Accepted Manuscript

MAC and application level performance evaluation of beacon message dissemination in DSRC safety communication

Xiaoyan Yin, Xiaomin Ma, Kishor S. Trivedi

PII: S0166-5316(13)00113-2
DOI: http://dx.doi.org/10.1016/j.peva.2013.10.001
Reference: PEVA 1739

To appear in: Performance Evaluation

Received date: 25 September 2012
Accepted date: 2 October 2013

Please cite this article as: X. Yin, X. Ma, K.S. Trivedi, MAC and application level performance evaluation of beacon message dissemination in DSRC safety communication, Performance Evaluation (2013), http://dx.doi.org/10.1016/j.peva.2013.10.001

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.
MAC and Application Level Performance Evaluation of Beacon Message Dissemination in DSRC Safety Communication

Xiaoyan Yin\textsuperscript{a,1}, Xiaomin Ma\textsuperscript{b}, and Kishor S. Trivedi\textsuperscript{a}

\textsuperscript{a}Department of Electrical and Computer Engineering, Duke University, Durham, USA
\textsuperscript{b}Department of Engineering and Physics, Oral Roberts University, Tulsa, USA

Emails: xy15@duke.edu, xma@oru.edu, kst@ee.duke.edu

Abstract—In this paper, an analytic model is proposed for both MAC and application-level performance evaluation of periodic beacon message dissemination in the dedicated short range communications (DSRC) system on highways. Each vehicle can periodically generate beacon messages, which contain the information including position, velocity and direction, etc. Out-dated message is replaced by the newly generated message. In order to develop a tractable analytic approximation, we first develop a semi-Markov process (SMP) model for the tagged vehicle to capture the periodic message generation, outdated message replacement, channel contention and backoff behavior in IEEE 802.11 broadcast ad hoc networks. Further, an SMP model with an absorbing state is constructed to derive the message service time distribution. Fixed-point iteration method is utilized to obtain the converged solutions to resolve interactions between the SMPs of different vehicles. Both MAC-level and application-level performance metrics are derived. MAC-level analytic-numeric results are verified through extensive simulations. Application-level analytic-numerical results are evaluated subsequently to provide insights on network parameter settings. To ensure the quality of service (QoS) for a VANET application, application requirements in terms of application-level metrics are specified. Three typical safety-applications are evaluated in order to assess whether their application requirements can be satisfied or not.

Keywords: application-level performance; dedicated short-range communications; Laplace–Stieltjes transform; MAC-level performance; periodic beacon message; semi-Markov process.

1. INTRODUCTION

Transportation safety is one of the most critical features addressed in Intelligent Transportation System (ITS). In wireless vehicular communications, various safety applications such as collision avoidance and safety warnings are expected to be enabled to prevent and reduce traffic accidents. To support these safety applications, periodic beacon message, \textit{i.e.}, the basic safety message (BSM) \cite{15}\cite{16}, is widely used in dedicated short range communications (DSRC). A beacon message contains information related to the state of vehicle (\textit{e.g.}, position, velocity and direction) and such messages are periodically exchanged with nearby vehicles in a broadcast fashion. Based on such information, various safety applications are able to assist drivers to take appropriate actions to prevent collisions when emergencies occur. Hence, beacon
messages are required to be transmitted both in a timely and reliable manner to ensure the quality of service (QoS) for various applications.

1.1. Related Work

The MAC-level performance of the beacon message dissemination in DSRC system has been studied in [2][3][4] based on discrete event simulations. Analytic models are more suited in comparing design alternatives and exploring a large parameter space. Several analytic models are developed in [5][6][7] to characterize the MAC layer behavior of beacon message transmission and important performance metrics are evaluated. However, Ma [5] used Poisson arrivals to approximate the periodic generations of beacon messages (i.e., routine message), which leads to inaccuracies. Vinel [6] considered the deterministic behavior of beacon generations by modeling each vehicle as a D/M/1 queuing system. Infinite queue size is assumed by them where out-dated information replacement is not considered. Bastani [7] accounted for the periodic nature of beacon message generations and new message canceling out old message phenomena, which is more reasonable and practical. However, their model does not accurately capture the periodic beacon message generation since it separates the periodic message generation from the message channel contention and transmission behavior, although these behaviors are closely correlated. According to [16], beacon messages are periodically generated to broadcast vehicle’s status information. However, for the model presented in [7], the time interval between the previous message finishing its transmission and the generation of the next message is fixed beaconing period, which implies that the message generation is no longer periodic since a message transmission time varies according to the channel’s status. Hence, model in [7] cannot precisely capture the periodic beacon message generations. In addition, all the above models are based on discrete-time Markov chain (DTMC) and ignore the continuous time system behavior, which leads to approximations.

Even though the MAC-level performance metrics play an important role for understanding the packet transmission behavior and evaluating the efficiency of a protocol, the application-level performance metrics are also essential to be addressed to ensure the QoS of safety and non-safety applications because performance requirements are typically given in terms of application-level metrics as opposed to packet-level metrics. Efforts have been made in previous work [19][20][21] to characterize the application-level performance metrics for specifying the application performance requirements. Bai et al. [19] characterized Region of Interest (ROI) of VANET applications into three qualitative levels: long, medium and short. In addition, the application-level latency and T-window reliability (TWR) (i.e., the probability to successfully receive at least one packet during tolerance time window T) were also proposed in [19] to guide performance requirements in terms of application-level metrics. An et al. [20] proposed that the application requirements can be specified as awareness range and awareness probability. The awareness probability generalized the idea of TWR as the probability to successfully receive at least n packets in the
tolerance time window $T$. Awareness range is defined as the maximum distance at which the awareness probability is greater than or equal to a certain threshold. The authors in [21] defined the invisible neighbor problem and proposed to minimize the total number of invisible neighbors over a certain time window to ensure the QoS of the vehicle’s safety applications.

### 1.2. Our Contributions

In this paper, we extend the work in [8][9] and develop a detailed and accurate interacting semi-Markov Process (SMP) model [10] to incorporate periodic generations as well as out-dated information replacement behavior for beacon messages. Compared to the M/G/1 FCFS (first come first served) queue model in our previous work [9], the generation and service of beacon messages in each vehicle in this paper is modeled by a D/G/1/1 LCFS (last come first served) queue with preemption as shown in Fig. 1(a). In this notation, “D” represents that the message inter-arrival time is deterministic. “G” represents that the service time has an arbitrary distribution. The first “1” represents that there is only one server. The second “1” represents that the queue size is one (i.e., no queuing). This queue is a LCFS preemption queue because out-dated beacon message will be replaced by the next message to ensure that the message to be sent in a vehicle is always up to date. Therefore, the overall model is a set of interacting D/G/1/1 LCFS queues, one for each vehicle. The interaction is that the server is shared as it is the contention medium/channel. To develop a tractable analytic approximation, we use a semi-Markov process model to capture the shared server’s behavior from one tagged vehicle’s perspective, which incorporates the behavior of this tagged vehicle and the influences from other vehicles as shown in Fig. 1(b). This SMP model is accurate and comprehensive since it closely follows the operational flow specified by DCF in IEEE 802.11 shown in Fig. 2. In addition, SMP model with absorbing state is constructed to derive the message service time distribution via Laplace–Stieltjes transform (LST). Due to the interactions between the SMP models of different vehicles, fixed-point iteration [11] method is utilized to obtain the converged solution in steady state. MAC-level performance metrics including mean delay, packet delivery ratio (PDR), packet reception ratio (PRR) and normalized channel throughput are derived. The proposed approximation is verified through extensive simulations and compared with previous models for Poisson packet arrivals and with infinite queue scenarios (i.e., M/G/1 FCFS queue) [9] and Bastani’s model in [7].

For application-level performance evaluation, we adopt the definitions of Region of Interest, application-level latency, $T$-window reliability, awareness probability and invisible neighbors from previous work to characterize the application performance requirements. The corresponding analytic results are derived based on the newly proposed interacting SMP model for periodic beacon messages. Insights are provided on network parameter settings to satisfy performance requirements for various applications. Furthermore, three typical safety applications are analyzed to assess whether their performance requirement can be satisfied or not under a given DSRC network parameters setting.
The key contributions of this paper are six-fold: 1) Periodic beacon message generation and out-dated message replacement behavior are incorporated into a detailed and accurate analytic model, which closely follows the operational flow specified by DCF in IEEE 802.11; 2) Beacon message service time distribution is first derived using Laplace–Stieltjes transform, based on which the mean service time is computed; 3) Normalized channel throughput is derived for better understanding of channel conditions under various input parameters; 4) Analytic results for a vehicle with periodic beacon message generation and out-dated message replacement are compared to those for a vehicle model in [9] with Poisson message arrivals and an infinite queue and Bastani’s model [7]; 5) Application-level performance metrics are derived based on the newly proposed interacting SMP model; 6) Three typical safety-applications are evaluated to assess whether their performance requirement can be met or not under a given DSRC network parameters setting.

This paper is organized as follows. Section 2 illustrates the system behavior in 802.11 MAC layer protocol and assumptions used in the system model. Section 3 presents the detailed analytic models and the fixed-point iteration algorithm. MAC and application-level performance metrics are derived in Section 4. Section 5 presents numerical results for MAC-level and application-level performance metrics. Three typical safety-applications are evaluated in Section 6. Conclusions are given in the last section.

2. SYSTEM DESCRIPTION AND ASSUMPTIONS

2.1. Broadcast Protocol

Fig. 2. Flow chart for DCF function
From [1], we know that the DSRC adopts IEEE 802.11 MAC layer specification based on the carrier sense multiple access with collision avoidance (CSMA/CA) with minor modifications. In the 802.11 MAC layer protocol [12], distributed coordination function (DCF) is the primary medium access control technique for broadcast services. Fig. 2 describes in detail the basic access mechanism of DCF for broadcast in the context of the safety communication.

According to Fig. 2, each vehicle in the network can generate messages and compete for the channel resource to transmit the message. If a vehicle does not have any message to transmit, it will wait for a packet to be generated. Then, for a newly generated packet, the vehicle senses the channel activity before it starts to transmit the packet. If the channel is sensed idle for a time period of distributed inter-frame space (DIFS), the packet can be directly transmitted. Otherwise, the vehicle continues to monitor the channel until channel is detected to be idle for DIFS time period. Subsequently, according to the collision avoidance feature of the protocol, the vehicle goes through the backoff process before transmitting the packet. It generates an initial random backoff counter from a uniform probability mass function (pmf) over the range [0, CW], where CW represents the contention window. The backoff time counter is decreased by one if the channel is sensed idle for a time slot of duration $\sigma$. Otherwise, the counter is frozen and reactivated when the channel is sensed idle again for more than DIFS duration. The packet is transmitted as soon as the backoff counter reaches zero. After this packet is transmitted, if there is no packet left in this vehicle, the process will start over again and the vehicle will wait for the next packet to be generated. Otherwise, if there are packets left, the vehicle repeats the procedure starting with sensing the channel for DIFS duration and going through the backoff procedure before transmitting the next packet. According to the protocol, a vehicle must go through the backoff process between two consecutive packet transmissions even if the channel is sensed idle for the duration of DIFS time for the second packet.

2.2. System Assumptions

Several assumptions are made in the broadcast system to produce analytically tractable yet a high fidelity model. The vehicular ad hoc network is considered to be one-dimensional (1-D) for traffic on highway [9]. The number of vehicles in a line is assumed to be Poisson distributed with parameter $\beta$ (vehicle density) [27], i.e., the probability $P(i, l)$ of finding $i$ vehicles in a lane of length $l$ is given by:

$$P(i, l) = \frac{(\beta l)^i}{i!} \cdot e^{-\beta l}$$

In addition, we assume all vehicles have the same transmission range, receiving range and carrier sensing range $R$ to simplify the analysis. Furthermore, an out-date beacon message is replaced by the new message. In other words, if the previous message has not been transmitted when the new message is generated in a vehicle, the previous message will be replaced by the next packet. This assumption is made
since a beacon message contains information such as the position, the velocity and the direction, etc., that needs to be updated periodically. Therefore, different from event-driven safety messages that are too important to be replaced, no queueing of messages is needed for the periodic beacon messages. Channel shadowing or fading, vehicle mobility and capture effect of transmissions are not considered in this paper.

3. ANALYTIC METHODS

3.1. Overall Method Description

The overall system behavior is that of multiple vehicles competing for the channel resource to transmit their own beacon messages. Due to such interactions between vehicles, the whole system can be captured by a Generalized Semi-Markov Processes (GSMP) [17][18], which is a set of interacting semi-Markov processes. Even assuming that all sojourn times are exponentially distributed, a single GSMP model to analyze system behavior faces state explosion problem when the number of vehicles is large. For example, suppose there are 100 vehicles in the system to be analyzed, and each vehicle’s maximal backoff counter is 15, the GSMP model will contain at least $15^{100}$ states (can be more due to channel sensing and backoff frozen states) to capture the whole system behavior. First, there is no known analytic solution to a GSMP; it can be either solved using discrete-event simulation or phase-type expansion for all non-exponential distributions, in which case the state space will get even bigger. We use model decomposition to develop a tractable analytic model of such a complex system, denoted as ISP (Interacting Stochastic Processes). This newly proposed ISP model and GSMP model are both based on a set of interacting semi-Markov processes. The distinction is that: in GSMP model, the interactions are at the level of events while for the ISP model, solutions of one SMP provide parameters to the others. This decomposition is what makes our proposed ISP approach analytically tractable while GSMP is not.

![Fig. 3. Import graph for the overall method](image)

The import graph for the proposed ISP approach is shown in Fig. 3 to illustrate the interactions between models and model parameters. First, we develop a detailed SMP model, referred as ISMP model, in Section 3.2 that directly captures the channel contention and backoff behavior from a single vehicle’s perspective. The influences from other vehicles are incorporated into this single vehicle SMP model through four model parameters ($P_f$, $p_b$, $q_b$, $r_b$ in Fig. 4) remaining to be determined. By solving the ISMP model, we can obtain the fully symbolic solution for the steady-state probability that a single vehicle transmits $\pi_{TX}$. Based on a single vehicle’s behavior, three model parameters ($p_b$, $q_b$, $r_b$ in Fig. 4) that
capture the degree of channel contention from multiple vehicles are derived. The one remaining model parameter ($P_f$ in Fig. 4) is related to the out-dated message replacement behavior, and hence the message service time distribution needs to be derived first to determine whether a message will be replaced by the next one or not. We propose Laplace–Stieltjes transform method over a modified SMP model, referred as SMPA model, in Section 3.3 to derive a formula for this service time distribution and subsequently derive the related model parameter. Fixed-point iteration method is used to obtain converged numerical solutions for the overall model in Section 3.4.

3.2. SMP Model

The behavior of a tagged vehicle is characterized using the irreducible SMP model in Fig. 4. The channel sensing, backoff and transmission behavior matches well with the flow chart shown in Fig. 2. The tagged vehicle is in idle state if there is no packet. After a packet is generated, the vehicle senses channel activity for DIFS time period, which is represented by state $CS_1$. If the channel is detected not busy during this period (with probability $1-q_b$), the vehicle goes from idle state to TX state, which means that a packet is transmitting. Otherwise, the vehicle will defer until channel is idle for DIFS duration represented by state $DCS$. Such deference behavior for the tagged vehicle includes two parts: waiting for the current packet in the channel finishing transmission and waiting for subsequent transmissions if any from other neighbors within its receiving range. The self-loop for state $DCS$ represents the phenomena in Fig. 6 that the tagged vehicle (vehicle $B$) waits for the current packet (from vehicle $A$) in the channel finishing transmission, and then senses the channel for DIFS time, which seizes the transmission from another vehicle (vehicle $C$) and leads to further deference for backoff procedure of vehicle $B$. The probability that the tagged vehicle detects another neighbor’s transmission during DIFS time is denoted as $r_b$. If no other neighbors’ transmission is detected, the tagged vehicle will start backoff procedure and randomly choose a backoff counter in the range $[0, W-1]$, where $W=CW+1$ is the backoff window size. The backoff counter will be decreased by one if the channel is detected to be idle for a time slot of duration $\sigma$ (with probability $1-p_b$), which is captured by the transition from state $W-i$ to state $W-i-1$. If the channel is busy during a backoff time slot of duration $\sigma$ (i.e., another vehicle starts to transmit a packet during this time slot), the backoff counter of the tagged vehicle will be suspended, which represented by the transition from state $W-i$ to $DW_{i-1}$ with probability $p_b$. Similar to state $DCS$, state $DW_{i-1}$ also contains self-loop because other neighbors’ transmission can lead to further deference of the tagged vehicle. When the backoff counter reaches zero, the packet will directly be transmitted (an SMP transition occurs from state 0 to state TX with probability one). In TX state, a packet is transmitting. To capture the out-dated packet replacement behavior, which can happen during any state except state idle, we simplify the model by considering the total replacement probability and placing it after state TX. If the current packet has not been replaced by the next packet (with probability $1-P_f$), the SMP goes to state idle. Otherwise, this current packet is out-dated and
replaced by the next incoming packet. Such simplification is reasonable since the packet transmission delay is usually much smaller than the packet generation interval and hence the replacement occurs extremely rare. Section 5.1 also verifies that this assumption leads to little impact on the numerical results. Next, the tagged vehicle starts the service for the next packet immediately and senses the channel for DIFS time (state $CS_2$). A new backoff procedure is started subsequently for the new packet instead of inheriting the backoff state of the old message. This is mainly because the out-dated message may finish the backoff procedure and is replaced during its transmission.

Fig. 4. SMP model for 802.11p beacon broadcast

Compared to the SMP model in [9], the newly proposed SMP model captures more detailed DCF behavior for periodic beacon message transmission by adding more states and self-loop structure. In addition, out-dated message replacement behavior is incorporated into the model by the newly introduced model parameter $P_f$. Therefore, the sojourn times and steady-state solutions are totally different from that in [9] although similar computation procedure is used.

Define the sojourn time in state $j$ as $T_j$. The mean and variance of $T_j$ in the SMP model are:

\[
E[T_j] = \tau_j = \begin{cases} 
A_1 & j = TX \\
A_2 & j = idle \\
A_3 & j = CS, CS_2 \\
A_4 & j = D_{CS} \\
A_5 & j = D_{\theta_1}, D_{\theta_2}, \ldots, D_{\theta_{W-2}} \\
0 & j = 0 \\
\sigma & j = 1, 2, \ldots, W - 1 
\end{cases}
\]  

(1)

\[
Var[T_j] = \theta_j^2 = \begin{cases} 
0 & j \in U \\
B_j & j = idle 
\end{cases}
\]  

(2)
where

\[
\begin{align*}
A_1 &= PL/R_d + T_d \\
A_2 &= \tau - E[S] \\
A_3 &= DIFS \\
A_4 &= (A_1 + DIFS)/2 \\
A_5 &= A_1 + DIFS
\end{align*}
\]

\(B_1 = \text{Var}[S]\)

\(PL\) represents the packet length. \(R_d\) presents the data rate. Hence, \(PL/R_d\) is the time to transmit the packet. \(T_d\) presents the time to transmit the packet header including physical layer header and MAC layer header. \(E[S]\) and \(\text{Var}[S]\) are the mean and variance of the overall message service time, which will be derived later. The sojourn time in state \(\text{idle}\) is the packet inter-arrival time excluding the packet service time. In addition, to simplify the model, the sojourn times for channel sensing states (\(CS_1, CS_2,\) and \(i=0,1,\ldots,W-1\)) are modeled as deterministic using the upper bound channel sensing time (\(i.e.,\) the sensing for each state only performs once). Such simplification may have impact on dense network in which channel contentions are severe. However, based on the analysis under various typical vehicle densities as shown in Section 5.1, the good match between the analytic model and the simulation results verifies that this assumption has very little impact. Moreover, the sojourn time in state \(D_{cs}\) is different from that in \(D_i\) \((i=0,1,\ldots,W-2)\) is because the packet transmission from another vehicle may already started before the new packet is generated from the tagged vehicle. Therefore, on average, the tagged vehicle only defers for a half of the packet transmission time plus an additional idle \(DIFS\) duration in state \(D_{cs}\). In contrast, for state \(D_i\) \((i=0,1,\ldots,W-2)\), the start point of packet transmission from another vehicle is detected within the backoff time slot (state \(i+1\)), and hence the tagged vehicle needs to defer for the whole packet transmission time plus an additional idle \(DIFS\) duration.

As in [9], the embedded DTMC is first solved for its steady-state probabilities for each state:

\[
\begin{align*}
v_j &= (W - j) \cdot v_{W-1} & j = 0,1,\ldots,W-1 \\
v_{0j} &= (W - j - 1) \cdot p_b \cdot v_{W-1} & j = 0,1,\ldots,W-2 \\
v_N &= W \cdot v_{W-1} \\
v_{a} &= \frac{1 - p_j}{P_j + q_b (1-P_j)} \cdot v_{W-1} \\
v_{cs} &= \frac{(1-P_j)W}{P_j + q_b (1-P_j)} \cdot v_{W-1} \\
v_{0cs} &= \frac{q_b (1-P_j)W}{P_j + q_b (1-P_j)} \cdot v_{W-1} \\
v_{W-1} &= \frac{2(1 - P_j)W}{(W+1)(1-r_b) + p_b (W-1)} \cdot v_{W-1}
\end{align*}
\]

Taking account of the mean sojourn time in each state, the steady-state probabilities of the SMP are:
Therefore, the steady-state probability that a vehicle is in the \( TX \) state is given by:

\[
\pi_{TX} = \frac{2A_i}{[P_f + q_b(1-P_f)](\sigma + \frac{P_b}{1-r_b}A_i)(W-1) + 2[A_i + A_s + (1-P_f)\left(A_i + \frac{q_b}{1-r_b}A_s\right)]}
\]

In Equation (5), four unknown parameters are:

- \( P_f \): the probability that the beacon message will be updated or replaced by the next beacon message.
- \( p_b \): the probability that the channel is detected busy (transmitting messages from other vehicles) in one time slot by the tagged vehicle.
- \( q_b \): the probability that the channel is detected busy (transmitting messages from other vehicles) in \( DIFS \) time by the tagged vehicle.
- \( r_b \): the probability that the channel is detected busy in \( DIFS \) time by the tagged vehicle after another neighbor of the tagged vehicle finishes transmission.

In the following two subsections, these parameters will be derived. Due to the dependences between these parameters, fixed-point iteration algorithm will be utilized to obtain the converged solutions. Since \( P_f \) depends on the service time to transmit a packet, the service time is derived first in the next subsection in order to obtain \( P_f \). Subsequently, the derivations of the other three parameters and the fixed-point iteration algorithm are presented in Section 3.4.

3.3. Service Time Computation

Fig. 5. Service time SMP model with an absorbing state
The MAC layer service time is defined as the time interval from the time instant when a packet starts to contend for transmission, until the time instant when the packet is transmitted. The SMP model in Section 3.2 describes the behavior of a tagged vehicle continuously transmitting packets taking into account the replacement of out-dated packets. The SMP model of Fig. 4 is modified to contain an absorbing state as shown in Fig. 5 to capture the transmission of only one packet. Hence, the time to reach the absorbing state, denoted as $TA$, will be the service time for a packet transmission. Let the cumulative distribution function (CDF) for $TA$ be denoted by $F_{TA}(t)$.

As mentioned earlier, $P_f$ is the probability that a packet is updated or replaced by the next packet. Since the beacon packets are generated periodically at a fixed time interval $\tau$, a packet is out-dated and will be replaced by the next packet only when its service time exceeds the message generation period $\tau$. Therefore,

$$P_f = P(TA > \tau) = 1 - P(TA \leq \tau) = 1 - F_{TA}(\tau) \quad (6)$$

According to Equation (6), we need to derive the CDF of the service time first to compute $P_f$. We use the Laplace transform for obtaining CDF of $TA$. From Fig. 5, we notice that a packet can only start the transmission service from two different states (either $CS_1$ or $CS_2$) according to two different scenarios. If a new packet does not replace the previous packet (with probability $1-P_f$), the packet will start service from state $CS_1$. Otherwise, if the packet replaces the previous packet (with probability $P_f$), the packet will start service from state $CS_2$. Denote $TA_{CS1}$ and $TA_{CS2}$ to be the time to reach absorbing state $END$ starting from state $CS_1$ or $CS_2$ respectively, and $q_{CS1}$ and $q_{CS2}$ to be the corresponding probabilities. Hence,

$$TA_j = \begin{cases} TA_{CS_1} & \text{w.p. } q_{CS_1} = 1 - P_f \\ TA_{CS_2} & \text{w.p. } q_{CS_1} = P_f \end{cases} \quad (7)$$

The service time for a packet transmission is then given by:

$$TA = (1 - P_f) \cdot TA_{CS_1} + P_f \cdot TA_{CS_2} \quad (8)$$

Since the sojourn time in each state is deterministic, their Laplace–Stieltjes transform (LST) can be easily determined:

$$L_{TA_j}(s) = E[e^{-sTA_j}] = \begin{cases} e^{-s\cdot TX} & j = TX \\ e^{-s\cdot CS_1} & j = CS_1, CS_2 \\ e^{-s\cdot DS} & j = D_{CS} \\ e^{-s\cdot D_{j}} & j = D_0, D_1, \ldots, D_{w-2} \\ 1 & j = 0 \\ e^{-s\cdot W-1} & j = 1, \ldots, W - 1 \end{cases} \quad (9)$$

Therefore, the LST of $TA_{CS1}$ and $TA_{CS2}$ can be calculated from Fig. 5:
From Equation (8), we know that the LST of TA is:

$$L_{TA}(s) = e^{-s(A+\lambda_{b})} \left[ (1-q_{b}) + q_{b} \cdot e^{-s\lambda} \sum_{k=0}^{W} \left( r_{b} \cdot e^{-s\lambda_{b}} \right)^{k} \right] \frac{1}{W}$$

$$L_{TA}(s) = e^{-s(A+\lambda_{b})} \cdot \frac{1}{W} \sum_{i=0}^{W-1} \left[ \left( 1-p_{b} \right) \cdot e^{-s\sigma} + p_{b} \cdot (1-r_{b}) e^{-s(\sigma+\lambda_{b})} \sum_{k=0}^{W} \left( r_{b} \cdot e^{-s\lambda_{b}} \right)^{k} \right]$$

Hence, the Laplace transform for $F_{TA}(t)$, denoted as $F^{*}(s)$ is:

$$F^{*}(s) = \frac{L_{TA}(s)}{s} = (1-P_{f}) \cdot \frac{L_{\Lambda_{b}}(s)}{s} + P_{f} \cdot \frac{L_{\Lambda_{b+1}}(s)}{s}$$

Upon inversion of such a Laplace transform, the service time distribution, $F_{TA}(t)$, can be easily obtained:

$$F_{TA}(t) = L^{-1}\left(F^{*}(s)\right) = (1-P_{f}) \cdot L^{-1}\left(\frac{L_{\Lambda_{b}}(s)}{s}\right) + P_{f} \cdot L^{-1}\left(\frac{L_{\Lambda_{b+1}}(s)}{s}\right)$$

Due to the packet replacement phenomena, if the service time of a packet exceeds the packet generation interval $\tau$, the packet will be replaced by the next packet. Therefore, the service time for the replaced packet can be interpreted as $\tau$. Thus, the service time distribution has to be adjusted to incorporate such packet replacement scenario:

$$F_{TA}(t) = \begin{cases} (1-P_{f}) \cdot L^{-1}\left(\frac{L_{\Lambda_{b}}(s)}{s}\right) + P_{f} \cdot L^{-1}\left(\frac{L_{\Lambda_{b+1}}(s)}{s}\right) & t \leq \tau \\ 1 & t > \tau \end{cases}$$

(13)

Since the sojourn time in every state of Fig. 5 is deterministic, we can easily conclude that the service time is a discrete variable.

After obtaining the service time distribution, the probability $P_{f}$ that a packet is replaced by the next packet can be computed as in Equation (6). To determine the sojourn time in idle state in Equation (2), the mean service time also needs to be calculated as follows:

$$E[S] = E[T_{A}] = \int_{0}^{\infty} \left[ 1 - F_{TA}(t) \right] dt$$

(14)

### 3.4. Fixed-point Iteration

As described in Section 3.2, four unknown model parameters need to be determined to obtain the system steady-state behavior (i.e., a vehicle transmits in steady-state). In the previous section, $P_{f}$ is shown to depend on the service time, which further depends on the other three model parameters $p_{b}$, $q_{b}$ and $r_{b}$. In this section, the channel busy probabilities $p_{b}$, $q_{b}$ and $r_{b}$ are derived first, each of them is shown to depend on the other three parameters. Therefore, fixed-point iteration algorithm is used to obtain final solutions.
Let $N_{cs}$ denote the average number of vehicles in the carrier sensing range of the tagged vehicle, and let $N_{tr}$ denote the average number of vehicles in transmission range of the tagged vehicle. Hence, without loss of generality, we have:

$$N_{cs} = N_{tr} = 2\beta R$$

The average number of vehicles in potential hidden area is:

$$N_{ph} = 4\beta R - N_{cs} = 2\beta R$$

From the tagged vehicle’s point of view, $p_b$ is the probability that it senses channel busy during one backoff time slot. Through the SMP model in Fig. 4, we know that state $I^i \in \{0,1,\ldots,W-1\}$ stands for the one time slot channel sensing during backoff process. Furthermore, before the system enters one of these channel sensing time slots, the channel has been idle for either at least one time slot or $DIFS$ time duration, which can be easily seen from the input transitions and states for state $I$. Therefore, to be more precise, $p_b$ stands for the probability that the channel becomes busy (i.e., goes from idle to busy) during one backoff channel sensing time slot of the tagged vehicle. The probability that a vehicle starts to transmit during one time slot (either just finishes backoff procedure or directly starts to transmit due to idle $DIFS$ channel sensing) is equivalent to $\pi_i (i=1,\ldots,W-1)$, because the sojourn time in these states is one time slot. Hence, we use $\pi_i$ to denote the probability that a vehicle starts to transmit during one time slot, which can be computed from Equation (4). Furthermore, the channel becomes busy if there is at least one neighbor (i.e., a vehicle in the transmission range of the tagged vehicle) that starts to transmit in a backoff channel sensing time slot of the tagged vehicle. Thus, we have:

$$p_b = 1 - \sum_{i=1}^{W-1} (1-\pi_i) \left( \frac{N_{tr}}{i} \right) e^{-N_{cs}/i} = 1 - e^{-N_{cs}/\pi_i}$$

Next, $q_b$ denotes the probability that the channel is detected busy by the tagged vehicle in $DIFS$ duration. Different from $p_b$ derivation, the channel does not necessarily become busy during $DIFS$ duration. In other words, the channel can becomes busy before $DIFS$ duration, and only been detected busy during this $DIFS$ duration. This is because the tagged vehicle is in state idle while waiting for a packet being generated and then it enters state $CS_1$ to sense the channel for $DIFS$ duration, which implies that the channel could have become busy before such $DIFS$ sensing time. As long as the channel busy status is captured by the $DIFS$ channel sensing time, with probability $q_b$, the vehicle will enter state $D_{CS}$ to defer transmission. Therefore, to be more precise, $q_b$ stands for the probability that channel busy status is captured by $DIFS$ duration. We can first define $Q_{TX}$ to be the probability that a neighbor’s transmission is captured in the $DIFS$ duration by the tagged vehicle, hence:

$$Q_{TX} = \frac{A + DIFS}{A} \pi_{TX}$$

Therefore, $q_b$ is given by:
Next we derive an expression for $r_b$. In Section 3.2, $r_b$ presents the probability that the following phenomena occurs: the tagged vehicle (vehicle $B$) waits for the current packet (from vehicle $A$) in the channel to finish transmission, and then sense the channel for DIFS time, which captures the transmission from another vehicle (vehicle $C$) and leads to further deference. As shown in Fig. 6, $A$, $B$ and $C$ present the vehicles on 1-D road. Two ovals present the transmission range of $A$ and $B$ respectively. For the tagged vehicle $B$, after it just received a packet from one of its neighbors $A$, it will sense the channel for DIFS time. During such DIFS time, only some of its neighbors are also in the DIFS channel sensing state (vehicles within $S_1$), whereas rest of its neighbors (vehicles within $S_2$, such as vehicle $C$) are not influenced by $A$’s transmission since they are outside $A$’s transmission range. Suppose the average number of these vehicles which are outside $A$’s transmission range but within $B$’s receiving range (i.e., space $S_2$) is $N$. Therefore, in the tagged vehicle $B$’s DIFS sensing time, the probability that it receives a neighbors transmission is:

\[
q_b = 1 - \sum_{i=0}^{\infty} (1-Q_{Tx})^i \frac{(N_q)^i}{i!} e^{-N_q} = 1 - e^{-N_q} q_{tx}
\]  

(19)

where $N$ is derived next. As shown in Fig. 6, let $x$ denote the distance between vehicle $A$ and $B$. Then, the 1-D distance in $S_2$ will also be $x$. Hence, the average number of vehicles in $S_2$ is given by:

\[
N = \int_{0}^{x} \beta x \frac{1}{R} dx = \frac{BR}{2} = \frac{N}{4}
\]  

(21)

Combining Equations (18)(20)(21), we obtain $r_b$.

From the above analysis, we know that the four parameters $P_f, p_b, q_b$ and $r_b$ are interdependent. Hence, the fixed-point iteration algorithm is utilized and outlined as follows to obtain the final converged solutions.

Step 1: Initialize $P_f=0, p_b=0, q_b=0, r_b=0$.

Step 2: With $P_f, p_b, q_b, r_b$, calculate new $P_f, p_b, q_b, r_b$ according to Equations (6)(17)(19)(20).

Step 3: If $P_f, p_b, q_b, r_b$ converge with the previous values, then stop the algorithm; otherwise, go to step 2 with the updated $P_f, p_b, q_b, r_b$.

Once the parameters $P_f, p_b, q_b, r_b$ are determined using the above algorithm, they are used for the performance-indices computation in the next section.
4. PERFORMANCE INDICES

4.1. MAC-level Performance Metrics

4.1.1. Mean Transmission Delay

One of the most important performance indices is the mean transmission delay of the beacon message. Different from the mean service time that takes into account all of the packets generated, the mean transmission delay only accounts for the packets transmitted and exclude those that have been replaced. Let $E[D]$ be the mean transmission delay, that is also the mean service time of transmitted packets. Since the service time for the packet that has been replaced is $\tau$ and the mean service time is given in (14), we have:

$$E[S] = (1-P_f) \cdot E[D] + P_f \cdot \tau$$

(22)

Therefore, the mean transmission delay is:

$$E[D] = \frac{E[S] - P_f \cdot \tau}{1-P_f}$$

(23)

4.1.2. Packet Delivery Ratio

The PDR [13] is the probability that all vehicles in the tagged vehicle’s transmission range successfully receive the broadcast packet from the tagged vehicle. Using the approach in [9], we have:

$$PDR = PDR_{cc} \cdot PDR_{ht}$$

(24)

Note that the formulas for $PDR_{cc}$ and $PDR_{ht}$ in [9] are adjusted according to the newly proposed model in this paper. $PDR_{cc}$ can also be interpreted as the non-concurrent transmission probability, i.e., two packets do not start transmission in the same time slot. Since the sojourn time in state 1 is one time slot, $\pi_1$ is equivalent to the probability that a vehicle starts to transmit in a time slot. Hence, $PDR_{cc}$ is given by:

$$PDR_{cc} = \sum_{i=0}^{\infty} (1-\pi_1)^i \frac{\left( N_{ns} - 1 \right)}{i!} e^{-\left( N_{ns} - 1 \right) \pi_1} = e^{-\left( N_{ns} - 1 \right) \pi_1}$$

(25)

The event that a transmission from hidden terminals collides with the tagged vehicle’s transmission only happens when hidden terminals start to transmit during the vulnerable period of duration $2 \cdot A_1$ [13]. Therefore, the probability that a vehicle starts to transmit during the vulnerable period of hidden terminal transmissions [13] is $2 \cdot \pi_{TX}$, and hence:

$$PDR_{ht} = \sum_{i=0}^{\infty} (1-2 \cdot \pi_{TX})^i \frac{\left( N_{ns} \right)}{i!} e^{-\left( N_{ns} \right) \pi_{TX}} = e^{-2 \cdot N_{ns} \pi_{TX}}$$

(26)

4.1.3. Packet Reception Ratio

Packet reception ratio (PRR) is defined as the percentage of nodes that successfully receive a packet from the tagged node among the receivers being investigated (i.e., vehicles within the receiving range of the tagged node) at the moment that the packet is sent out [13]. Using an approach similar to that presented in our previous work [9], we adjusted and simplified the formulas according to the new model by
considering the impacts from both concurrent transmissions \( PRR_{cc} \) and hidden terminals problem \( PRR_{ht} \). Hence:

\[
P RR = P RR_{cc} P RR_{ht}
\]

where the impact of the concurrent transmission is:

\[
P RR_{cc} = \frac{e^{-\beta R}}{\beta R \pi_i} (1 - e^{-\beta R})
\]

and the impact of the hidden terminal is:

\[
P RR_{ht} = \frac{1}{RC} (1 - e^{-RC})
\]

with \( C = 2\beta \cdot \pi_{TX} \).

### 4.1.4. Normalized Channel Throughput

The normalized channel throughput is the ratio of the time used for successful transmitted packets (i.e., the packet is received by all vehicles within the tagged vehicle’s transmission range) and the entire time. From a channel’s perspective, during one packet generation interval \( \tau \), \( N_r \) packets are generated in total for all vehicles sensing this channel, one packet from each vehicle. The transmission time for each packet is \( PL/R_d \) (\( PL \) represents the packet length and \( R_d \) represents the data rate). Hence, the normalized channel throughput can be easily computed as:

\[
S = \frac{N_r \cdot PL}{\tau \cdot R_d \cdot PDR}
\]

### 4.2. Application-level Performance Metrics

Besides MAC-level metrics, application-level performance metrics are also important to capture the performance of broadcast-based safety applications. Furthermore, the QoS requirements are typically expressed in terms of application-level performance metrics. Therefore, derivation of the application-level performance metrics is essential for QoS assessment of safety applications. To derive the application-level performance metrics, the evaluation of point-to-point reception probability needs to be conducted. Therefore, the node reception probability is first computed, based on which the application-level performance metrics including T-window reliability, application-level delay, awareness probability and average number of invisible neighbors are derived.

### 4.2.1. Node Reception Probability

Given a transmitting node \( O \) placed at the origin (see Fig. 7), \( U \) is one of the receivers within transmission range \( R \) of node \( O \). \( U \) is placed on 1-D line with certain distance to \( O \), which is denoted as \( x \) (\( 0 < x < R \)). The probability that the node \( U \) receives the broadcast message from the tagged node \( O \) successfully is the node reception probability (NRP) at distance \( x \), which is denoted as \( P_s(x) \).
There are two factors affecting the node reception probability: hidden terminal problems, collisions due to concurrent packet transmissions:

### a. Impact of Hidden terminals

Based on the SMP model and its solution, we have the probability that node $U$'s reception of the broadcast message from node $O$ is free from the hidden terminals:

$$P_e(x) = \sum_{i=0}^{\infty} \left(1-2\pi x^2\right) \frac{\left(\beta y\right)^i}{i!} e^{-\beta y} = e^{-2\pi x^2 \beta y}$$  \hspace{1cm} (31)

### b. Impact of concurrent collisions

In addition to collisions caused by the hidden nodes, transmissions from nodes within interference range from the tagged node in the meantime at which the tagged node transmits may also cause collisions. When the tagged node transmits in a slot time, collisions will take place if any node in the interference range of the tagged node transmits in the slot.

Given that as both $O$ and $U$ sense the channel idle, $O$ will transmit within the duration of a slot. In order to prevent interference due to concurrent collisions to $U$’s receiving the broadcast message sent by $O$, no transmission in $[-(R-x), R]$ is allowed. The average number of nodes transmitting in the concurrent slot in area $[0, x]$ is $\beta x \pi$.

Suppose node $W$ is $y$ away from $O$, $x<y<R$. The probability that concurrent transmission occurs resulting from node $W$ is the probability that node $W$ starts to transmit during the concurrent slot and all nodes in $[R+x, R+y]$ are not in transmitting state, which is expressed as Equation (32). The transmissions from nodes in $[R, R+x]$ have been taken into account for the analysis of hidden terminals in Equation (31), and hence, we do not need to consider such transmissions for concurrent transmission impact analysis.

$$P_s(y, x) = \pi s \sum_{i=0}^{\infty} \left(1-\pi y^2\right) \frac{\left(\beta (y-x)^2\right)}{i!} e^{-\beta (y-x)^2} = e^{-\beta (y-x)^2}$$  \hspace{1cm} (32)

Hence, the average number of nodes located in area $[x, R]$ that start transmission during the concurrent slot that collides with the transmission from $O$ is:

$$\bar{n_s} = \beta \int_0^R P_s(y, x) dy = \beta \int_0^R \pi s e^{-\beta y^2} dy = \frac{\pi}{\pi y} (1-e^{-\beta y^2 (R-x)})$$  \hspace{1cm} (33)

Suppose node $V$ is $z$ away from $O$, $-(R-x)<z<0$, the probability that concurrent transmission occurs resulting from node $V$ is the probability that node $V$ starts to transmit during the concurrent slot and all nodes in $[z-R, -R]$ are not in transmitting state, which is expressed as
Therefore, the average number of nodes located in area \([-\{(R-x), 0\}\] that start transmission during the concurrent slot that collides with the transmission from \(O\) is

\[
P_s(z, x) = \pi \sum_{i=0}^{\infty} (1 - \pi_{s_{\text{TX}}}) \left(\frac{\beta_1}{i!}\right) e^{-\beta_1} = \pi e^{-\beta_1}\]

Therefore, the average number of nodes located in area \([-\{(R-x), 0\}\] that start transmission during the concurrent slot that collides with the transmission from \(O\) is

\[
\bar{n}_T = \beta \int_{x-1}^{0} P_s(z, x)dz = \beta \int_{x-1}^{0} \pi e^{-\beta_1} dz = \frac{\pi}{\pi_{s_{\text{TX}}}} (1 - e^{-\beta_1})
\]

Hence, the total average number of nodes that may transmit concurrently is:

\[
\bar{n}_\Sigma = \bar{n}_T + \bar{n}_p + \beta \pi_1
\]

Therefore, given Poisson node distribution, the probability that no nodes within the reception range of \(U\) start transmission during the slot that collides with the transmission from \(O\) is

\[
P_{\text{con}}(x) = \left(\frac{\pi}{\pi_{s_{\text{TX}}}}\right)^0 \exp(-\bar{n}_\Sigma) = \exp(-\bar{n}_\Sigma)
\]

Taking hidden terminal and possible packet collisions, the node reception probability that the node \(U\) receives the broadcast message from the tagged node \(O\) is:

\[
P_s(x) = P_{\text{con}}(x)P_{\text{app}}(x)
\]

4.2.2. Application-level T-window Reliability

Application-level T-window reliability is defined in [19] as the probability of successfully receiving at least one packet out of multiple packets from a broadcast vehicle at distance \(x\), within a given time \(T\) (referred to as application tolerance window):

\[
P_{\text{app}}(x,T) = 1 - (1 - P_s(x))^T
\]

where \(T\) is the beacon generation interval and \(P_s(x)\) is the node reception probability given in (38).

4.2.3. Application-level Delay

Application-level latency [19] \(T_d\) is the duration between the time when a broadcast packet is generated at application layer of transmitting vehicle and the time at which the first successful packet is received by the application layer of receiving vehicle. Suppose the distance between the transmitting vehicle and the receiving vehicle is \(x\), the average application-level delay is given by:

\[
E_{d_{\text{app}}}(x) = \sum_{i=1}^{\infty} [(i-1)T + E[D]] P_s(x)(1 - P_s(x))^{i-1} = E[D] + \frac{1}{P_s(x)} - 1
\]

where \(E[D]\) is the mean transmission delay of the beacon message, which is given in (23).

4.2.4. The Awareness Probability

The awareness probability [20] is the probability of successfully receiving at least \(n\) packets in the tolerance time window \(T\).
\[ P_A(x, n) = \sum_{k=1}^{\lfloor \frac{T}{\tau} \rfloor} P_x(k)(1 - P_x(x))^k \]  \( (41) \)

where \( x \) is the distance between the sender and receiver. It is noted that the awareness probability \( P_A(x, n) \) becomes the application-level T-window Reliability \( P_{app}(x, T) \) as \( n \) is equal to 1.

### 4.2.5 Average Number of Invisible Neighbors

Region of interest (ROI) has been proposed in [19] to present the size of the geographical region covered by entities participating in an application. For a safety application to work properly, a vehicle needs to be aware of the neighbors’ status within ROI. Different applications may have different ROI. The invisible neighbor problem is defined in [21]: if vehicle A has not received any broadcast packet from vehicle B for a certain time interval, vehicle B is an invisible neighbor of A. Therefore, under the concept of ROI, the number of invisible neighbors needs to be under a certain constraint to satisfy the QoS of an application.

We adopt this concept and newly define another application-level performance metric: average number of invisible neighbors within range \( x \) from the receiver after a tolerant time window \( T \).

\[ N_{invis}(x) = 2\beta x \left( 1 - \int_0^1 \frac{1}{x} P_{app}(s, T) ds \right) = 2\beta x - 2\beta \int_0^1 P_{app}(s, T) ds \]  \( (42) \)

### 5. NUMERICAL RESULTS

Matlab is used for the computation of analytic models and simulations. The first subsection describes the simulation procedure. The second subsection compares the analytic results with the simulations, which show the high accuracy of our decomposition-based analytic approximation. The last subsection compares the model in [9] (Poisson arrival with infinite queue) and the model in [7] with the new proposed model (periodic arrival with packet replacement strategy).

#### 5.1. Numerical Results for MAC-level Performance Metrics

##### 5.1.1. Simulation Description

Discrete-event simulation is conducted in Matlab to assess the approximation error in the decomposition and fixed-point iteration used in solving the analytic model. Different from the decomposition method used for the analytic model, the simulation process simulates the overall system behavior of the GSMP model. To better illustrate the simulation procedure, a brief simulation flow chart is presented in Fig. 8. Note that this flow chart is drawn from the whole system’s perspective, whereas each individual vehicle’s behavior is consistent with the operation flow in Fig. 2. In Fig. 8, neighbors of the transmitting vehicle refer to the vehicles within the transmitting vehicle’s transmission range. Possible backoff behavior of the neighbors is considered while deferring or resetting their next event time. This simulation procedure is performed multiple times based on independent replications, from which confidence intervals of the output measures are computed.
5.1.2. Analytic Vs. Simulation Results
The same typical DSRC parameter settings as that in Table 1 in [9] are used for the proposed model.

Table 1. DSRC Communication Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission range $R$</td>
<td>$500 m$</td>
<td>Slot time $\sigma$</td>
<td>$16 \mu s$</td>
</tr>
<tr>
<td>PHY preamble $T_{p}$</td>
<td>$40 \mu s$</td>
<td>DIFS</td>
<td>$64 \mu s$</td>
</tr>
<tr>
<td>MAC header $T_{h}$</td>
<td>$272 bits$</td>
<td>CWMin $W-1$</td>
<td>$15$</td>
</tr>
<tr>
<td>PLC header $T_{pl}$</td>
<td>$4 \mu s$</td>
<td>Propagation delay</td>
<td>$0$</td>
</tr>
</tbody>
</table>

The time to transmit the header is: $T_t = T_{p} + T_{h} / R_c + T_{pl}$. The numerical results from the analytic solution including mean transmission delay, PDR, PRR and normalized throughput are plotted in Figs. 9-12 respectively, vs. the vehicle density $\beta$ (# vehicles per meter), data rate $R_d$ (Mbps), packet generation interval $\tau$ (second) and packet length $PL$ (byte).

We conducted 30 runs of simulation and each run last 5 seconds. Due to the Central Limit Theorem, normal distribution is assumed to compute confidence intervals for the population means of the output measures. Equation (43) shows the $100(1-\alpha)\%$ confidence interval for population mean $x$, where $\bar{x}$ stands for the sample mean, $\sigma$ for sample variance, $n$ for sample size (i.e., number of simulation runs) and $z$ for critical value of normal distribution.

$$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq x \leq \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$  \hspace{1cm} (43)

The simulation results for 95% confidence interval are also illustrated in Figs. 9-12 to compare with analytic-numerical results. The good match of the analytic and the simulation results verifies the accuracy of our proposed model. The packet replacement probability $P_r$ is extremely low from our numerical analysis resulting from the fact that packet transmission delay is much smaller than the packet generation interval as shown in Fig. 9. Fig. 9-11 show that mean transmission delay, PDRs and PRRs benefit from high data rate and short packet length. Furthermore, PDRs are less than PRRs given the same network parameters. Fig. 12 shows that normalized channel throughput increases as vehicle density increases resulting from more severe channel contentions. It may become saturated due to more severe packet collisions.
In this section, the model in [9] and the model in [7] are compared with our proposed model. These three models are comparable since all of them concentrate on MAC layer behavior of safety message transmissions. However, the model in [9], denoted as model 1, assumes Poisson packet arrivals and that each vehicle has an infinite queue to store generated packets, which may be unrealistic for beacon messages. The proposed model, denoted as model 2, relaxes these assumptions and focuses on periodic message generation and no queue scenario (i.e., out-dated message is replaced by the new message), which is more practical. Bastani’s model proposed in [7] accounted for the periodic nature of beacon message generations and new message canceling out old message phenomena. However, their model does not accurately capture the periodic beacon message generation since it separates the periodic message generation from the message channel contention and transmission behavior, although these behaviors are closely correlated. Since the output measures computed in [7] are different from those we calculated in this paper, Bastani’s model is slightly modified, denoted as model 3, to obtain PDR for comparison.
Equation (11) in [7] for probability of successful transmission is modified to compute packet delivery ratio (PDR) of beacon message only as follows:

\[
PDR = \left(1 - \tau^b\right)^{N_b - 1 + N_w} \frac{p^T}{p_{t} + (1 - p)w}
\]

where

\[
\tau^b = \frac{2}{W + 1 + 2\tau / \sigma} ; p = 1 - \left(1 - \tau^b\right)^{N_w} ; T^b = 2T^r; T_b = PL / R_d + T^b + \delta
\]

Figs. 13-15 present the analytic-numerical results comparison of these three models. Notice that packet generation rate $\lambda$ in model 1 is set to be equivalent to the reciprocal of the packet generation interval $\tau$ in model 2 and model 3. The results show that the mean transmission delay obtained from model 2 is slightly higher than that from model 1, whereas the PDR and PRR are slightly lower than that from model 1. Taking into consideration the variations in the simulations results (i.e., 95% confidence intervals) in Figs. 9-11, we can conclude that model 1 leads to similar results with model 2. In other words, Poisson arrival with infinite queue scenario can be used to approximate the periodic beacon message generation without any queue case. Such phenomena may result from the fact that the packet generation interval is much larger than the packet transmission delay, which further implies that a packet is out-dated and replaced by the next packet with a very low probability. Therefore, queue length does not have significant influence on packet transmissions. Fig. 14 also shows that PDR obtained from model 3 has a good match with model 1 and model 2 when the beacon message generation interval is longer. However, when the beacon message is generated more frequently, model 3 has much higher PDR than model 1 and model 2. Compared with the simulation results in Fig. 10, we conclude that model 3 is not as accurate as model 2 proposed in this paper. The possible reason is already stated earlier.
5.2. Numerical Results for Application-level Performance Metrics

5.2.1. Analytic-numerical Results for Fixed Network Parameters

Table 2. Network parameter settings

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission range $R$</td>
<td>500 m</td>
<td>Packet length $PL$</td>
<td>400 bytes</td>
</tr>
<tr>
<td>Time window $T$</td>
<td>1 s</td>
<td>Beacon message interval $\tau$</td>
<td>0.1 s</td>
</tr>
<tr>
<td>Data rate $R_d$</td>
<td>24 Mbps</td>
<td>Vehicle density $\beta$</td>
<td>0.1 vehicles/m</td>
</tr>
</tbody>
</table>

In this section, the application-level performance metrics are evaluated for a given network parameter settings as shown in Table 2.

Fig. 16 shows that NRP decreases linearly with the distance from the sender. The awareness probability also decreases with the distance to the sender. Note that the Application-level T-window Reliability is equivalent to the awareness probability with packet requirement $n=1$. The application with stricter packets requirements has lower awareness probability. During the time window $T=1s$, there are 10 beacon packets sent out from the sender since the beacon message interval is 0.1s. If the packet
requirement is less than or equals to 5 packets, the awareness obtained is larger than 99%, which can ensure the application works appropriate even though the NRP in the network layer is not very high. Otherwise, if the packet requirement is larger than 5 among 10 packets transmitted within 1 second, the application layer awareness probability may be even lower than the NRP in the network layer as seen from PA\(n=8\) case. Hence, for some applications that require more than 5 packets within \(T=1s\), the reliability requirement may not be met.

Fig. 17 shows that the application-level delay increases almost linearly with the distance. Furthermore, compared to the MAC-level transmission delay in Fig. 9, the application-level delay is much higher, especially when the distance between the sender and receiver is large. Such phenomena results from the fact that NRP decreases linearly according to the distance and the application-level delay is inversely proportional to NRP as shown in Equation (40). Fig. 18 shows that the number of invisible neighbors increases fast when the distance exceeds 300m. However, the number of invisible neighbors is quite small (in the order of 1E-6), which means that the receiver can receive at least one packet from almost every neighbor within time window \(T=1s\) and hence all neighbors are visible to the receiver.

5.2.2. Analytic-numerical Results for Different Network Parameters

In order to evaluate the influences of different network parameters such as data rate, beacon message interval and packet length on the application-level metrics, we vary three input parameter value in Table 2 and analyze the output measures as shown in Figs. 19-22. The legend represents the data rate \(R_d\) (Mb/s), beacon message interval \(\tau\) (s) and data length \(PL\) (bytes), respectively.
Fig. 19 and Fig. 20 show that increasing the data rate helps increasing NRP and awareness probability (more obvious when the distance to the sender is large). Another important observation is that decreasing the beacon message interval (i.e., messages are sent more frequently) will decrease the NRP, but increase awareness probability. This implies that even though more collisions occur in the MAC layer resulting from higher beacon message generation rate, we may still obtain satisfied QoS in the application layer. Furthermore, increasing the packet length leads to decreasing NRP and decreasing awareness probability. Therefore, we can improve the awareness probability by increasing the data rate, decreasing the data length or decreasing the beacon message interval within the acceptable ranges for these parameters based on performance of IEEE 802.11p protocol.

Fig. 21 shows that increasing data rate or decreasing data length helps reducing the application-level delay. In addition, even though decreasing beacon message interval has significant impact on NRP and awareness probability (i.e., decrease NRP whereas increase awareness probability), it has little impact on the application-level delay. Therefore, to reduce the application-level delay, we can increase data rate or decrease the data length reasonably.
Fig. 22 shows that the number of invisible neighbors is relatively small for the given four sets of input parameters, which is in the order of 1E-4. We observe that the number of invisible neighbors has the inverse trend as the awareness probability. Hence, based on the observations for the awareness probability, we conclude that increasing data rate, decreasing data length or decreasing the beacon message interval helps reduce the number of invisible neighbors.

6. VANET Applications Evaluation

6.1. Application Requirements

In this paper, we propose to specify the performance requirements for applications in terms of the following aspects to provide satisfied QoS:

- Range of interest (ROI) \([19]\): intended message delivery range \(R_I\);
- Delay requirement: the time for a vehicle to receive beacon messages from any one of its neighbors within range \(R_i\) has to be less than \(D\) second;
- Awareness probability \([20]\): the probability that a vehicle successfully receives at least \(n\) packets from any one of its neighbors within range \(R_i\) in the tolerance time window \(T\) has to be larger than \(p_a\);
- Average number of invisible neighbors \([21]\): the average number of invisible neighbors within range \(R_i\) in tolerance time window \(T\) has to be less than \(N\);

Given the above application-level performance requirements, our proposed analytic models can be used to evaluate whether the given network parameters can satisfy the QoS requirement or not for an application. In addition, we can provide insights to tune the network parameters to meet the application requirements.

6.2. Case Studies for VANET Applications

Vehicles are able to track surrounding vehicles’ status by their periodically broadcasted beacon messages containing position, velocity and acceleration. Many safety applications judge the risk based on such information and provide corresponding warnings to the driver to prevent potential accidents. In this section, three typical safety applications \([25]\) are analyzed.

6.2.1. Emergency Vehicle Warning

Vehicles can receive route information of emergency vehicles (e.g., police cars, ambulances, fire trucks etc.) from the beacon messages received. Hence, based on such information, emergency vehicle warning application \([24]\) enables drivers to be aware of the emergency vehicle and take appropriate actions to reduce accidents and save time. Some reasonable assumptions are made for the performance requirements of the emergency vehicle warning application: The ROI is 500\(m\) \([23]\); Delay requirement is 1000\(ms\) \([23]\); Awareness probability that a vehicle successfully receives at least 1 packet in the tolerance time window \(T=1s\) is larger than 99.9%; Average number of invisible neighbors is less than 1.

Fig. 23 shows that the application-level delay, awareness probability and average number of invisible neighbors requirement can all be satisfied with given input parameters \(R_{c}=24Mbps, \tau=0.2s, PL=200bytes\).
6.2.2. Slow Vehicle Indication

Slow vehicle indication application [25] is able to provide alerts to the driver about potential hazard if a slow vehicle is detected based on the beacon messages received. For the slow vehicle indication application to work properly, the vehicle has to receive beacon messages from the slow vehicle timely and reliably. This application may have stricter awareness requirements than the emergency vehicle application.
warning application. The performance requirement assumptions can be made as follows: The ROI is 100m; Delay requirement is 50ms; Awareness probability that a vehicle successfully receives at least 3 out of 5 packets in the tolerance time window $T=1s$ is larger than 99.9%; Average number of invisible neighbors is less than 1.

Fig. 24 shows that application-level delay, awareness probability and average number of invisible neighbors requirement can all be satisfied with given input parameters $R_d=24Mbps$, $\tau=0.2s$, $PL=200$ bytes.

6.2.3. Rear-end Collision Warning

Rear-end collision warning application should have stricter awareness requirements than the previous two applications since it is more critical for safety. Therefore, we suggest the performance requirements to be: The ROI is 50m; Delay requirement is 20ms [6]; Awareness probability that a vehicle successfully receives 4 out of 5 packet in the tolerance time window $T=1s$ is larger than 99.9%; Average number of invisible neighbors is less than 1.
Fig. 25 shows that the application-level delay and average number of invisible neighbors can satisfy the requirement given the input parameters. However, the awareness probability is lower than 99.9% when the distance to the sender is larger than 20m. Therefore, the rear-end collision avoidance application cannot provide the satisfactory QoS for the given input parameters. To meet the application requirement, based on the conclusions obtained in Section 5.2.2, we can improve the awareness probability by increasing the data rate, decreasing the data length or decreasing the beacon message interval.

7. CONCLUSIONS AND FUTURE WORK

In this paper, an analytic model is developed to characterize the periodic beacon message dissemination in DSRC for highway safety communications. Instead of assuming Poisson arrivals and infinite queue as in most of the literature, the periodic packet generation and out-dated information replacement are taken into consideration in the new proposed model. Important MAC-level performance indices such as the mean transmission delay, PDR, PRR and normalized channel utilization are analytically derived and computed. Detail simulations for the overall system in the MAC layer are conducted to verify the accuracy of the decomposed model. The model is also compared with previous model that assumed Poisson arrival and infinite queue and Bastani’s model.

Besides MAC-level performance evaluation, application-level performance metrics including T-window reliability, application-level delay, awareness probability and average number of invisible neighbors are analytically derived based on the node reception probability. The analytic-numerical results are evaluated for application-level performance metrics under various network parameters. Such analysis can be very useful for tuning network parameters in order to obtain satisfied QoS for many safety applications. Three typical safety applications including emergency vehicle warning, slow vehicle indication and rear-end collision warning are assessed to check whether their performance requirements can be met for a given network parameter setting.
In future, we will take into account the channel switching behavior \cite{14} between the control channel and service channels, where the control channel is used for beacon message and service channels are used for other types of messages. Furthermore, the assumptions in Section 2.2 will also be relaxed to reflect real world traffic scenarios. For example, the MAC layer and application layer performance metrics will be analyzed incorporating fading/shadowing channel and vehicle mobility. In addition to 1-D highway scenarios, 2-D models for urban, suburban and intersections will also be developed in the future.

ACKNOWLEDGMENTS

The authors would like to thank NSF grants (CNS-1018605 and CNS-1017722) to support this research.

REFERENCES

Xiaoyan Yin received B.S. in Electrical Engineering and Computer Science at Peking University, China, and M.S. in Electrical and Computer Engineering at Duke University. Currently, she is a Ph. D. student in Electrical and Computer Engineering at Duke University. Her research interests include performance and reliability evaluation of computer and communication systems.

Xiaomin Ma (M’03-SM’08) received B.E. and M.E. degrees in electrical engineering in 1984 and 1989, respectively. He got the Ph.D. degree in Information engineering at the Beijing University of Posts & Telecommunications, China, in 1999. From 2000 to 2002, he was a post-doctoral fellow in the Department of Electrical and Computer Engineering, Duke University, USA. Currently, he is a professor in the College of Science and Engineering at Oral Roberts University in U.S. He has published more than 90 papers in peer-reviewed journals and conferences. He also holds a US patent. His research interests include stochastic modeling and analysis of computer and communication systems, physical layer and MAC layer of vehicular ad hoc wireless networks, computational intelligence and its applications to coding, signal processing, and control, and Quality of service (QoS) and call admission control protocols in wireless networks.
Kishor S. Trivedi (M’86-SM’87-F’92) received M.S. and Ph.D. degrees in Computer Science from University of Illinois, Urbana-Champaign. He holds the Hudson Chair in the Department of Electrical and Computer Engineering at Duke University, Durham, NC. He has been on the Duke faculty since 1975. He is the author of a well known text entitled, Probability and Statistics with Reliability, Queuing and Computer Science Applications, published by Prentice-Hall; a thoroughly revised second edition (including its Indian edition) of this book has been published by John Wiley. He has also published two other books entitled, Performance and Reliability Analysis of Computer Systems, published by Kluwer Academic Publishers and Queueing Networks and Markov Chains, John Wiley. He is a Fellow of the Institute of Electrical and Electronics Engineers. He is a Golden Core Member of IEEE Computer Society. He has published over 480 articles and has supervised 45 Ph.D. dissertations. He is the recipient of IEEE Computer Society Technical Achievement Award for his research on Software Aging and Rejuvenation. His research interests are in reliability, availability, performance, performability and survivability modeling of computer and communication systems. He works closely with industry in carrying our reliability/availability analysis, providing short courses on reliability, availability, performability modeling and in the development and dissemination of software packages such as SHARPE and SPNP.