performed independently on each \( N \times N \) block using the CG technique described above but only the central \( M \times M \) sub-
Fig. 1. A flowchart of the proposed off-resonance correction method. Table I compares the computational complexity of the proposed scheme and the CPR method.

block is restored in the final corrected image. The $N \times N$ block from the blurred image is used as the initial approximation required in CG. The algorithm proceeds block by block till the entire image has been exhausted.

There are two reasons for correcting a larger block and restoring only the central sub-block in the final image. Firstly to prevent any potential blocking artifact and secondly, the pixels lying at the edge of each block are getting contributions from the blurring of the neighboring pixels from the adjacent blocks. Since the region of support for the blurring kernel (say $r$ pixels) is only local any contribution from pixel’s beyond $r$ can be neglected. So correction will be accurate for central $M \times M$ sub-block as long as $N - M < r$.

3.1. Experimental Methods

Experiments were performed on a Signa EXCITE 3T system (GE Healthcare, Waukesha, WI) with maximum gradient amplitude of 40 mT/m, maximum slew rate of 150 T/m/s and $4\mu s$ sampling. Field maps were obtained using the conventional approach of acquiring two data sets at different echo times and using the phase difference between the two reconstructed images as the estimate. Resolution phantom data was acquired with a 16 interleaf spiral, 4096 points per interleaf and 20 cm FOV [15]. A strong linear shim was used to induce steeply varying off-resonance (-800 Hz to 800Hz) across the phantom from left to right.

Two healthy volunteers were scanned with the approval of the institutional review board of USC. Cardiac images were acquired with a 16 interleaf spiral, 2048 points per interleaf and 30 cm FOV. Localisation of the right coronary artery was performed using product sequences. Acquisition of both the echo times was performed within a single breathhold.

The CG technique, for the phantom was applied with an outer block size ($N \times N$) of $32 \times 32$ and an inner block size ($M \times M$) of $16 \times 16$ while for the volunteer the inner block size was increased to $24 \times 24$, since the resonance offsets have a smaller range (-150Hz,150Hz) implying a more compact blurring kernel. Regularisation of 0.01 was used in both the cases. In order to improve computational efficiency, entries in $H$ with an amplitude lower than $0.1\%$ of the maximum were zeroed out. There were no visible differences in the images reconstructed with and without the zero out.

4. RESULTS

Resolution phantom images from CPR and the proposed method are shown in figure 2. The proposed method corrects most of the visible blurring while the images from CPR still have residual blurring. This is because the field map has a steep slope (10Hz per pixel). Note that conjugate phase performs inadequately at the center of the phantom (character ‘A’) where...
the resonance offset is less than 80Hz but has a large slope.

Figure 3 contains the coronary artery images from one of the volunteers, which was reconstructed using the proposed method and conjugate phase. The distal end of the right coronary artery (RCA) appears sharper when using the proposed method than when using CPR. This is presumably because the off-resonance varies sharply in that region as shown in the field map. The region around the RCA is deblurred most effectively using the proposed reconstruction technique.

The CG technique on average converged within 5 iterations. Each iteration of CG approximately requires $N^4$ complex multiplications for an $N \times N$ block, but with the zero out, on average 90% of the entries in $H^\dagger H$ are zero which reduces the computational complexity without any visible loss in image quality. A comparison of the computational complexity of the proposed method and CPR is shown in the table in Figure 1

5. DISCUSSION

Presented here, is a new technique for fast and accurate off-resonance correction for arbitrary field map variations. As shown, the proposed method is more effective than conjugate phase in regions of steeply varying off-resonance. The proposed method also provides the flexibility to deblur quickly and exactly over specific regions of interest (ROI) which is particularly important in cardiac or real time imaging where nearly exact correction is required over certain ROI. Note that the proposed reconstruction scheme will be exact if no entries in the blurring matrix are neglected.

Finally, this framework can be extended to the correction of flow-related artifacts. It is known that rapid in-plane flow during data acquisition also causes a phase related to the first moment of the gradient waveform and the velocity vector [2]. The correction of flow-related phase may be addressed using a similar deconvolution to undo artifacts from fast flow.

6. REFERENCES


Fig. 3. Gated cardiac images of the right coronary artery (thin arrow). The proposed method produces a sharper depiction of the distal end of the artery (thick arrow).