Improved Data Model of TR-UWB System Using Multichannel Autocorrelation Receivers Under NBI

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Abstract—Transmitted reference Ultra-wideband (TR-UWB) system in conjunction with autocorrelation receivers (AcR) is a popular low cost and low complexity radio system for low and mid-data rate applications. The scheme is in general robust against multipath fading. At the receiver front-end, however, both the desired UWB signal and narrowband interference (NBI) can enter to the radio receiver in which case several co-terms and cross-terms are produced after correlation. Previous studies provided an approximate data model of TR-UWB under NBI. However, analytically computed covariance matrix based on the approximate data model showed inferior performance to that of the estimated covariance matrix when used to compute bit error rate (BER) by using linear minimum mean squared error (MMSE) method. In this paper we provide an improved data model taking into account some of the neglected factors in the previous data model derivation. Finally, we show that the analytic covariance matrix derived from the improved data model achieves a closely matching BER with that achieved by the estimated covariance matrix.

I. INTRODUCTION

TR signaling is a signaling scheme in which a portion of the transmitted energy is used to sound the channel. The TR signal, composed of a data-carrying signal and a reference signal, is transmitted through a random and time-varying channel. The information on the data carrying signal is coded with respect to the reference signal and at the receiver side the information is extracted by correlating the data-carrying signal with the reference signal. By doing so, the channel estimation stage and the required circuit is avoided to yield low complexity receiver.

The general concept of TR signaling dates back to the 1950’s [1] where the reference signal was required to be locally stored or transmitted through an auxiliary channel. In [1], the reference signal is the copy of the data carrying signal. In [2], a transmitted reference system is described where a reference carrier signal and a modulated version of that reference carrier is sent over a common (ideally identical) random or unknown channel. The basic objective in [2] is to have a reference carrier that serves as an exact reference, or pilot tone, for the received modulated carrier. An implicit or explicit assumption is usually made that the reference and modulated carriers are offset from one another in frequency and/or in time in order that they can be separated at the receiver side.

In the context of UWB, Hoctor and Tomlinson (HT) proposed a TR-UWB scheme, where the transmitted signal is simply a train of pulses consisting of reference pulses and data-carrying pulses [3]. Then data detection is carried out by correlating the received signal with its delayed version. The fundamental idea here is to use the delayed signal as a template in the demodulation block, without requiring any kind of channel estimation. This can be achieved using an AcR. The basic functional unit of an AcR is nothing but a pulse pair correlator whose delay $D$ is matched to the time interval between the reference and data pulse as illustrated in Fig. 1. Among the main driving forces behind the continued research in TR-UWB systems are efficiency, noise suppression and NBI suppression.

The main appealing point for the choice of TR-UWB AcRs over coherent receivers is their low implementation complexity. One drawback of AcR is the performance degradation in the presence of NBI. During the TR-UWB transmission, both the reference pulse and the data pulse are corrupted by noise and NBI. Then the AcR produces co-term and cross-term of noise and NBI in addition to the UWB signal co-term. The cross-terms reduce the signal-to-interference-noise (SINR) and degrade the receiver performance.

Fig. 1. The basic functional unit of an AcR with a delay element $D$ and TR-UWB signaling
essential to evaluate the receiver performance under NBI. In the past data models for TR-UWB signaling in AcR have been developed [4], [5], [6], [7]. In these works, the models in general consider a data component and a noise component which comprise the noise-by-noise co-term and noise-by-data cross-terms. In the presence of NBI, the AcR produces additional cross-terms due to the NBI. Models that take into account this factor is presented in [8], [9], [10]. In [8], a data model for low duty cycle (LDC) type differential transmitted reference (DTR) is presented. In [9], a more general data model of TR-UWB signaling is presented. In particular, the later work and also [11] studied the second-order statistics of the terms forming the data model. According to these studies, the NBI co-term and cross-terms show strong correlation among the AcR channels under certain system design constraints.

Following the observation of strong correlation among the NBI co-terms and cross-terms, linear signal processing was applied to suppress the interference in which a significant NBI suppression [10] was reported. In this paper we provide an improved data model in particular the NBI co-term. By using the improved data model, we compute the covariance matrix of received signal. We apply the newly computed covariance matrix to perform MMSE data combining. The resulting BER matches closely to the BER obtained from the MMSE combining that uses an estimated covariance matrix.

The rest of the paper is organized as follows. In Section II, a brief description of the system and derivation of the approximate data model is presented. In Section III derivation of the improved data model and the covariance computation is presented. In section IV simulation result and discussion is presented. Section V presents conclusive remarks.

II. APPROXIMATE DATA MODEL

The TR-UWB signaling and its detection by multichannel AcR works as follows. As shown in Fig. 1, the delay line in an AcR front-end is matched to the lag between reference and data pulse. The transmitted signal of the original HT TR-UWB scheme is a train of pulses consisting of reference pulses and data-carrying pulses [3]. The basic building block of such signal is the doublet, which consists of two pulses and is called a frame. One of the pulses in the frame has always fixed polarity and serves as a reference, while the second pulse’s polarity is modulated according to the data symbol and the spreading code. One or more frames can be grouped together to form a chip. Therefore, a given data symbol can be represented either by a group of chips or frames in order to collect more energy per symbol at the receiver. Fig. 2 illustrates the UWB signal described above. Its analytic representation is given by

\[ s(t) = \sum_{i=-\infty}^{\infty} \sum_{j=1}^{N_{ch}} \sum_{q=1}^{N_f} \left[ \tilde{w}(t - iT_{sym} - jT_{ch} - qT_f) + d[i]b_j \tilde{w}(t - iT_{sym} - jT_{ch} - qT_f - D_j) \right] \]

where \( i \) is the data index, \( q \) is the frame index, \( j \) is the chip index, \( N_{ch} \) is the number of chips in a symbol, and \( N_f \) is the number of frames in a chip. \( D_j \) is the delay of the pulse doublet and changes from chip to chip. \( T_f \) is a fixed duration between successive frames, \( T_{ch} = N_f T_f \) is the chip period, and \( T_{sym} = N_{ch} T_{ch} \) is the symbol period. Finally, \( \tilde{w}(t) \) is the transmitted UWB pulse. The second pulse is modulated by data \( d[i] \in \{+1, -1\} \) and the spreading code \( b_j \in \{+1, -1\} \). For the transmission, we consider a simple and generic UWB channel model \( h(t) \) of an indoor environment. A non-line-of-sight (NLO), exponentially decaying channels, and with a homogeneous Poisson process of ray-arrivals is considered.

The transmitted pulse \( \tilde{w}(t) \) in (1) is considered to represent the transmitted UWB pulse shape, effects of TX and RX antennas, up to a front-end filter at the receiver input. Then \( g(t) = \tilde{w}(t) * h(t) \) becomes the channel response for a UWB pulse. The received UWB signal \( r(t) \) is written as

\[ r(t) = \sum_{i=-\infty}^{\infty} \sum_{j=1}^{N_{ch}} \left[ g(t - t_{ij}) + d[i]b_j g(t - t_{ij} - D_j) \right] \]

where \( t_{ij} = iT_{sym} + jT_{ch} \) is the timing of chip \( j \) in symbol \( i \). A TR-UWB transmitted signal with the signaling scheme as described above can be received and detected by an AcR with a bank of delays [3]. The front-end of such a receiver can be viewed as parallel operating pulse pair correlators hence the name multichannel AcR. The delays in the correlators are matched to the delays of the delay hoping (DH) code used in the transmitted signal. A case where the AcR delays are short w.r.t. the integration interval is considered here.

The property of the multichannel AcR output is analyzed by considering the following input signal \( \hat{r}(t) \)

\[ \hat{r}(t) = r(t) + n(t) + \beta(t) \]

where \( \beta(t) \) is an NBI signal, \( n(t) \) is additive white Gaussian noise (AWGN) and \( r(t) \) is an UWB signal. The NBI is modeled as a block spectrum with bandwidth \( W_{\beta} \), signal power of \( P_{\beta} \) with carrier frequency \( f_{\beta} \). The noise \( n(t) \) has a power spectral density (PSD) determined by the front-end filter \( f_{rx}(t), S_n(f) = N_0/2 |f_{rx}(f)|^2 \), where \( f_{rx}(f) \) is the Fourier transform of \( f_{rx}(t) \) and \( N_0/2 \) is the double sided PSD of the input noise.

For the sake of analysis, the NBI is more specifically characterized as follows,

\[ \beta(t) = \sqrt{2} Re \{ x(t) e^{j(2\pi f_{\beta} t + \theta)} \} \]

where \( x(t) \) is the complex equivalent baseband signal of the NBI. Assuming the power spectral density (PSD) to take a Note that only one frame per chip is shown in (2)
rectangular shape, the autocorrelation (ACF) is given by

\[ R_{\beta\beta}(\tau) = E\{\beta(t + \tau)\beta(t)\} = P_{\beta} \cos(2\pi f_{\beta}\tau) \text{sinc}(W_{\beta}\tau). \]  

(5)

Given the input signal (3), we can write the \( k \)th correlator output for the \( j \)th chip and \( r \)th symbol as

\[ \left( y_{j}[i] \right)_{k} = \int_{t_{ij}}^{t_{ij}+T_{j}} \tilde{r}(t+D_{k})\hat{r}(t)dt \]  

(6)

where \( T_{j} \) is the integration interval chosen in accordance with the channel delay spread. The value of \( T_{j} \) is often chosen as twice of the room mean square (rms) channel delay spread. A vector \( y_{j}[i] \) of samples is acquired for each chip \( j \) of symbol \( i \), consisting of samples \( \{y_{j}[i]\}_{k}, k=1,\ldots,N_{cr} \) for each of the \( N_{cr} \) correlator channels. Replacing the received signal \( \hat{r}(t) \) in (6) with (3), we obtain

\[ \left( y_{j}[i] \right)_{k} = \int_{t_{ij}}^{t_{ij}+T_{j}} (r(t+D_{k}) + \beta(t+D_{k}) + n(t+D_{k})) \times (r(t) + \beta(t) + n(t))dt. \]  

(7)

The above equation yields nine product terms summarized in Table I from which the approximate data model is derived.

The UWB-by-UWB product per chip can be modeled as a contribution comprising a bias term \( g_{j} \) plus a linearly data dependent term \( d[i]h_{j} \) [4], [6], [7] (see Appendix). This model requires the assumption that ISI is negligible. The dominant NBI term is represented by an NBI signature \( c \) scaled by the instantaneous power of the NBI. This property comes from the ACF of the NBI in (5). The product terms \( v^{(n,r)} \) and \( v^{(r,n)} \) are zero-mean random variables since \( n(t) \) is a zero-mean Gaussian process. Furthermore it is assumed that the noise ACF is zero for the ACR lags selected, \( R_{in}(D_{k}) = 0 \) and hence \( v^{(n,n)} \) is also a pure noise contribution.

Taking into account the randomness and zero-mean property of the noise-by-NBI, noise-by-UWB, noise-by-noise and UWB-by-NBI terms, they can be combined and taken as random contribution in a noise vector \( v[iN_{ch} + j] \). The approximate data model [8], [9], [10] is thus given by

\[ y_{j}[i] = h_{j}d[i] + g_{j} + c\phi_{x}[iN_{ch} + j] + v[iN_{ch} + j]. \]  

(8)

The effects of the UWB radio channel and the radio front end filter on the NBI can be incorporated as a scaling factor in the NBI power envelope.

### III. Improved Data Model

The NBI-by-NBI product, which is approximated as an amplitude modulated signature function \( c\phi_{x}[iN_{ch} + j] \), is the most dominant term of the correlation terms. Thus, any improvement on the modeling of this term can have significant impact on the overall data model. To identify the neglected terms in the approximation, we review the derivation of the NBI-by-NBI term as follows

\[ \left( y_{j}^{(\beta)}[i] \right)_{k} = \int_{t_{ij}}^{t_{ij}+T_{j}} \beta(t+D_{k})\beta(t)dt. \]  

(9)

By substituting (4) into (9), and using trigonometric identities, we obtain

\[ \left( y_{j}^{(\beta)}[i] \right)_{k} = \left( \int_{t_{ij}}^{t_{ij}+T_{j}} \cos(2\pi f_{\beta}D_{k})\text{Re}\{x(t)x^{*}(t+D_{k})\}dt \right) + \left( \int_{t_{ij}}^{t_{ij}+T_{j}} \text{Re}\{\Lambda\cos(2\pi f_{\beta}[2t+D_{k}]+2\theta)dt \right) - \left( \int_{t_{ij}}^{t_{ij}+T_{j}} \text{Im}\{\Lambda\sin(2\pi f_{\beta}[2t+D_{k}]+2\theta)dt \right). \]  

(10)

where \( \Lambda = x(t)x(t+D_{k}) \). The above equations can be interpreted as follows. The first term in (13) consists of the NBI signature vector \( c \), which is simply the normalized expectation of (9),

\[ (c)_{k} = \frac{1}{T_{j}P_{\beta}} \int_{t_{ij}}^{t_{ij}+T_{j}} R_{\beta\beta}(D_{k})dt \approx \cos(2\pi f_{\beta}D_{k})\text{sinc}(W_{\beta}D_{k}) \]  

(11)

and the signal \( \phi_{x}[iN_{ch} + j] \), which modulates the signature waveform corresponding to the instantaneous power of the NBI during the integration window,

\[ \phi_{x}[iN_{ch} + j] \approx \int_{t_{ij}}^{t_{ij}+T_{j}} |x(t)|^{2}dt. \]  

(12)

In the approximate model, only the second part of 9 is considered. By rewriting (10) further, we obtain

\[ \left( y_{j}^{(\beta)}[i] \right)_{k} = \cos(2\pi f_{\beta}D_{k})\int_{t_{ij}}^{t_{ij}+T_{j}} \text{Re}\{x(t)x^{*}(t+D_{k})\}dt \]  

\[ + \cos(2\pi f_{\beta}D_{k})\int_{t_{ij}}^{t_{ij}+T_{j}} \text{Re}\{|\Lambda|e^{i(4\pi f_{\beta}t+\theta')}\}dt \]  

\[ - \sin(2\pi f_{\beta}D_{k})\int_{t_{ij}}^{t_{ij}+T_{j}} \text{Im}\{|\Lambda|e^{i(4\pi f_{\beta}t+\theta')}\}dt \]  

(13)

where

\[ \theta' = 2\theta + \angle \{x(t)x(t+D_{k})\}. \]  

(14)

The last two terms in (13) can be written in more compact form as follows

\[ c(\text{Re}\{\phi_{x}[iN_{ch} + j]\}) + s(\text{Im}\{\phi_{x}[iN_{ch} + j]\}) \]  

(15)
where $\phi_k[iN_{ch} + j] = \mathcal{F} \{ |x(t)x(t + D_k)| \} e^{-j\theta^t}$ and $\mathcal{F}$ is a short time Fourier transform for the integration timings between $t_{ij}$ and $t_{ij} + T_I$ at frequency $2f_B$. The vector $s$ defines a new NBI signature phase-shifted to $c$, where $(s)_k = \sin(2\pi f_B D_k)$ for $k = 1, 2, ..., N_{ch}$.

Finally, the AcR output for the NBI-by-NBI co-term is written as

$$y^{(b)}_j[i] = c\phi_x[iN_{ch} + j] + c(Re\{\phi_x[iN_{ch} + j]\}) + s(Im\{\phi_x[iN_{ch} + j]\}). \quad (16)$$

The above result will be a basis for the improved data model given bellow

$$y_j[i] = h_j[d[i] + g_j + c(\phi_x[iN_{ch} + j] + Re\{\phi_x[iN_{ch} + j]\}) + s(Im\{\phi_x[iN_{ch} + j]\}) + v[iN_{ch} + j]. \quad (17)$$

IV. Covariance Matrix

Computation of the covariance matrix of the received data matrix is an important step in the design of linear combiner for instance MMSE. The numerical approach to compute the covariance matrix, we independently generate the covariance matrix contributions stated in the Table I. Assuming the terms are uncorrelated, the overall covariance matrix is given by the sum of each covariance matrix. The covariance of these terms including the approximate model of NBI-by-NBI term can be computed from the analytic expressions in [10] (see the Appendix). Since the improved data model has modification only in the NBI-by-NBI co-term we limit the analysis to this term.

A. NBI-by-NBI co-term

Let’s define covariance matrix of a given matrix $Y$ as $cov\{Y\} = E\{YY^T\} - E\{Y\}E\{Y\}^T$. Then, the covariance matrix of the matrix containing only the NBI-by-NBI co-term of chip $j$ is written as

$$cov\{Y^{(b)}_j\} = cE((\phi_x[j])^2) - cE(\phi_x[j])^2 \quad (18)$$

where $Y^{(b)}_j = [y_j[1]y_j[2]...y_j[N_d]]$ and $N_d$ is number of symbols. The first and second moments of $\phi_x[j]$ are given by

$$E\{\phi_x[j]\} = P_B T_I \quad (19)$$
$$E((\phi_x[j])^2) = \kappa(P_B T_I)^2 \quad (20)$$

where the constant $\kappa$ depends on the ratio of the integration interval and the bandwidth reciprocal of the NBI. It is bounded between $0 < \kappa \leq 1$; approaching $\kappa = 1$ for $T_I \ll 1/W_B$. In a similar fashion, the covariance matrix for (17) follows,

$$cov\{Y^{(b)}_j\} = cE((\phi_x[j])^2) + cE(Re\{\phi_x[j]\})^2 + \kappa(P_B T_I)^2 \quad (21)$$

The other co- and cross terms such as UWB-by-UWB, UWB-by-NBI, UWB-by-noise, noise-by-noise are computed in [10]. We have listed the results in the Appendix section for reference. Although in terms of power, the NBI-by-NBI co-term is the most significant one. The cross terms by NBI can also become significant since they are proportional to the amount of NBI.

B. MMSE Combining

To obtain the decision statistic, we need to combine the received data matrix with an appropriately designed weighting vector. The straightforward solution is combining the data matrix with the spreading vector $b$ which coherently combines the $N_{ch}$ data chips. The test statistic for symbol $i$ is then given by

$$z[i] = \sum_{j=1}^{N_{ch}} b_j(y_j[i] - \hat{g}) \quad (22)$$

An estimate of the bias component $\hat{g}$ should be subtracted before forming the decision variable. The data model presented in (17) is linear. Hence a weighting vector designed based on the linear property can suppress interference and achieve better result. A general linear combining to obtain the test statistic for symbol $i$ is given by

$$z[i] = w^T(y[i] - \hat{g}) \quad (23)$$

where $w$ is a linear combiner. To solve an optimum linear combiner in MMSE sense, we formulate it as follows

$$w_{MMSE} = \arg \min_{w} E\{(d_i - w^T y[i])^2\} = (hh^T + R)^{-1} h, \quad (24)$$

where $R$ is the autocorrelation matrix of all noise and NBI interference terms and $h$ collects the channel vectors $h_j$. The transmitted data symbols are considered to possess unit variance $\sigma_d^2 = 1$. The key parameter to implement the MMSE solution is to obtain $R$. As we stated before, for analyzing the receiver performance under NBI, $R$ can be computed from the given analytic expressions. However this requires prior knowledge of all the involved signals and transmission channel parameters. In this case, $R$ can be estimated by using some training symbols and adaptive techniques. In the result section we discuss the BER obtained by the MMSE combiners from the approximate and improved data models. Furthermore we compare both results with the BER obtained by using estimated covariance matrix.

V. RESULTS AND DISCUSSION

In order to evaluate the improved data model, we performed BER simulation by using MMSE combiner and the despreader. The despreader performs as stated in (22). The MMSE combiner is computed analytically from the improved data model and is used to combine the received data matrix. For comparison, the performance of the MMSE combiner by using the approximate data model is also included. The simulation is performed as follows.
A. Simulation

To generate a UWB pulse, a second derivative Gaussian monocycle, with an effective pulse-width of $\tau_m = 0.29$ ns is used. Two of such UWB pulses, where one is data bearing and another one reference, represent a doublet or frame. The TR-UWB system considered consists of one frame per symbol with 1.3 ns delay between the reference and data pulse. A symbol period of $T_{sym} = 50$ ns is considered. On the receiver side four correlators with lags $D_k \in \{1.3, 1.6, 2.9, 3.1\}$ are employed. An integration interval $T_j = 20$ ns is considered to collect the received signal energy. The transmission channel assumed is a NLOS, exponentially decaying channel, with $\tau_{rms} = 10$ ns. Since we have a single frame transmission, the amplitude spreading code is a four elements vector with the first element to be 1 and the rest zeros. Each data burst of the transmission represents 512 bits. The simulation result is averaged over 100 independent channel realizations. The NBI generated is an OFDM signal with a center frequency $f_\beta = 5.125$ GHz, 48 sub-carriers with QPSK modulation, and with a total bandwidth of 16 MHz.

B. Discussion

1) BER performance results: We compare performances of 4 combiners namely; despreader, analytic MMSE based on the approximate model, Analytic MMSE based on the improved data model and MMSE based on estimated covariance matrix. Fig. 3 illustrates the average BER as a function of SIR for each combiner. We measure the relative performance gain of the combiners compared to that of the despreader at $10^{-2}$ BER. The analytic MMSE, based on approximate model shows approximately 7 dB gain. The analytic MMSE combiner with the improved data model and the simulated MMSE, achieve approximately 17 dB and 18 dB respectively. The small deviation between the simulated MMSE and the analytic MMSE based on the improved data model can be accounted to the approximations in the NBI cross-terms analysis and the UWB channel. Slight variation in the UWB channel during each realization can contribute for the deviation. In the simulated MMSE, this effect is taken care of while estimating the MMSE combiner from the received data matrix during each channel realization. However, in the analytic solution, only an average knowledge of the covariance matrix of the UWB-by-NBI term and the channel is considered. Nevertheless, the results obtained confirm that the improved data model shows closer performance with that of the simulated MMSE. Such model can be useful to study the performance the receiver under NBI and investigate different linear NBI suppression schemes.

VI. Conclusion

An improved data model for DH TR-UWB system using a multichannel AcR, taking into account NBI, has been derived. The NBI is modeled as a block spectrum which could be representative for different types of narrowband interferences including single-tone interference. The NBI coefficients are represented by two components. One component is a column vector of a scaled NBI signature, a cos-sequence with fixed phase. The second component consists of additional scaled NBI signature composed of two orthogonal components, which corresponds to a cos-sequence of a random phase. The improved data model is validated by computing the BER based on MMSE combining and comparing the result with simulation. This demonstrates the usefulness of the data model to generate the covariance matrix analytically and study the impact of interference numerically. Furthermore by using the data model, different NBI interference suppression mechanisms for instance linear signal processing, can be investigated.

REFERENCES


**VII. APPENDIX**

The key purpose of this Appendix is just for the sake of completeness as the information will be used to compute the analytical covariance matrix. Full derivation of the terms is given in [10].

A. UWB-by-UWB

This term comprises the desired UWB signal to be detected. The linear model of this term for chip \( j \) of the \( k^{th} \) correlator and symbol \( d \), is written as

\[
(g_j)_k + d(h_j)_k = \int_0^{T_j} [g(\xi) + db_j g(\xi - D_j)] \times [g(\xi + D_k) + db_j g(\xi - D_j + D_k)] d\xi
\]

where the average channel is given by

\[
E \{ (h_j)_k \} = b_j \int_0^{T_j} E \{ g(\xi) g(\xi - D_j + D_k) \} d\xi
\]

\[
\approx b_j \int_0^{T_j} P_h(\xi) \phi_w(D_k - D_j) d\xi
\]

\[
\approx b_j E \phi_w(D_k - D_j)
\]

and \( P_h(t) \equiv E \{ h^2(t) \} \) is the average power delay profile of the channel, and \( \phi_w(\tau) \) is the autocorrelation function of the UWB pulse prototype \( w(t) \). For the matched AcR channel

\[
E \{ (h_j)_j \} \approx b_j E
\]

B. UWB-by-NBI

The UWB-by-NBI cross product for the correlator \( k \) and chip \( j \) is given by

\[
\nu^{(r,\beta)}_{j,k} = \int_0^{T_j} \left( [g(\xi) + db_j g(\xi - D_j)] \beta(\xi + jT_{ch} + D_k)
\]

\[
+ \beta(\xi + jT_{ch}) [g(\xi + D_k) + db_j g(\xi - D_j + D_k)] \right) d\xi.
\]

where the time-variable has been substituted with \( \xi = t - jT_{ch} \) and synchronization of the integration start is assumed. Averaged over the random data, the variance of the AcR channels becomes

\[
E \{ \text{var} \nu_{j,k}^{(r,\beta)} \} \approx 8EP_{\beta}S_w(f_{\beta}) \cos^2(2\pi f_{\beta} D_k)
\]

where \( E = \int_0^{T_j} P_h(t) dt \).

1) NBI-by-noise: The cross-correlation of the NBI-by-noise product terms is given by

\[
E \{ \nu_{j,k}(n,\beta) \nu_{j,k'}(n,\beta) \} \approx 4P_{\beta} S_n(f_{\beta}) T_I \cos(2\pi f_{\beta} D_k) \cos(2\pi f_{\beta} D_k')
\]

where \( S_n(f_{\beta}) \) is the noise power spectral density evaluated at \( f_{\beta} \).

C. UWB-by-noise component

The variance of the UWB-by-noise component is written as

\[
\text{var} \nu_{j,k}^{(r,n)} \approx 4E \int_{-\infty}^{\infty} S_w(f) S_n(f) df.
\]

The integral is equivalent to \( N_0 W_{\pi w} \), where \( W_{\pi w} \) is an equivalent bandwidth of the filtered UWB pulse \( \bar{w}(t) \).

D. Noise-by-noise component

The co-variance of the noise-by-noise component is given by

\[
\text{var} \nu_{j,k}^{(n,n)} \approx T_I \int_{-\infty}^{\infty} S_n^2(f) df = T_I \frac{N_0^2}{4} \int_{-\infty}^{\infty} |F_{\bar{w}}(f)|^4 df.
\]

Expressing the integral by an equivalent bandwidth \( 2W_{\bar{w}} \), we can write \( \text{var} \nu_{j,k}^{(n,n)} \approx T_I W_{\bar{w}} \frac{N_0^2}{2} \).