Analysis of Adaptive Cyclic ADCs Performance under Nonidealities of Their Internal ADCs

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Abstract — The paper presents results of investigation on new adaptive Cyclic A/D Converters (CADCs) behaviour in conditions of possible nonidealities of their components, particularly of their internal ADCs transfer characteristics. The simulation results show that the adaptive CADCs do not require high quality internal components and work properly even when the components characteristics differ significantly from their nominal forms.

Keywords: A/D conversion, adaptive cyclic ADC.

I. INTRODUCTION

Recently, the cyclic or sub-ranging analog-to-digital converters belong to one of the most widely developed classes of A/D converters [1]. In [2-4] and other works, a new analytical approach to optimization and design of sub-optimal adaptive CADCs was developed. The approach indicates a possibility to design a new class of adaptive CADCs with digital computing of estimates (codes) of the input samples, which would have the highest, close to theoretically achievable performance. The main principle of the approach is introduction of a digital part computing estimates (codes) and using adaptive algorithms [5] for joint data-processing and control of converter analogue blocks ensuring the most effective, in the given conditions, conversion of input samples. In earlier works the mathematical model of the adaptive CADC was introduced. The model enables exact mathematical and simulation analysis of CADC performance depending on parameters of input signals, analogue elements, noises and other disturbances. It also enables detailed investigations of CADCs functioning and permits us to formulate practical recommendations for their design.

This paper deals with the problem of application of non-ideal components in CADC structure. One of the most important CADC components, having the significant influence on its performance, is the internal A/D converter (ADC_{In}). Therefore, in the paper the consequences of internal ADC characteristic distortions such as offset, gain and nonlinearity errors are investigated.

II. ADAPTIVE CADC FUNCTIONING AND PROPERTIES

General structure of the adaptive CADC is presented in Fig. 1. Each sample $V$ of the input signal is held at the output of sample and hold block during the time $T$ necessary to complete $n$ cycles of conversion. In each $k$-th cycle ($k=1,...,n$), the data-processing block calculates a new estimate $\hat{V}_k$ of the sample $V$ using the previous estimate $1\hat{V}_{k-1}$ and current observation $y_k$ at the output of the internal fast low-bit A/D converter ADC_{In}:

$$\hat{V}_k = \hat{V}_{k-1} + L_k y_k \quad (k = 1,...,n).$$

Fig. 1: General structure of the adaptive CADC.

Observations $\tilde{y}_k = C_1 e_k + \xi_k$ are formed from residual signal $e_k = V - \hat{V}_{k-1} + v_k$. The residual signal $e_k$ is amplified and converted by the internal ADC_{In}. The ideal transfer characteristic of ADC_{In} is presented in Fig. 2.

Fig. 2: Ideal characteristic of the internal ADC_{In}.

Noise $\xi_k$ is the quantization noise of ADC_{In} with the variance $\sigma^2 = \Delta^2 / 12 = D^2 \cdot 2^{-2N_{ADC}} / 3$, where $D$ describes the boundaries of the ADC_{In} input range, $N_{ADC}$ is its resolution (in bits). $\Delta$ is the ADC_{In} quantization interval (the least significant bit - LSB). Internal noise
of amplifier is not considered as negligibly small in comparison with the quantization noise $\xi_k$. Parameter $C_k$ describes the gain of amplifier which increases with the number of conversion cycle. Noise $v_k$ is a summary noise of the feedback D/A converter (DAC), subtracting block and other possible analogue part noises.

The proposed in [2-4] approach permits determining the optimal structure and parameters of analogue and digital parts of the sub-optimal adaptive CADC with MSE $P_k$ of conversion and resolution $N_k$:

$$P_k = E[(\hat{V}_k - V)^2], \quad N_k = \frac{1}{2} \log_2 \frac{\sigma_0^2}{P_k},$$

(2)
close, for each $k=1,...,n$, to theoretically achievable boundaries. The algorithm of sub-optimal conversion is as follows [2-5]:

$$\hat{V}_k = \hat{V}_{k-1} + L_k \hat{y}_k,$$

(3)
where

$$L_k = \frac{C_k P_k}{\sigma_k^2 + C_k \sigma_v^2}, \quad C_k = \frac{\alpha}{\sqrt{\sigma_v^2 + P_{k-1}}},$$

(4)
with initial conditions: $\hat{V}_0 = V_0$, $P_0 = \sigma_0^2$. Parameter $\alpha$ is determined by the probability $\mu$ of CADC saturation and satisfies the equation: $\Phi(\alpha) = (1 - \mu)/2$, where $\Phi(\alpha)$ is the Gaussian error function.

For CADC employing the algorithm (3)-(5), there is the initial phase of conversion, when the MSE of conversion diminishes exponentially and resolution increases with maximal theoretically achievable rate [2-5]. When $P_k$ reaches the value of $\delta^2$, the exponentially fast diminution of conversion errors slows down and takes the hyperbolical form.

### III. CADC performance analysis under Internal ADC nonidealities

The analytical analysis of influence of ADC characteristics on CADC performance is impossible due to their complicated nonlinear models. Therefore, the effects occurring in such situations ought to be investigated by simulation tools. The experiments presented below were carried out using full mathematical model of sub-optimal CADC as in Fig. 1 employing the algorithm (3)-(5) for estimates $\hat{V}_k$ calculation and analogue part control. The analogue part of CADC was modelled taking into account a step-wise form of ADCIn characteristic under its different distortions [6]. Input signals were generated as a sequence of random samples $V^m$ ($m=1,...,M$) each of the length $n$ enabling realisation of $n$ cycles of conversion. Resolution and MSE of CADC conversion were measured using their empirical evaluation according to formulas:

$$\hat{P}_k = \frac{1}{M} \sum_{m=1}^{M} (\hat{V}_k^m - V^m)^2, \quad \hat{N}_k = \frac{1}{2} \log_2 \frac{\sigma_0^2}{P_k},$$

(6)
where $\hat{V}_k^m$ is the estimate of $m$-th sample $V^m$ at $k$-th cycle of conversion.

It was assumed that noise $v_k$ is conditioned only by quantization noise of the feedback DAC. Then, its variance $\sigma_v^2 = \Delta_{DAC}^2 / 12 = D_{DAC} \cdot 2^{-2N_{DAC}} / 3$, where $N_{DAC}$ is DAC resolution and DAC output range $[-D_{DAC}, D_{DAC}]$ is equal to the input range of CADC. Computer simulations were carried out for the following values of parameters: $v_k = 0$, $\sigma_0^2 = 4$, $\alpha = 5$, $D = 1.25$, $N_{DAC} = 16$, $D_{DAC} = 10$, $M = 1000$.

#### A. Offset error

In the first series of experiments the influence of ADCIn offset errors on CADC performance was investigated. The ADCIn characteristic with the assumed offset error model is presented in Fig. 3. The offset error was modelled as the displacement of the whole characteristic given by $\delta \cdot \Delta$.

![Fig. 3: Offset error model of ADCIn characteristic.](image)

The empirical MSEs of CADC conversion (6) obtained for different values of $\delta$ (changing from 0 to 1.5) and $N_{ADC} = 2, 4, 8$ are shown in Fig. 4. The respective empirical resolutions of CADC $\hat{N}_k$ are presented in Fig. 5. The plots presented in Fig. 4 and 5 show that CADC performance is kept in wide interval of offset error value until $0.9 \Delta$ for $N_{ADC} = 8$. The CADC offset errors tolerance increases with $N_{ADC}$ growth. Identical results were obtained for $\delta < 0$.

#### B. Gain error

The goal of the second series of experiments was the analysis of CADC functioning under ADCIn gain errors. The gain error of ADCIn was modelled as in Fig. 6. The variable $\gamma$ determines the level of ADCIn characteristic slope deviation.

![Fig. 6: Gain error model of ADCIn characteristic.](image)

The empirical MSEs of CADC conversion obtained for different values of $\gamma$ are presented in Fig. 7.
The range of tolerance of the ADC\textsubscript{in} characteristic slope deviation changes with the ADC\textsubscript{in} resolution, i.e. for $N_{\text{ADC}}=2 : \gamma \in [-1.2,1.4]$ , for $N_{\text{ADC}}=4 : \gamma \in [-1.4,1]$ and for $N_{\text{ADC}}=8 : \gamma \in [-2,1.8]$ .

C. Nonlinearity errors

The third series of experiments is devoted to differential nonlinearity (DNL) errors of ADC\textsubscript{in}. The assumed model of errors is presented in Fig. \ref{fig:DNL_model}.

![DNL errors model of ADC\textsubscript{in} characteristic.](image)

Nonlinearity errors were modelled as the independent random displacements of all ADC\textsubscript{in} quantization levels. The displacements were uniformly distributed around their nominal values. The interval $\varepsilon \cdot [-\Delta / 2, \Delta / 2]$ of displacements distribution is determined by $\varepsilon$ . For large DNL ($\varepsilon > 1$) the overlapping of the quantization levels can occur and then missing code errors appear \cite{6}.

The empirical MSEs of CADC conversion obtained for different values of $\varepsilon$ are presented in Fig. \ref{fig:DNL_MSE}. The results show a wide range of distortions of the ADC\textsubscript{in} quantization levels, in which CADC does not lose its quality. The range increases with the $N_{\text{ADC}}$ growth. The admissible values of $\varepsilon$ are: for $N_{\text{ADC}}=2 : \varepsilon \in [0,1.4]$ , for $N_{\text{ADC}}=4 : \varepsilon \in [0,1.8]$ and for $N_{\text{ADC}}=8 : \varepsilon \in [0,2]$.

In the fourth series of experiments integral nonlinearity (INL) errors of ADC\textsubscript{in} were investigated. The assumed model of errors is presented in Fig. \ref{fig:INL_model}. The nonlinearity was modelled by the nonlinear $\mu$-low compressing function used in nonlinear quantizers \cite{7}:

$$f(x) = D \text{sgn}(x) \frac{\log(1 + \mu |x| / D)}{\log(1 + \mu)} ,$$

where $\mu$ determines the nonlinearity. Formula (7) describes the line binding the code centres of the ADC\textsubscript{in} characteristic.

![INL errors model of ADC\textsubscript{in} characteristic.](image)

![DNL errors model of ADC\textsubscript{in} characteristic.](image)

![Empirical MSE of CADC conversion under gain errors for different values of $\gamma$ and $N_{\text{ADC}}=2,4,8$.](image)
The empirical MSEs of CADC conversion obtained for different values of $\mu$ are presented in Fig. 11. The tolerance range of $\mu$ values, for which the CADC performance is preserved, decreases significantly with $N_{ADC}$ growth. For $N_{ADC} = 2$ the CADC performance is changing very slightly in the analysed very wide range of $\mu$ values. But for $N_{ADC} = 4$ and $N_{ADC} = 8$, the admissible values of $\mu$, for which CADC does not lose its performance, are $\mu \in [0, 2.4]$ and $\mu \in [0, 0.07]$ respectively.

IV. CONCLUSIONS

The results of simulation analysis of the new adaptive CADCs behaviour under distortions of their internal ADCs characteristics confirm complicated dependencies between CADC parameters and degree of ADC$_{in}$ characteristic distortions. The developed simulation tools allow the exact analysis of these dependencies and can be very helpful in design of real CADCs. Especially, they help to define admissible deviations from ideal characteristics for which CADCs do not lose their performance. Moreover, these tools can estimate how CADC performance changes when we apply components with given tolerance or accuracy.

The obtained results show that the close to theoretically achievable performance of the adaptive CADC is preserved under weak requirements for the internal ADC. The boundaries of the requirements for analysed CADC examples were also presented.

REFERENCES