VOLTAGE REGULATION AND STABILITY ANALYSIS OF A PHOTOVOLTAIC SYSTEM WITH A BOOST CONVERTER INTERFACE

Konstantinos F. Krommydas, Antonio T. Alexandridis

Department of Electrical and Computer Engineering
University of Patras, Rion 26500, Greece
Email: krommydas@ece.upatras.gr, a.t.alexandridis@ece.upatras.gr

ABSTRACT
A main control task in photovoltaic (PV) power generation is the output voltage stabilization under varying irradiation or load changes. Taking into account the input voltage nonlinear dependence from the input current of a PV system as well as the nonlinear dynamics of a DC/DC boost converter, an appropriate PI controller is proposed. To this end, the passivity-based control analysis as it has been developed for switched power converters is suitably modified for this case. Hence, as it is proven in the paper, this controller is capable to guarantee system stability and convergence to the desired operating point. Extensive simulation results verify the system good performance and the controller efficiency.

KEY WORDS
Photovoltaic systems, voltage regulation, DC/DC boost converter, stability analysis, passivity-based PI control

1. Introduction
In modern distributed power systems, power generation based on photovoltaic (PV) systems is rapidly increasing [1]. In grid-connected PV systems, it is very important for the electrical power produced from the PV system to satisfy the requirements of the transmission system operator. Similarly, in stand-alone PV systems the load determines the required operation characteristics. To achieve these requirements, power electronic devices play a key role since they interface the PV modules with the electrical grid or the load.

One of the most basic power electronic device utilized in PV systems is the DC/DC boost converter, which is used to boost the voltage of the PV modules at the desired voltage level [2]. Therefore, its performance is crucial and can be achieved by applying proper control schemes that guarantee stability and convergence to the desired operating point. However, this is not an easy task due to both: the nonlinear nature of the PV operation (since the voltage is a nonlinear function of the current) and the nonlinear dynamics of the boost converter.

The most common technique of modeling and controlling a PV system is the small signal analysis. This technique unfortunately is very sensitive to model uncertainties and changes of the operating point [3]. To overcome this problem, other control schemes have been proposed such as fuzzy control or sliding mode control [4], [5], [6]. Although these schemes seems to improve system performance, their implementation is rather heuristic and stability cannot be guaranteed.

Other techniques such as passivity based control designs can exhibit remarkable transient performance and at the same time guarantee closed-loop stability [7]. In [8] the theory of passivity based PI controllers is extended to switched power converters while in [9], an adaptive PI stabilization is proposed for a quadratic boost converter. A methodology to design PI controllers for a large class of switched power converters operating under constant input voltage, is based on the fact that an input affine nonlinear system is passifiable via constant control action without requiring the knowledge of the constant term [8]. However, when the voltage input of the boost converter is not constant, that is the case of the PV module where a nonlinear voltage source appears, a similar design and stability analysis has not yet been addressed.

In this paper, we consider the nonlinear model of the PV module, i.e. we take into account the input voltage-current nonlinear dependence. For the DC/DC boost converter we use its average model that includes its accurate nonlinear dynamics [7], while a single resistance load is considered. Aim of the control design is to stabilize the output voltage at a desired level, regardless of the irradiance or the resistance load. To this end, we extend the passivity-based PI method presented in [8] to the case of a non-constant input voltage. We prove that also in the case of the nonlinear input voltage the passivity based PI controller guarantees stability and convergence to the desired equilibrium point. Therefore, precise voltage regulation can be achieved. Simulations results exhibit satisfactory transient performance and verify the closed loop stability analysis.

The remaining of the paper is organized as follows. In Section 2, we present the theory for the passivity based PI controller for the case of switched power converter. In Section 3 we describe the nonlinear dynamic model of the PV system, while in Sections 4 and 5, we propose a suitable passivity-based PI controller and we prove the stability and convergence at the desired operating point of the closed loop system. In Section 6, simulation results are presented and finally, in Section 7, some conclusions are drawn.
2. Preliminaries

Let the nonlinear system

$$\dot{x} = f(x) + g(x)u$$  \hspace{1cm} (1)$$

with $x \in \mathbb{R}^n$ and control $u \in \mathbb{R}^m$ and an admissible equilibrium point $x_\star \in \mathbb{R}^n$.

In the following we present some preliminaries for the design of the passivity based PI controllers, as presented in [8].

Proposition 1 (Passifiability via constant control): Assume there exist $h(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ and $V(\cdot) : \mathbb{R}^n \to \mathbb{R}$ with $V$ bounded from below and radially unbounded, and a constant vector $\theta \in \mathbb{R}^n$ such that:

$$\begin{align*}
\left( \frac{\partial V}{\partial x} \right)^T g(x) &= h^T(x) \hspace{1cm} (2) \\
\left( \frac{\partial V}{\partial x} \right)^T [f(x) + g(x)\theta] &\leq 0 \hspace{1cm} (3)
\end{align*}$$

Then for all initial conditions $(x(0), z(0))$, the trajectories of the system (1) in closed-loop with the PI controller

$$\begin{align*}
y &= h(x) \\
u &= -K_p y + z \hspace{1cm} (4) \\
\dot{z} &= -K_y y
\end{align*}$$

with $K_p, K_y \in \mathbb{R}^{n \times n}$ symmetric positive definite matrices, are bounded and $\lim_{t \to \infty} y(t) = 0$.

Proposition 2 (Detectability): The equilibrium $x_\star$ is locally detectable from the output. That is, for any solution $x(t)$ of the system (1) which belongs to some open neighbourhood of the equilibrium for all $t \geq 0$ the following implication is true:

$$h\left(x(t)\right) = 0 \Rightarrow \lim_{t \to \infty} x(t) = x_\star$$

In the following sections a PI controller of the form (4) is developed and analysed in accordance to proposition 1 for a PV system feeding a load via a DC/DC boost converter.

3. System Modeling

The system, as shown in Figure 1, consists of a series of photovoltaic modules connected to a DC/DC boost power converter, while the converter is driving a single resistance load.

3.1 PV cell circuit model and equations

The PV cell can be modeled as shown in Figure 2 by a parallel connection of a photo-current source $I_{SC}$ (proportional to the irradiance), a diode and a resistor $R_p$, which is then connected in series with a resistor $R_s$.

The equation of the current $I_{PVcell}$ and the voltage $V_{PVcell}$ of the PV cell are given from the following equations [10]:

$$I_{PVcell} = I_{SC} - I_D - \frac{V_D}{R_p}$$

$$I_D = I_0\left(e^{V_{PV}/V_T} - 1\right)$$

$$V_{PVcell} = V_D - R_s I_{PV}$$

3.2 PV module model integration with the DC/DC boost converter

A PV module consisting of 36 PV cells in series is considered. We define the output voltage and current of the PV module as $V_{PV}$ and $I_{PV}$ respectively. A $V_{PV} - I_{PV}$ characteristic is shown in Figure 3. A PV module string is considered as input of the DC/DC boost converter. The output voltage of the string equals with the sum of all the PV module voltages $V_{PV}$. As for the current of the string, it is equal with the PV module current $I_{PV}$. 

![Figure 1. Model of the PV system](image1)

![Figure 2. PV cell circuit model](image2)
Using the average model of the boost converter we obtain the following state space model of the system in the form of (1):

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
\frac{E(x)}{L} & -\frac{x_2}{RC} \\
\frac{x_1}{C} & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
x_2 \\
x_1
\end{bmatrix}
\]

where \( x_1 \) is the current of the induction, \( x_2 \) is the voltage of the capacitor and \( E(x) \) is the output voltage \( V_{PV, string} \) of the PV module string. The converter inductance and capacitance is represented by \( L, C \) respectively while \( R \) is the resistance load. Finally \( \mu \), where \( \mu = 1 - s \), is the duty ratio representing the switching cycle ratio in a period. It should be noted that \( \mu \) is a continuous function that ranges in \([0,1]\).

\[ y = h(x) = Mx_2 - x_1 \]  

(7)

We will prove now that the two expressions given by (2), (3) hold true. For the first one we obtain:

\[ \begin{bmatrix}
\frac{\partial V}{\partial x}^T \\
\frac{\partial f}{\partial x} + g(x)\theta
\end{bmatrix} =
\begin{bmatrix}
aLx_1 - MLx_2 - Cx_2 - C \left( \frac{x_1}{C} \right) \\
-a\frac{x_1^2}{R} + (M\theta + \frac{1}{R})x_2 + (aE(x) - \theta)x_1 - ME(x)
\end{bmatrix} \]

\[ Mx_2 - x_1 = h(x) \]

and for the second one:

\[ \begin{bmatrix}
\frac{\partial V}{\partial x}^T \\
\frac{\partial f}{\partial x} + g(x)\theta
\end{bmatrix} =
\begin{bmatrix}
-a\frac{x_1^2}{R} + (M\theta + \frac{1}{R})x_2 + (aE(x) - \theta)x_1 - ME(x)
\end{bmatrix} \]

(8)

In order to prove that (3) holds true, we proceed by finding the maximum value of (8). This is hard to be done analytically due to the dependence of the function \( E(x) \) from \( x_1 \). Thus, we express \( E(x) \) as a function of the ratio \( M/a \) which always satisfies the following inequality:

\[ (ax_1 - M)E(x) \leq (ax_1 - M)E \left( \frac{M}{a} \right) \]

since \( E(x) : [0, I_{sc}] \to [0, V_{oc}] \) and \( \frac{dE(x)}{dx_1} < 0 \)

It should be noted that \( I_{sc} \) represents the short circuit current and \( V_{oc} \) the open circuit voltage of the PV module string. Thus, from (8) we obtain the inequality:

\[ \begin{bmatrix}
\frac{\partial V}{\partial x}^T \\
\frac{\partial f}{\partial x} + g(x)\theta
\end{bmatrix} \leq
\begin{bmatrix}
-a\frac{x_1^2}{R} + (M\theta + \frac{1}{R})x_2 + (aE(x) - \theta)x_1 - ME \left( \frac{M}{a} \right)
\end{bmatrix} \]

If we choose \( \theta = aE \left( \frac{M}{a} \right) \) the above inequality becomes:

\[ \begin{bmatrix}
\frac{\partial V}{\partial x}^T \\
\frac{\partial f}{\partial x} + g(x)\theta
\end{bmatrix} \leq
\begin{bmatrix}
-a\frac{x_1^2}{R} + (M\theta + \frac{1}{R})x_2 - ME \left( \frac{M}{a} \right)
\end{bmatrix} \]

Now, let us define the function:

\[ V(x) = aH(x) - M(aLx_1 - Cx_2) \]

(6)

\[ y = h(x) = Mx_2 - x_1 \]  

(7)

4. Passifiability via constant control

In this Section, we prove that conditions (2) and (3) of Proposition 1 can be satisfied. Hence, a PI controller of the form (4) can be effectively applied.

First we start from the quadratic Hamiltonian storage function, which represents the total system energy:

\[ H(x) = \frac{1}{2}Lx_1^2 + \frac{1}{2}Cx_2^2 \]

Then a bounded from below and radially unbounded storage function can be determined as follows:

\[ V(x) = aH(x) - M(aLx_1 - Cx_2) \]

(6)

where \( a \) and \( M \) are positive constants. Finally, we define as output of the system:

\[ y = h(x) = Mx_2 - x_1 \]  

(7)

We will prove now that the two expressions given by (2), (3) hold true. For the first one we obtain:

\[ \begin{bmatrix}
\frac{\partial V}{\partial x}^T \\
\frac{\partial f}{\partial x} + g(x)\theta
\end{bmatrix} =
\begin{bmatrix}
aLx_1 - MLx_2 - Cx_2 - C \left( \frac{x_1}{C} \right) \\
-a\frac{x_1^2}{R} + (M\theta + \frac{1}{R})x_2 + (aE(x) - \theta)x_1 - ME(x)
\end{bmatrix} \]

\[ Mx_2 - x_1 = h(x) \]

and for the second one:

\[ \begin{bmatrix}
\frac{\partial V}{\partial x}^T \\
\frac{\partial f}{\partial x} + g(x)\theta
\end{bmatrix} =
\begin{bmatrix}
-a\frac{x_1^2}{R} + (M\theta + \frac{1}{R})x_2 + (aE(x) - \theta)x_1 - ME(x)
\end{bmatrix} \]

(8)

In order to prove that (3) holds true, we proceed by finding the maximum value of (8). This is hard to be done analytically due to the dependence of the function \( E(x) \) from \( x_1 \). Thus, we express \( E(x) \) as a function of the ratio \( M/a \) which always satisfies the following inequality:

\[ (ax_1 - M)E(x) \leq (ax_1 - M)E \left( \frac{M}{a} \right) \]

since \( E(x) : [0, I_{sc}] \to [0, V_{oc}] \) and \( \frac{dE(x)}{dx_1} < 0 \)

It should be noted that \( I_{sc} \) represents the short circuit current and \( V_{oc} \) the open circuit voltage of the PV module string. Thus, from (8) we obtain the inequality:

\[ \begin{bmatrix}
\frac{\partial V}{\partial x}^T \\
\frac{\partial f}{\partial x} + g(x)\theta
\end{bmatrix} \leq
\begin{bmatrix}
-a\frac{x_1^2}{R} + (M\theta + \frac{1}{R})x_2 + (aE(x) - \theta)x_1 - ME \left( \frac{M}{a} \right)
\end{bmatrix} \]

If we choose \( \theta = aE \left( \frac{M}{a} \right) \) the above inequality becomes:

\[ \begin{bmatrix}
\frac{\partial V}{\partial x}^T \\
\frac{\partial f}{\partial x} + g(x)\theta
\end{bmatrix} \leq
\begin{bmatrix}
-a\frac{x_1^2}{R} + (M\theta + \frac{1}{R})x_2 - ME \left( \frac{M}{a} \right)
\end{bmatrix} \]

Now, let us define the function:
\[ F(x_2,a) = -a \frac{x_2^2}{R} + (M \theta + \frac{1}{R})x_2 - ME \left( \frac{M}{a} \right) \]  \tag{9}

Proceeding with the maximum value of the function (9) with respect to \( x_2 \), we have:

\[ \frac{\partial F(x_2,a)}{\partial x_2} \bigg|_{x_2=x_2^*} = 0 \Rightarrow x_2^* = \frac{R}{2a} \left( MaE \left( \frac{M}{a} \right) + \frac{1}{R} \right) \]

For the value \( x_2^* \), we determine:

\[ \tilde{F}(x_2^*,a^*) = 4aF(x_2^*,a) = R \left( MaE \left( \frac{M}{a} \right) + \frac{1}{R} \right)^2 - 4aME \left( \frac{M}{a} \right) \]

If we choose:

\[ a^* = \frac{1}{ME \left( \frac{M}{a} \right) R} \]  \tag{10}

we can easily see that:

\[ \tilde{F}(x_2^*,a^*) = 0 \]

and as a result:

\[ \exists a^*: \text{max} \tilde{F}(x_2,a^*) = 0 \]

which indicates that inequality (3) holds true and that the system can be passified with constant control.

We need to investigate the existence of \( a^* \) satisfying (10). Thus, we introduce the variable \( k = \frac{M}{a} \) and after some calculations, (10) becomes:

\[ M^2E(k)R - k = 0 \]  \tag{11}

In order to find the solution \( k \) of the equation (11) we introduce the function:

\[ f(k) = M^2E(k)R - k \]

from which we can easily obtain:

\[ f(I_{\infty}) = -I_{\infty} < 0 \]

\[ f(0) = M^2V_{i_{\infty}}R > 0 \]

Therefore there exists at least one \( k^* \in (0, I_{\infty}) : f(k^*) = 0 \), which ensures that there exists at least one \( a^* \) satisfying (10).

5. Obtaining Detectability

The passivity based PI controller guarantees that \( h(x(t)) = 0 \Rightarrow Mx_2 - x_1 = 0 \Rightarrow Mx_2 = x_1 \). After some calculations we obtain for the closed loop system:

\[ \left(1 + \frac{C}{M^2L}\right) \dot{i}_1 + \frac{1}{LRM_2} x_1 = \frac{E(x_1)}{L} \]

\[ \left(1 + \frac{M^2L}{C}\right) \dot{x}_2 + \frac{1}{RC} x_2 = \frac{ME(x_1)}{L} \]

Taking the time derivative of the above differential equations and defining \( \rho_1 = \dot{x}_1 \) and \( \rho_2 = \dot{x}_2 \) we obtain:

\[ \left(1 + \frac{C}{M^2L}\right) \dot{\rho}_1 + \frac{1}{LRM_2} - \frac{1}{L} \frac{dE(x_1)}{dx_1} \rho_1 = 0 \]

\[ \left(1 + \frac{M^2L}{C}\right) \dot{\rho}_2 + \frac{1}{RC} - \frac{M^2}{L} \frac{dE(x_1)}{dx_1} \rho_2 = 0 \]  \tag{12}

since \( \frac{dE(x_1)}{dx_1} < 0 \) the system (12) is asymptotically stable, therefore:

\[ \lim_{t \to \infty} \rho_1(t) = 0 \Rightarrow \lim_{t \to \infty} \dot{x}_1(t) = 0 \]

\[ \lim_{t \to \infty} \rho_2(t) = 0 \Rightarrow \lim_{t \to \infty} \dot{x}_2(t) = 0 \]

which indicates that system (12) will be stabilized at the equilibrium point:

\[ x_{t_1} = E(x_{t_1})RM^2 \] and \( x_{t_2} = RME(x_{t_2}) \)

This completes the proof of the detectability from the output. Now, if we choose \( M = \frac{x_{t_2 \text{ref}}}{E(x_{t_1})R} \), where \( x_{t_2 \text{ref}} \) is the desired output voltage and \( \dot{x}_1 \) is the equilibrium point corresponding to \( \tilde{x}_1 = x_{t_2 \text{ref}} \), then we arrive after some calculations at:

\[ \tilde{x}_1 = \frac{x_{t_2 \text{ref}}^2}{E(x_{t_1})R} \] and \( \tilde{x}_2 = x_{t_2 \text{ref}} \)  \tag{13}
which indicates that the output voltage will be stabilized at the desired voltage achieving precise voltage regulation for any $E(x_t)$.

The stability analysis of Section 4, completed with the convergence proof given in Section 5, guarantees that the PI control scheme (4) with $y$ given by (7) stabilizes the PV string-boost converter and leads the system to the desired operating point. In order to avoid computing the constant $M$, a simple PI comparator is used to provide the proper value of $M$. The system and the complete control scheme is shown in Figure 4.

![Figure 4. Block diagram of the control scheme](image)

6. Simulations Results

In order to verify the stability analysis and the transient performance of the passivity based PI controller, the response of the system is simulated for two different cases.

In the first case at time $t = 1$sec the irradiance drops from 1000 $W/m^2$ to 600 $W/m^2$ and while at time $t = 2$sec the resistance load changes from $R = 800 \Omega$ to $R = 1.1k\Omega$. The reference value is set at $x_{2,ref} = 600V$. Aim of the passivity based PI controller is to stabilize the output voltage at the reference value regardless from the irradiance or the resistance load.

A string of ten PV modules is used as the input voltage of the DC/DC boost converter. The parameters of the DC/DC boost converter are $C = 200 \mu F$ and $L = 20mH$.

The response of the PV system for this case is shown in Figures 5, 6, 7, 8.

In the second case we change the reference value of the output voltage from 600$V$ to 580$V$ at time $t = 1$sec and we obtain the response of the output voltage as shown in Figure 9. In this case the converter drives a resistor load of a constant value $R = 1.1k\Omega$.

In both cases one can easily see the excellent response of the controller. Fast transient and effective convergence to the desired reference voltage are observed.

7. Conclusion

In this paper, a passivity-based PI controller for a DC/DC boost converter fed by a PV system has been designed and extensively simulated. Nonlinear stability analysis is presented for this particular case, where a varying voltage input is considered. The theoretical results are fully verified by the simulation results obtained for different cases: irradiation or load variations and reference output voltage step changes.

![Figure 5. Output voltage of the DC/DC converter in the case of irradiation and load changes](image)

![Figure 6. Current of the PV string in the case of irradiation and load changes](image)

![Figure 7. Voltage of the PV string in the case of irradiation and load changes](image)
Figure 8. Duty ratio in the case of irradiation and load changes

Figure 9. Output voltage of the DC/DC converter in the case of a step change of the reference voltage value

References


