Dynamics of oscillating pulse edges in two-dimensional switch lines

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Abstract: The oscillating motions of a wavefront on a two-dimensional (2D) switch line, i.e., a transmission line periodically loaded with electronic switches (the switch is open for voltages greater than a fixed threshold and closed otherwise), are discussed. In a two-dimensional switch line, the amplitude of the wavefront decays more rapidly than in a one-dimensional switch line, and the oscillation frequency depends on the propagation orientation. The rapid decay of the wavefront, supported by inherent frequency entrainment, is useful for generating high-frequency voltage waves.

Keywords: nonlinear waves, tunneling diodes, switch lines, frequency entrainment

Classification: Science and engineering for electronics

References

1 Introduction

We investigate a switch line [1], defined as a lumped transmission line containing a series inductor, series resistor, shunt capacitor, and shunt switch (open for voltages greater than a fixed threshold and closed otherwise), in each section. In one-dimensional (1D) lines, a unique property of the wavefront is observed when a step pulse is input: the wavefront travels forward at the beginning and then returns to the input port. The returned wavefront is reflected at the input port; therefore this oscillating behavior continues [2].

The wavefront oscillates in two dimensions as well. However, its qualitative properties greatly differ from those in one dimension. The fundamental frequency of the oscillating wavefront is unique in one dimension, while it depends on the propagation orientation and the location of the input cells in two dimensions.

2 Two-dimensional switch lines

Figure 1 (a) shows a unit cell of a two-dimensional (2D) switch line. $L$, $R$, $C$, and $I_{sw}$ represent the series inductor, series resistor, shunt capacitor and current in the shunt switch, respectively. Throughout this article, the switch

![Diagram](image)

Fig. 1. Fundamental properties of 2D switch lines. (a) 2D switch line, (b) a model of the shunt switches, (c) Spatio-temporal development of the wavefront with linearly aligned inputs at an angle of 0 with respect to the $x$-axis, and (d) Voltage transients for the linear inputs at an angle of 0 (black) and $\pi/4$ (red).
is modeled by the current source whose current–voltage relationship is shown in Fig. 1 (b). The switch opens for voltages greater than a threshold $V_{th}$ and exhibits a constant conductance $G_0$ for voltages less than $V_{th}$. Tunneling diodes [3] could successfully simulate the switches by taking their peak voltage as $V_{th}$. In 1D switch lines, the wavefront was found to oscillate, when a rising step pulse crossing $V_{th}$ is input. When the wavefront travels forward, an exponential wave is expected to develop when the voltages are less than $V_{th}$, while an ordinary sinusoidal wave develops for voltages greater than $V_{th}$. The sinusoidal wave can combine continuously with the leading exponential wave. This exponential-sinusoidal wavefront is unstable; it becomes attenuated with transmission and finally disappears. At this point, an exponential wave newly develops at voltages greater than $V_{th}$, and the wavefront starts to travel backward.

Since both unstable sinusoidal-exponential and stable exponential-exponential waves develop, a similar oscillating wavefront can be observed in 2D switch lines. However, two properties are inherent in two dimensions. The first one being the attenuation rate of the wavefront. In one dimension, the amplitude of the wavefront decays as $r^{-1}$ at a distance $r$ from the input, while it decays as $r^{-2}$ in two dimensions. This rapid attenuation of the wavefront reduces the turnaround distance, such that the oscillation frequency becomes higher in 2D lines than in one dimension.

The other property concerns the propagation orientation of a wavefront. Consider an exponential wave with the wave vector $(\kappa_x, \kappa_y)$ for line voltages less than $V_{th}$ and a sinusoidal wave with the wave vector $(k_x, k_y)$ for voltages greater than $V_{th}$. The normalized phase velocities $[4]$ of the exponential ($v_e$) and sinusoidal ($v_s$) waves are given by

$$v_e = -\frac{\beta}{\kappa} + \frac{1}{\kappa} \sqrt{\beta^2 + 4 \left( \frac{\sinh^2 k_x}{2} + \frac{\sinh^2 k_y}{2} \right)},$$

$$v_s = \frac{2}{k} \sqrt{\frac{\sin^2 k_x}{2} + \frac{\sin^2 k_y}{2}},$$

where $\beta = G_0 \sqrt{L/C}$, $\kappa = \sqrt{\kappa_x^2 + \kappa_y^2}$, and $k = \sqrt{k_x^2 + k_y^2}$. Since both waves travel at a common velocity to preserve the wavefront, $v_s$ must be nearly equal to $v_e$. Evidently, this common velocity depends on $\theta = \tan^{-1}(k_y/k_x)$. Evaluating the intersection of the dispersion curves of the exponential and sinusoidal waves using Eqs. (1) and (2) reveals that the intersection velocity decreases monotonically as $\theta$ varies from $\theta = 0$ to $\pi/4$ [4]. The decreased velocity increases the turnaround time of the wavefront, reducing its oscillation frequency.

### 3 Numerical evaluations

First, we examined the above-mentioned properties of 2D switch lines. We then considered step pulse inputs at simply connected finite cells of a 2D switch line, and described how the wavefront achieves limit-cycle oscillation. We numerically solved the transmission equations of a 2D switch line [4] in
the time domain. We set $L$, $R$, $C$, $G_0$, and $V_{th}$ to 2.0 $\mu$H, 6.0 $\Omega$, 250.0 pF, 25.0 mS, and 0.2 V, respectively. Figure 1(c) shows the steady-state waveforms monitored at cells on the $y$-axis when step pulses were applied at linearly aligned cells along the $x$-axis. To obtain these, the total number of cells was set to $100 \times 100$, and a periodic boundary condition was employed to uniquely define the propagation orientation. The wavefront reaches the turnaround point at $t = 0.66 \mu s$, returns to the input cell, highlighted by the dotted line, at $t = 1.10 \mu s$ and starts traveling forward again. A similar calculation was done for step pulses applied at linearly aligned cells at an angle of $\pi/4$ with respect to the $x$-axis to examine the dependence of fundamental frequency on propagation orientation. Figure 1(d) compares the waveforms for these two different input arrangements. The black and red curves show the waveforms monitored at a fixed cell for inputs at an angle of 0 and $\pi/4$, respectively. Both exhibit periodicity, and the period of the black waveform $T_0$ is shorter than that of the red one $T_{\pi/4}$. The fundamental frequency consistently decreases as $\theta$ increases.

When a common step pulse is applied at linearly aligned but finite cells,

![Figure 2](image-url)

**Fig. 2.** Entrainment of oscillation frequency in 2D switch lines. (a) Development of frequency entrainment observed for inputs along a finite-length line, (b) Temporal variation of pulse durations, and (c) Contour map of the line voltages for finite-length linear input cells.
the oscillation properties of the wavefront at the edges of the input line differ from those at other parts. To examine the dynamics, we applied a step pulse at cells located at \((i, 40)\) for \(i \in [100, 3100]\) in a \(3200 \times 80\) 2D switch line. The wavefront is expected to decay more rapidly around the edge of the input line; therefore, the oscillation period is expected to become shorter than that observed at the side. Figure 2 (a) shows the waveforms immediately after the step pulse is input. Note that the wavefront motion possesses linear symmetry with respect to \(x = 1600\). Waveforms monitored at nine different cells along \(y = 35\) are plotted. The waveform at the bottom of the figure is that of the cells adjacent to the edge at \((100, 40)\). On the other hand, the waveform at the top is that of the cells adjacent to the center of the input line located at \((1600, 40)\). Waveforms monitored at seven cells evenly spaced are shown. We can see that the initial oscillation period at the edge of the input line is typically estimated by the pulse train observed in circle \(A\) in Fig. 2 (a), while the pulse train in circle \(B\) exhibits the natural oscillation period at the side of the input line. As expected, the period in \(A\) is shorter than that in \(B\). It is noteworthy that high-frequency motion spreads rapidly to cover all the cells. This is caused by the frequency entrainment established through the finite couplings among neighboring cells. The boundary between natural and entrained oscillation is represented by the red dashed line. The entrained region of cells spreads at a constant speed in the \(x\) direction. As in other pulse-coupled relaxation oscillators [5], the wavefront motions are entrained such that the fundamental frequency increases.

The input line keeps generating oscillating wavefronts that are incoherent with the entrained periodic motions; therefore, the periodic waves at the cells neighboring the input must be disturbed, and the duration of the pulse varies cycle by cycle. Because a high-frequency wavefront develops, the edges of the input line influence the short-term periodicity. The disturbances become smaller at cells farther from the edge. Figure 2 (b) shows the per-cycle pulse duration for each pulse observed at three distinct cells. The orange, blue, and light blue circles correspond to pulse durations monitored at \(x = 100, 400,\) and 1200, respectively. The variance in circle distribution clearly becomes smaller for larger \(x\).

Figure 2 (c) shows a contour map of the line voltages for a \(400 \times 100\) 2D switch line, recorded at five distinct temporal points with an increment of \(0.11\ \mu s\) after frequency entrainment is complete. A step pulse was applied at cells located at \((i, 50)\) for \(i \in [100, 200]\) as shown by the yellow line in the leftmost map. A 2D pulse develops at the edge of the input line and then starts to travel along the input line. The wavefront located at \(P_1\) reaches \(P_5\) in 0.44 \(\mu s\). Moreover, the 2D pulse disappears at the midpoint of the input line due to interaction with that traveling from another edge.

Finally, we consider how the fundamental frequency of oscillating wavefronts depends on the signal-input arrangements. A step pulse is input into cells included in a square with a side of nine distinct values \((N = 1, 2, 3, 5, 7, 9, 11, 13, \) and 15) centered in a \(100 \times 100\) 2D switch line. For all values of \(N\), the wavefront motion finally achieves a steady periodic oscillation.
Figure 3 (a) shows a contour map of the line voltage recorded at six distinct temporal points in a single period for \( N = 13 \). The wavefront starts to travel forward at \( t = 0 \), reaches the turnaround point at \( t = 0.495 \mu s \), and then turns back. Although the wavefront exhibits a complicated motion, every cell oscillates at a unique period due to entrainment. Wavefronts originating near the apexes of the input square exhibit higher frequency than those originating from the sides. Hence, the apex wavefronts dominate the steady-state oscillation. Because the contribution of the apex wavefronts becomes small, the fundamental frequency decreases when \( N \) increases. The red circles in Fig. 3 (b) show this dependence. The frequency decreases very rapidly when \( N \) increases for the present model (about 60% at \( N = 10 \) in comparison with that at \( N = 1 \)).

In conclusion, we characterize the oscillating wavefronts in a 2D switch line using a simplified model of electronic switches. For practical switches such as tunneling diodes, the dynamics become much more complicated. Our observations contribute to securing a solid foundation in preparation for the development of an actual 2D switch line.