Experimental Validation of Controller Switching Strategies for Constrained Systems

K. Kogiso¹, T. Matsumoto², K. Hirata²

¹ Department of Information Systems, Nara Institute of Science and Technology, Takayamacho 8916-5, Ikoma 630 0192, JAPAN
kogiso@is.naist.jp
² Department of Computer-controlled Mechanical Systems, Osaka University, Yamadaoka 2-1, Suita 565 0871, JAPAN
matumoto@newton.mech.eng.osaka-u.ac.jp
hirata@mech.eng.osaka-u.ac.jp

Abstract: Practical control systems necessarily have pointwise-in-time constraints such as a physical limit on state variables and/or such as an actuator bound on control variables. For such systems, switching control strategy techniques are proposed, but few results about experimental evaluations are done. This paper, therefore, considers a switching control strategy using a maximal output admissible set, and investigates the strategy for systems with state and control magnitude constraints. In experimental evaluations of the strategy we utilize practical constrained systems of a DC-motor position servomechanism.

Keywords: switching control, constrained system, state and control constraint, discrete-time system, linear programming, experimental evaluation

1. Introduction

Practical control systems necessarily have pointwise-in-time constraints. They appear most commonly as not only actuator bounds on control variables but also physical limits on state variables. It is known that violations of such constraints drastically degrade system performance, and in the worst cases, lead to instability [2].

To escape such performance degradation, switching control strategy techniques [5,6,8] have been proposed. Their main and common features are appropriately switching controllers, which are prepared in advance and designed to achieve each specification. The primary purpose of a switching control strategy is to meet performance objectives such as fast responses and good disturbance rejections, simultaneously fulfilling specified state and control constraints.

On the other hand, few experimental researches of such switching control strategies have been reported. See [7], for example. It is important to experimentally confirm its effectiveness and limitations in the presence of modeling uncertainties and state observation noises. Therefore, this paper investigates the experimental evaluations of controller switching strategies for systems with state and control magnitude constraints [5], utilizing an experimental DC-motor position servomechanism. The structure of this paper is as follows. In Section 2, we formulate systems with state and control constraints and in Section 3, explain a control strategy for their constrained systems by switching designed dynamic controllers. Instead of the original strategy, we show in Section 4 that an algorithm suitable for real-time computation for the sake of implementation. In Section 5, the switching control algorithm’s effectiveness is demonstrated by using a practical DC-motor position servomechanism. Finally, Section 6 concludes this paper.

2. Switching Control System

Consider a linear discrete-time switching control (closed-loop) system with k controllers (see Fig. 1):

\[ x(t+1) = A_i(t)x(t), \]
\[ z_0(t) = C_0(t)x(t), \]
\[ z_1(t) = C_1(t)x(t), \]

where subscription \( i \in \{1, 2, \ldots, k\} \) denotes an index of the \( i \)th controller \( C_i \). \( x = [x'_p \ x'_c]^T \in \mathbb{R}^n \) \((n = n_p + n_c)\) is a state vector of a closed-loop system, \( x_p \in \mathbb{R}^{n_p} \) and \( x_c \in \mathbb{R}^{n_c} \) are a state vector of a plant and a controller respectively. An initial state of the plant is given by \( x_p(0) = x_{p0} \in \mathbb{R}^{n_p} \), and the plant state is measurable. \( z_1 \in \mathbb{R}^{n_1} \) is a controlled output, while \( z_0 \in \mathbb{R}^{n_0} \) is a vector to be constrained within a prescribed subset \( Z \subset \mathbb{R}^{n_0} \) as a constraint condition:

\[ z_0(t) \in Z, \quad \forall \ t \in \mathbb{Z}^+, \]

where \( \mathbb{Z}^+ \) is a set of a nonnegative integer and \( Z = \{ z_0 \in \mathbb{R}^p \mid M_z z_0 \leq m_z \} \), \( M_z \in \mathbb{R}^{n_x \times n_z} \) and \( m_z \in \mathbb{R}^{n_z} \). The control problem to be considered here is regulation under fulfillment of the constraint condition (2).
3. Switching Control Strategy

Switching operation can be basically constructed with a concept of a maximal output admissible set.

**Definition 1.** Let $z_0(t; x_0, C_i)$ denote an output (1b) of the closed-loop system with the $i$th controller $C_i$ without any switching for an initial condition $x(0) = x_0$. Define a maximal output admissible set $O^i_\infty$ as

$$O^i_\infty = \{ x_0 \in \mathbb{R}^n \mid z_0(t; x_0, C_i) \in \mathcal{Z} \forall t \in \mathbb{Z}^+ \}.$$  

**Remark 1.** In this paper we describe control systems (1) that are already designed by applying appropriate controller design method. With fixed $i$ the description of systems (1) means a simple closed-loop system, and with time-varying $i \in \{1, \cdots, k\}$ a state moves following with each dynamics of controllers and switching.

The control problem we consider here is a regulation that a system state converges from an initial state to zero (equilibrium) and at the same time is fulfillment of constraint condition (2). The basic idea of switching strategy can be basically constructed with the corresponding controller. Depending on measured plant state $x_p$, the supervisor simultaneously operates and executes the tasks so that it does not violate constraint condition (2). The basic idea of switching strategies follows the method of [5] and is illustrated in the next section.

**Remark 2.** Consider a closed-loop system with the $i$th controller $C_i$ under no switching. The necessary and sufficient condition for constraint fulfillment with the fixed controller is $x(0) \in O^i_\infty$. Moreover, from the definition of $O^i_\infty$ we can also say that for any step $T \in \mathbb{Z}^+$, $x(T) \in O^i_\infty \Leftrightarrow z_0(t) \in \mathcal{Z}, \forall t \geq T$. These mean that it is useful to introduce the concept of maximal output admissible sets as a tool to assure constraint fulfillment.

**Remark 3.** Linear programming based computational procedures of $O^i_\infty$ have been proposed [3,4]. The maximal output admissible set $O^i_\infty$ is a convex polyhedral set and may be realized in the form of

$$O^i_\infty = \{ x \in \mathbb{R}^n \mid M_i x \leq m_i \},$$

where $M_i \in \mathbb{R}^{n \times n}$ and $m_i \in \mathbb{R}^n$ are the matrices and vectors to describe linear constraints.

Naturally, we must choose and design low gain controllers so as large state area as possible to emphasize stability. On the other hand, to request control performance of noise rejection we design high gain controllers, but then the area of corresponding maximal output admissible set is sufficiently small. Therefore, it is difficult to realize the stability and control performance simultaneously by conventional control method. However, switching control method can realize such specifications simultaneously, and moreover can achieve constraint fulfillment as well in this approach.

Here it is assumed that the maximal output admissible sets calculated after controller design processes are labelled in ascending order as their volume becomes smaller on the state space as illustrated in Fig. 2(a) in case of $k = 3$. We pile a maximal output admissible set $O^i_\infty$ corresponding to the higher gain controller $C_i$ on $O^{i-1}_\infty$ to lower $C_{i-1}$. Switching control strategy can be realized by using the piled sets. Before statement of the strategy, we introduce a projection of a maximal output admissible set $O^i_\infty$ onto $x_p$ subspace of a state space. The projection set is defined by

$$O^i_p = \{ x_p \in \mathbb{R}^{n_p} \mid \exists x_{ci} \in \mathbb{R}^{n_{ci}}, \begin{bmatrix} x_p \\ x_{ci} \end{bmatrix} \in O^i_\infty \},$$

where the projection set can be calculated by Fourier-Motzkin algorithm in [1].

Utilizing $O^i_\infty$ and $O^i_p$, as shown in Fig. 2, we construct a switching control strategy in a supervisor that has two functions. The first function selects a controller and can be realized by using the following

$$i(t) = \max_{i \in \{1, \cdots, k\}} i \text{ subject to } x_p(t) \in O^i_p, \quad (3)$$

at time $t$. The second is a decision of an initial state of a plant and can be realized by the following linear
programming. Given plant state \( x_p(t) \) at time \( t \),

\[
\min_{x_{ci} \in \mathbb{R}^{n_c}} ||x_{ci}(t)||_1 \text{ subject to } \begin{bmatrix} x_p(t) \\ x_{ci}(t) \end{bmatrix} \in O_{\infty}^{(t)} \tag{4}
\]

provides an initial state of the controller used in switching. Here \( ||x||_1 \) denotes a \( l_1 \)-norm of a vector \( x \). Solving on-line (3) and (4) makes it possible to fulfill constraint condition (2), and we summary the feature as the following theorem.

**Theorem 1.** Assume that \( x_{p0} \in O_1 \) holds. A control system (1) with a supervisor that is a switching control strategy based on (3) and (4) can achieve fulfillment of constraint conditions (2).

**Proof.** The proof is omitted, and see [5] in details. \( \Box \)

**Remark 4.** From Theorem 1 we can see that because constraints are not violated at all in control systems equipped with the switching control strategy based on (3) and (4), any degradation of control performance or instability does not occur. Therefore, this approach can safely operate control systems.

Following an approach by using (3) and (4) of the literature [5], we must prepare and calculate the sets \( O_{P} \) because of its explicit use in (3). It takes much time to calculate their sets as a dimension of controllers is increased because Fourier-Motzkin algorithm in [1] is utilized. Actually, we confirm that it is too much time to calculate their sets with even third-order controllers. Then, in the next section we show an algorithm more suitable for practical implementation instead of calculating projection sets \( O_{P} \).

### 4. Implementation of Switching Control Strategy

We employ the following algorithm, which equivalently works in comparison with the method based on (3) and (4), without preparing \( O_{P} \).

**[Implementation algorithm]**

**step0.** Initialize \( t = t_0 \), \( x_p(t_0) = x_{p0} \) and \( i(t_0) = 1 \).

**step1.** Measure state \( x_p(t) \in O_{P}^{(t)} \) at current time \( t \).

**step2.** Solve linear programming problem, \( LP(x_{P}) \):

\[
\min_{x_{ci} \in \mathbb{R}^{n_c}} ||x_{ci}||_1 \text{ subject to } \begin{bmatrix} x_p(t) \\ x_{ci}(t) \end{bmatrix} \in O_{\infty}^{(t+1)}. \tag{5}
\]

**step3.** If \( LP(x_{P}) \) is feasible, then \( x_{ci(t+1)} = x_{ci}^* \) and go to step1 with \( t = t + 1 \) and \( i = i + 1 \).

**step4.** If \( LP(x_{P}) \) is not feasible, then \( x_{ci(t)}(t + 1) = A_{ci}x_{ci}(t) \) and go to step1 with \( t = t + 1 \).

Here, \( t_0, A_{ci}, \) and \( x_{ci}^* \) respectively denote start time of controls, dynamics of the \( i \)th controller, and an optimal solution to \( LP(x_{P}) \). Moreover, although the set \( O_{P}^{(t)} \)

is described in step1 for the sake for a clarity, there is no need to calculate it. The key is forcibly solving a linear programming \( LP(x_{P}) \) in step2, and feasibility of \( LP(x_{P}) \) corresponds to renewing the solution to the optimization (3). If \( LP(x_{P}) \) is feasible, then the algorithm provides the same optimal solution \( x_{ci}^* \) as the optimization (4). Therefore, we can see that the algorithm realizes the switching control method stated in the previous section.

**Remark 5.** The implementation algorithm does not consider how to calculate controller initial state \( x_{ci(0)}(0) \). Therefore, before performing the algorithm we must solve the following linear programming:

\[
\min_{x_c \in \mathbb{R}^{n_c}} ||x_c||_1 \text{ subject to } \begin{bmatrix} x_{p0} \\ x_c \end{bmatrix} \in O_1 \tag{5}
\]

Otherwise, by appropriate method we must set the initial state of the 1st controller.

We summarize in this note about difference between the method based on (3) and (4) and the implementation algorithm that we employ in this paper. As mentioned before, the former method requires calculation process of \( O_{P} \), before its implementation and for each sampling period we necessarily must check judgement of inclusion of \( x_p \in O_{P} \) in the problem (3). If \( i(t) \) is updated by the judgement, then we need solve another problem (4) to decide controller states. Namely, the former approach requires off-line calculation of \( O_{P} \), the judgement and a linear programming problem by on-line. On the other hand, the implementation algorithm always solve only one problem \( LP(x_{P}) \) each sampling interval. Apparently, computation burden of the algorithm is lighter and it is obvious that an approach with the implementation algorithm is more suitable for practical control under limited computer performance.

### 5. Experimental Validation

This section performs experimental validations of a switching control strategy using implementation algorithm shown in the previous section and illustrates experimental results using a practical DC-motor position servomechanism.

#### 5.1 Design of practical switching control system

Consider that a controlled plant shown in Fig. 3(a) is a DC-motor position servomechanism, which consists of a DC-motor, an encoder, a belt, pulleys, and rigid shafts, and from corresponding dynamic equations its model in state-space representation is derived as follows,

\[
\dot{x}_p(t) = \begin{bmatrix} 0 & 1 \\ 0 & -3.48 \end{bmatrix} x_p(t) + \begin{bmatrix} 0 \\ 62 \end{bmatrix} u(t),
\]

\[
z_1(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p(t),
\]
and higher C in Fig. 4 and Fig. 5 when the lower gain controller C is RT-Linux v3.2, to construct the practical control system into a personal computer, whose operation system hold and implemented them and the switching strategy. For the plant model, we designed three third order controllers by using loop-shaping method, $C_1(s) = \frac{0.108s + 1}{s} \frac{s + 1.6}{s + 160}$, $C_2(s) = \frac{0.001s + 1}{s} \frac{s + 2.25}{s + 225}$, $C_3(s) = \frac{0.080s + 1}{s} \frac{s + 3.0}{s + 300}$, and $C_1(s) = \frac{0.014s + 1}{s} \frac{s + 100}{s + 225}$, respectively.

5.2 Experimental results

We illustrate two time-responses of closed-loop systems in Fig. 4 and Fig. 5 when the lower gain controller $C_1$ and higher $C_3$ are respectively used. From an output response in Fig. 4(a) the lower gain controller $C_1$ makes the red (thick) solid line be slow convergence to zero although it does not almost occur the saturation in Fig. 5(b). Furthermore, in case of the higher gain controller $C_3$ the red (thick) solid line in Fig. 5(a) has a good fast response although control response in Fig. 5(b) results a saturation. Therefore, we can see that it is difficult to meet both good tracking performance and constraint fulfillment by a conventional control method.

6. Conclusion

This paper performed experimental validations of controller switching strategies for systems with state and control magnitude constraints. From experimental results we confirmed that the considered implementation algorithm of a switching control strategy is more effective for practical controls of constrained systems.
Fig. 6: Experimental result about time responses of control systems with switching control strategy in comparison with time responses of a normal closed-loop system with the lower gain controller $C_1$.

References


