Resource Allocation for Selection-Based Cooperative OFDM Networks

Kianoush Hosseini and Raviraj Adve
Department of Electrical and Computer Engineering, University of Toronto
10 King’s College Road, Toronto, ON, Canada M5S 3G4
Email: \{kianoush, rsadve\}@comm.utoronto.ca

Abstract

This paper considers resource allocation to achieve max-min fairness in a selection-based orthogonal frequency division multiplexing network wherein source nodes are assisted by fixed decode-and-forward relays. Crucial design questions such as whether to relay, relay assignment and power allocation form a combinatorial problem with exponential solution complexity. The first set of problems assume perfect source-relay channels and that sources distribute power equally across subcarriers. The solutions based on these simplifications help illustrate our general methodology and also why these solutions provide tight bounds. We then formulate the general problem of transmission strategy selection, relay assignment, and power allocation at the sources and relays considering all communication channels, i.e., imperfect source-relay channels. In both sets of problems, transmissions over subcarriers are assumed to be independent. Given the attendant problems of synchronization and the implementation using a FFT/IFFT pair, resource allocation at the subcarrier level appears impractical. We, therefore, consider resource allocation at the level of the entire OFDM block. While optimal resource allocation requires an exhaustive search, we develop tight bounds with lower complexity. Finally, we propose a decentralized block-based relaying scheme. Simulation results using the COST-231 channel model show that this scheme yields close-to-optimal performance while offering many computational benefits.

Index Terms

Cooperative communication, orthogonal frequency division multiplex (OFDM), resource allocation.
I. INTRODUCTION

In a cooperative network, geographically distributed nodes share the available resources to achieve the benefits of multiple-input multiple-output systems and combat the impact of fading through relaying. The initial work in [1]–[3] sparked much research activity in this area. Of specific interest here is the decode-and-forward (DF) protocol where the relay node decodes and re-encodes the source’s data [2]. If multiple relays are available, selection, wherein sources choose one “best” relay, has been shown to provide almost all the benefits of the cooperative diversity while minimizing overhead. Most importantly, selection avoids issues of synchronization across relays. Selection cooperation has now been studied in various context [4]–[8]. However, relay selection becomes more crucial in multi-source networks when simultaneous data flows are allowed. Since each relay must split its available power amongst all the source nodes it supports, the individually optimal relay allocation scheme may not be globally optimal. Hence, the relay assignment is a combinatorial optimization problem with exponential complexity. Without addressing power allocation at the relays, Beres and Adve presented low complexity relay selection schemes for multi-source networks in [5]. In [9], Kadloor and Adve investigated the performance of a single-carrier cellular network assuming a perfect source-relay channel.

In a separate track, orthogonal frequency division multiplexing (OFDM) is an increasingly popular technique to mitigate the impact of multipath fading and enables high data rates for current and emerging wireless communication technologies. Furthermore, because each subcarrier experiences a different channel realization, resource allocation can significantly enhance performance [10]–[13]. OFDM benefits from the crucial implementation advantage that the transmitted signal can be obtained from an Inverse Fast Fourier Transform (IFFT) of the data. This IFFT is paired with a FFT at the receiver. However, as we will see, this pairing also restricts how nodes can cooperate.

The combination of OFDM and cooperative diversity has attracted an intense interest. Specifically, optimal relay assignment and dynamic subcarrier and power allocation have received significant attention. Li et al. developed a graph-theoretical approach to maximize the sum rate under fairness constraints [14]; here fairness is imposed by limiting the number of sources a single relay can help. The work in [15] maximizes the minimum rate in a two-hop cooperative network while allowing for subcarrier permutation. In [16], Ng and Yu constructed a utility
maximization framework for solving relay selection, relaying strategies, and power allocation in cellular OFDMA-based networks. Using the decomposition method and assuming a finite discrete set of rates, authors apply an exhaustive search to deal with the optimization problem. The same approach is used in [17] in order to minimize power subject to the data rate constraints on each flow. The authors of [18] proposed the resource allocation scheme for a two-hop clustered-based cellular network with relays chosen \textit{a priori}.

All these works deal with the OFDM transmission on a per-subcarrier basis, i.e., as if each were an \textit{independent} transmission. Given the importance of time and frequency synchronization in OFDM, it is unrealistic to expect distributed nodes to relay individual subcarriers independently. Furthermore, in OFDM, the raw data is channel encoded before data modulation and the IFFT; the data is spread over all subcarriers. This implies that DF requires decoding all subcarriers. Most of the subcarrier-based resource allocation is, therefore, theoretically optimal, but impractical.

In this paper, we consider a selection-based cooperative OFDM network of access points (AP) where relays use the DF protocol. We begin with the assumption that sources distribute their power equally across subcarriers and that all relay nodes can always decode each individual data streams. While this may be valid in a few practical scenarios, e.g., when relays are installed close enough to the source nodes, this is clearly not a universally valid assumption. However, as we will see, the solution based on this simplifying assumptions provide useful insights to finding the near-optimal solution for a subcarrier-based selection scheme. It has been claimed that selection is the \textit{optimal} power allocation solution [19]. Building on the work in [9], we show that this is true for \textit{most, not all}, subcarriers. Using the Karush-Kuhn-Tucker (KKT) conditions, we characterize an upper bound to the original problem which leads to the joint relay and power allocation for each subcarrier. We also derive a simple tight lower bound on the solution of the original problem. We then deal with selection for an entire OFDM block and propose a simple selection scheme, but with performance close to using an exhaustive search and not much different from the per-subcarrier relaying scheme. To the best of our knowledge, there has been little published work on selection and resource allocation at the level of an entire OFDM block.

In the next section, we solve the general form of the relay selection and resource allocation problem for OFDM-based networks irrespective to the positions of the relays, i.e., unlike previous works, we take source-to-relay (S-R), source-to-destination (S-D), and relay-to-destination (R-D)
channels into account. Furthermore, our scheme allows for direct transmission if that were optimal. By introducing time-sharing coefficients, we transform the original combinatorial problem into a standard convex optimization problem, resulting in an upper bound on performance. In addition, using the same approach, we formulate block-based selection for multi-source mesh networks and characterize the upper bound to the achievable rate of this scheme. A tight lower bound for both of these schemes can be achieved by imposing the selection constraint, i.e., each subcarrier/block is transmitted either via a single relay or directly to the destination. Finally, we propose a distributed selection scheme which offers large computational advantages, but with close-to-optimal performance. It is worth emphasizing that unlike most of the previous works, neither the transmission strategy nor the relay nodes are chosen a priori.

The remainder of this paper is organized as follows. Section II present our system model in some detail. Section III investigates node selection and resource allocation under the assumption that all relays can decode. Section IV deals with the optimization problem for both subcarrier-based and block-based schemes by considering all communication channels while also taking both selection and per-node power constraints into account. Section V presents simulation results that quantify the performance of different relaying and resource allocation algorithms. Finally, we wrap up this paper in Section VI with some conclusions.

II. SYSTEM MODEL

This paper considers a OFDM-based static mesh network of access points (APs) as shown in Fig. 1. The network comprises $K$ sources assisted by $J$ dedicated relays. Each source node has its own destination node which is not within the set of sources and relays. Let $\mathcal{K} = \{1, 2, ..., K\}$, $\mathcal{J} = \{1, 2, ..., J\}$, and $\mathcal{N}_k = \{1, 2, ..., N\}$ be the set of source nodes, relay nodes, and subcarriers of source $k$, respectively. All sources and relays are attached to the power supply and transmit with constant and maximum total power of $P$. We consider the DF relaying wherein each relay receives, decodes, and re-encodes the information with the same codebook as the transmitter, and forwards it to the destination. Nodes meet a half-duplex constraint. All transmissions use OFDM within their own frequency band, i.e., simultaneous transmissions do not interfere. We further assume that OFDM blocks are synchronized; hence, distributed transmission is possible. The inter-node wireless channels are modeled as frequency-selective. Since individual subcarriers of each source node experiences different channel realizations, adaptive transmission strategy and
implementing power allocation at the sources and relays can enhance the system performance. We furthermore assume that all inter-node channels vary slowly enough for the channel state information (CSI) to be fed back to a centralized unit with limited overhead, making resource allocation possible.

To meet the half duplex constraint, we implement a two-stage version of the DF protocol. In Fig. 1, the solid arrows indicate the first, time-sharing, stage wherein each source broadcasts its data using $N$ subcarriers and each relay receives the OFDM block from all source nodes on orthogonal channels. During the second stage, represented by dashed arrows, only those relays that can fully decode the received information are nominated to participate. Finally, the destination node combines messages received in the two phases to decode the original information. This paper considers two cooperative scenarios; treating each subcarrier as an independent transmission and cooperation at the level of an entire OFDM block.

The focus of this paper is to achieve max-min fairness in a multi-source mesh network, i.e., to maximize the minimum rate across all source nodes. In the next section, we present resource allocation schemes in such networks under the assumption that all relay nodes can successfully decode received symbols.

III. RELAY ASSIGNMENT AND RESOURCE ALLOCATION WITH PERFECT S-R CHANNELS

This section develops optimal relay selection and power allocation in a subcarrier-based fashion to achieve max-min fairness. Furthermore, two block-based relaying schemes with different complexities are proposed. The assumption of a perfect S-R channel is valid when the relays are close to the sources or the S-R channels have a line-of-sight component. However, for our purposes, this is largely a simplifying assumption that helps us develop insight to solution methodologies for the general problem in Section IV.

A. Subcarrier-Based Resource Allocation with Perfect S-R Channels

In this section, each subcarrier is treated as an independent transmission. In practice, the total number of subcarriers in a network, $KN$, is much higher than the number of relay nodes, $J$. Hence, a relay is most probably required to support multiple subcarriers. In order to meet its power constraint, a relay must distribute its available power amongst all subcarriers that it supports.
Under these conditions, the achievable rate of source $s_k$ over its $n^{th}$ subcarrier is

$$R_k^{(n)} = \max_j \min \left\{ I_{s_k r_j}^{(n)}, I_{s_k r_j d_k}^{(n)} \right\},$$

$$I_{s_k r_j}^{(n)} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{k j}^{(n)} \alpha_{0k}^{(n)} | h_{kj}^{(n)} |^2 \right),$$

$$I_{s_k r_j d_k}^{(n)} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{0k}^{(n)} \alpha_{0k}^{(n)} | h_{0k}^{(n)} |^2 + \text{SNR}_{jk}^{(n)} \alpha_{jk}^{(n)} | h_{jk}^{(n)} |^2 \right),$$

where $\text{SNR}_{0k}$, $\text{SNR}_{k j}$, and $\text{SNR}_{jk}$ are the ratios of the total transmitted power to the power of noise; $h_{0k}^{(n)}$, $h_{kj}^{(n)}$, and $h_{jk}^{(n)}$ denote the complex S-D, S-R, and R-D channels over the $n^{th}$ subcarrier of $s_k$. $\alpha_{0k}^{(n)}$ and $\alpha_{jk}^{(n)}$ are, respectively, the fraction of the allocated power to the $n^{th}$ subcarrier of source $k$ at the source node and relay $j$. Eqn. (1) declares that the rate of each source node over its individual subcarriers is the minimum of the S-R rate (Eqn. (2)) and the compound S-R-D rate (Eqn. (3)), i.e., the cooperative rate requires that both the relay and destination fully decode the received data. The total rate then is $R_k = \sum_{n_k} R_k^{(n)}$.

We first assume that all S-R channels are strong enough that

$$I_{s_k r_j}^{(n)} \geq I_{s_k r_j d_k}^{(n)} \Rightarrow R_k^{(n)} = I_{s_k r_j d_k}^{(n)}, \quad \forall k, n.$$

It is important to note that optimal power allocation at the source nodes can improve the communication rates on some channels while degrading the performance over others. Thus, considering the assumption that relays and destination nodes are required to fully decode the received symbols, only equal power allocation is applicable at the sources, i.e., $\alpha_{0k}^{(n)} = 1/N$. We, therefore, investigate optimal power allocation problem only at the relays.

In keeping with its many benefits described earlier, we impose a selection constraint in the second, relaying, phase, i.e., each subcarrier of a source node is relayed through at most one of the relays in the network. Therefore, the optimization problem we wish to solve is

$$\max_{\alpha} \min_k R_k$$

$$\text{s.t. } C_1 : \alpha_{j1k}^{(n)} \times \alpha_{j2k}^{(n)} = 0, \quad \forall k, n, \text{ and } j_1 \neq j_2,$$

$$C_2 : \alpha_{jk}^{(n)} \geq 0, \quad \forall j, k, n,$$

$$C_3 : \sum_k \sum_{n_k} \alpha_{jk}^{(n)} = 1, \quad \forall j,$$

wherein $R_k = \sum_{n_k} \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{0k} | h_{0k}^{(n)} |^2 + \sum_{j \in J} \text{SNR}_{jk} \alpha_{jk}^{(n)} | h_{jk}^{(n)} |^2 \right)$.
Constraint $C_1$ enforces selection by allowing only one node to devote power to each subcarrier. Constraints $C_2$ and $C_3$ state that the amount of allocated power must be non-negative and that the total available power of all relays is limited. Due to the selection constraint, (4)-(7) is an, essentially intractable, mixed-integer programming optimization problem. One proposed solution [11], [13] separates the power allocation and selection problems. First, subcarriers are selected assuming equal power allocation; then, power is distributed based on this selection. However, with $K$ sources, $J$ relays, and $N$ subcarriers, there are $J^{KN}$ relay assignments to be checked. Therefore, even this scheme is infeasible for realistic values of $K$, $J$, and $N$. We build on an alternative approach developed in [9] to form an approximate solution that is also an upper bound.\[1]\*\*2\**

1) An Approximate Solution and Upper Bound: The objective function of the optimization problem is increasing and concave in $\alpha_{jk}^{(n)}$. Other than the integer constraint of (5), the constraints in the original problem of (4)-(7) are convex. In order to find an approximate, tractable, solution we first ignore the selection constraint. Since we relax a constraint, the solution to this modified optimization problem will be an upper bound to the original subcarrier-based (UBSB) resource allocation problem. The revised formulation, stated here in epigraph form, is a concave maximization problem solvable in polynomial time using available efficient solvers [20].

$$\max_{\{t, \alpha\}} t$$

$$C_1 : \sum_{N_k} \frac{1}{2} \log_2 \left(1 + \frac{\text{SNR}_{0k} |h_{0k}^{(n)}|^2}{2} + \sum_{J} \frac{\text{SNR}_{jk} \alpha_{jk}^{(n)} |h_{jk}^{(n)}|^2}{2} \right) - t \geq 0, \ \forall k,$$

$$C_2 : \alpha_{jk}^{(n)} \geq 0, \ \forall j, k, n, \ C_3 : \sum_{K} \sum_{N_k} \alpha_{jk}^{(n)} = 1, \ \forall j.$$

The solution to this convex optimization problem is characterized by the KKT conditions [20]. The Lagrangian is given by

$$L(\alpha_{jk}^{(n)}, \gamma_k, \mu_j, \lambda_{jkn}) = t + \sum_{K} \sum_{N_k} \frac{1}{2} \log_2 \left(1 + \frac{\text{SNR}_{0k} |h_{0k}^{(n)}|^2}{2} + \sum_{J} \frac{\text{SNR}_{jk} \alpha_{jk}^{(n)} |h_{jk}^{(n)}|^2}{2} \right) - t$$

$$+ \sum_{J} \mu_j \left(1 - \sum_{K} \sum_{N_k} \alpha_{jk}^{(n)} \right) + \sum_{J} \sum_{K} \sum_{N_k} \lambda_{jkn} \alpha_{jk}^{(n)}.$$

\[1\] It is worth emphasizing that while the solution methodology here is similar to that of [9], both our problem formulation and solution are significantly different. The development here, using the epigraph form, leads to effective solutions to OFDM-based relaying and allows us to show that selection is a sub-optimal solution to the resource allocation problem.
For the sake of clarity, let us assume that $J = 2$, i.e., a cooperative network comprising two relays. Since the problem is a standard convex optimization problem and satisfies Slater’s conditions, any solution for the amount of power that relays $r_1$ and $r_2$ allocate to the $n^{th}$ subcarrier of source $k$ satisfies the KKT conditions. Let us suppose that both relays, $r_1$ and $r_2$, allocate some power to the $n^{th}$ subcarrier of $s_k$. Therefore, the convex problem satisfies the complementary slackness condition if, $\lambda_{r_1 kn} = \lambda_{r_2 kn} = 0$. Now, we can conclude that $\frac{\mu_1}{\mu_2} = \frac{|h_{1 r_1 k}|^2}{|h_{2 r_2 k}|^2}$. Similarly, if the same two nodes contribute to relaying the $i^{th}$ subcarrier of source $k$, using the same KKT conditions, $\frac{\mu_1}{\mu_2} = \frac{|h_{1 r_1 k}|^2}{|h_{2 r_2 k}|^2}$. These two equations cannot be simultaneously satisfied since channel gains are continuous random variables. Thus, contradicting the claim in [19], at most one subcarrier of each source can be helped by more than one relay.

Now, let us evaluate all the possible relay selections in the network with $J = 3$, assuming the $n^{th}$ subcarrier of source $k$ is being helped by all relay nodes. The KKT conditions state that $\frac{\mu_1}{|h_{1 r_1 k}|^2} = \frac{\mu_2}{|h_{2 r_2 k}|^2} = \frac{\mu_3}{|h_{3 r_3 k}|^2}$. Now suppose that the $i^{th}$ subcarrier of the same source is relayed through $r_1$ and $r_2$, i.e., $\frac{\mu_3}{|h_{1 r_1 k}|^2} = \frac{\mu_2}{|h_{2 r_2 k}|^2}$. Thus, we have $|h_{r_1 k}|^2/|h_{r_2 k}|^2 = |h_{r_1 k}|^2/|h_{r_2 k}|^2$, which is a zero-probability event.

Consider, again, the scenario in which none of the subcarriers can be helped with all three relays. As an example, consider the case wherein the $n^{th}$ subcarrier is relayed via node $r_1$ and $r_2$ and the $i^{th}$ subcarrier can be helped by node $r_1$ and $r_3$. Applying the same KKT conditions, it follows that $\frac{\mu_3}{|h_{1 r_1 k}|^2} = \frac{\mu_2}{|h_{1 r_1 k}|^2}$ and $\frac{\mu_3}{|h_{2 r_2 k}|^2} = \frac{\mu_2}{|h_{2 r_2 k}|^2}$. Now, the $m^{th}$ subcarrier can be helped by node $r_2$ and $r_3$ only if $\frac{\mu_3}{|h_{1 r_1 k}|^2 |h_{1 r_1 k}|^2} = \frac{\mu_2}{|h_{2 r_2 k}|^2 |h_{2 r_2 k}|^2}$, which happens with zero probability. Therefore, when two subcarriers are relayed by two nodes, all others can be helped by at most one node.

Generalizing this to the network with $K$ sources and $J$ relays, one concludes that at most $J - 1$ subcarriers of each source can be helped by more than one relay and selection is imposed on $(N - J + 1)$ subcarriers. In practice, $N \gg J$ which means that a large fraction of subcarriers meet the selection criterion, i.e., selection is an approximate, though not optimal solution, to the relaxed optimization problem.

2) A Heuristic Algorithm and a Lower Bound: By neglecting the selection constraint, the solution to the modified problem provides an upper bound to that of the original optimization problem in (4)-(7). Here, we use this to develop a heuristic solution to the original problem. We force the (maximum of $J - 1$) subcarriers that do not meet the constraint to receive power only
from the single relay that achieves a higher data rate. Mathematically speaking

\[ r_k^{(n)} = r_m^{(n)}, \quad m = \arg \max_j \left\{ \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{ok} \left| h_{0k}^{(n)} \right|^2 + \text{SNR}_{jk} \alpha_j^{(n)} \left| h_{jk}^{(n)} \right|^2 \right) \right\}, \]

where \( r_k^{(n)} \) is the relay node which contributes to the transmission of source \( k \) on the \( n^{th} \) subcarrier. Since this solution meets all the constraints of the original problem, this is also a lower bound on the subcarrier based (LBSB) optimization problem. In Section V, we will show that the upper and lower bounds are indistinguishable. As a result, this heuristic approach provides almost the exact solution to the original mixed-integer optimization problem with significantly reduced solution complexity.

**B. Block-Based Resource Allocation with Perfect S-R Channels**

The optimization problem and solution detailed so far is in keeping with existing literature. It allows different subcarriers within an OFDM block to be helped by different relays. This is problematic for two reasons. One, while not explicitly stated, most of the previous work assumes a relay can treat each subcarrier as an independent transmission. In DF-based relaying, the decoding constraint is at the level of a subcarrier, e.g., (1). However, in OFDM, the data is first protected by a channel code, modulated and then a block of \( N \) subcarriers is formed. It is not possible to decode information without receiving and decoding an entire OFDM block. Second, practical OFDM systems depend heavily on accurate time and frequency synchronization. This would be extremely difficult in a distributed mesh network.

In a multi-source network, as long as each relay has to divide its available power amongst all allocated sources, the solution to the relay assignment problem is not immediate. Here, we separate the problem into selection followed by power allocation (via waterfilling) across subcarriers. As in [5], two different selection schemes with different levels of complexity are proposed and results will be compared in terms of the max-min rate in Section V.

1) **Optimal Relay Selection**: In a network with \( K \) sources and \( J \) relays, there are \( J^K \) different possible relay assignments. The optimal scheme is exhaustive search over all possible relay selections and pick the one which provides the maximal minimum rate. This is clearly impossible for any reasonable \( K \) and \( J \).

2) **Decentralized Relay Selection**: The decentralized or simple relay selection scheme ignores all other sources. Each source chooses its best relay with the assumption that the corresponding
relay distributes its power equally over all subcarriers of only that source. In particular

\[ r_k = r_m, \quad m = \arg \max_j \left( \sum_{N_k} \log_2 \left( 1 + \text{SNR}_{jk} |h_{jk}^{(n)}|^2 \right) \right). \]

With each source having selected the relays, power is allocated via waterfilling, to the assigned sources. Note that since each source-destination pair only needs local CSI and selection is performed independently of all other sources, this scheme can be implemented in a decentralized manner. In a network with \( J \) dedicated relays, only \( J \) water-filling problems need to be solved.

### IV. Relay Assignment and Resource Allocation with Imperfect S-R Channels

The previous section developed solutions under the assumption of a perfect S-R channel. In this section we consider the general case of resource allocation across the S-D, S-R, and R-D channels. The solution to this optimization problem also chooses the best transmission strategy for each source, i.e., direct transmission is a valid solution if that were optimal. Our approach also allows us to move beyond heuristics for block-based relaying.

#### A. Subcarrier-Based Resource Allocation with Imperfect S-R Channels

Given the fact that each source is allowed to switch between DF and direct transmission, one concludes that in a network with \( K \) sources and \( J \) relays

\[ R_k = \sum_{N_k} R_k^{(n)}, \quad R_k^{(n)} = \max \left\{ \mathcal{I}_{s_kd_k}^{(n)}, \max_j \min \left\{ \mathcal{I}_{s_kr_j}^{(n)}, \mathcal{I}_{s_kr_jd_k}^{(n)} \right\} \right\}, \]

where \( \mathcal{I}_{s_kd_k}^{(n)} = \log_2 \left( 1 + \text{SNR}_{0k} \alpha_{0k}(n) |h_{0k}^{(n)}|^2 \right) \). Eqn. (8) declares that the rate of each source node over its individual subcarriers is the maximum of the direct and cooperative transmission rates; in turn, the cooperative rate requires that both the relay and destination fully decode the received data. The total rate is then the sum of achievable rates of all subcarriers. In addition, by taking the S-R channel into account, optimal power allocation at the source nodes may alter the relay selection and further enhance the performance of the network.

Let \( \mathcal{J}_+ = \{0, 1, 2, ..., J\} \) be the extended set of relays. Therefore, the formal optimization
problem is
\[
\max \min_k R_k \tag{9}
\]
\[
s.t. \quad C_1 : \alpha_{j_1k}^{(n)} \times \alpha_{j_2k}^{(n)} = 0, \ \forall k, n, \{j_1, j_2\} \in J, \tag{10}
\]
\[
C_2 : \alpha_{jk}^{(n)} \geq 0, \ \forall k, n, j \in J_+, \tag{11}
\]
\[
C_3 : \sum_k \sum_{j \notin J_+} \alpha_{jk}^{(n)} \leq 1, \ \forall j \in J, \tag{12}
\]
\[
C_4 : \sum_k \alpha_{0k}^{(n)} = 1, \ \forall k, \tag{13}
\]
Eqns. (10)-(11) are equivalent to the constraints (5)-(7) of the original optimization problem of the previous section. Unlike the previous optimization problem, since source nodes are allowed to transmit directly, some relays might stay silent during the second time-slot. Hence, as stated in Eqn. (12), the power constraint can be satisfied by inequality. Eqn. (13) limits the available power of each source node. Similar to the previous scenario, since each relay must split its available power amongst all source nodes which it supports, transmission strategy selection, relay assignment, and power allocation problem is combinatorial and needs to be solved jointly.

1) An Approximate Solution and Upper Bound: To make the problem mathematically tractable, we introduce $KN(J + 1)$ indicator variables to the objective function. Therefore, the new optimization problem can be expressed as
\[
\max \{\alpha, \rho \in \{0, 1\}^J\} \min_k R_k \\
\text{s.t.} \quad C_1 : \alpha_{jk}^{(n)} \geq 0, \ \rho_{jk}^{(n)} \geq 0, \ \forall k, n, j \in J_+, \quad C_2 : \sum_k \sum_{j \notin J_+} \rho_{jk}^{(n)} \alpha_{jk}^{(n)} \leq 1, \ \forall j \in J, \\
C_3 : \sum_{j \notin J_+} \sum_{k} \rho_{jk}^{(n)} \alpha_{0k}^{(n)} = 1, \ \forall k, \quad C_4 : \sum_{j \notin J_+} \rho_{jk}^{(n)} = 1, \ \forall k, n.
\]
From this modified problem, one can conclude that if $s_k$ allocates a fraction of its available power to the $n^{th}$ subcarrier, $\alpha_{0k}^{(n)} \neq 0$, for any set of $\alpha_{jk}^{(n)}$ satisfying (10)-(12)
\[
\rho_{jk}^{(n)} = \begin{cases} 
1, & \alpha_{jk}^{(n)} \neq 0, \\
0, & \alpha_{jk}^{(n)} = 0.
\end{cases} \tag{14}
\]
Eqn. (14) along with the fact that only one indicator variable of each source can be non-zero at a time enforces the selection constraint of the original problem in this revised problem. Moreover,
the total rate of $s_k$ is

$$R_k = \sum_{N_k} \rho_{0k}^{(n)} \log_2 \left( 1 + \text{SNR}_{0k}\alpha_{0k}^{(n)} |h_{0k}^{(n)}|^2 \right) +$$

$$\sum_{\mathcal{J}} \sum_{N_k} \rho_{jk}^{(n)} \min \left\{ \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{kj}\alpha_{0k}^{(n)} |h_{kj}^{(n)}|^2 \right), \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{0k}\alpha_{0k}^{(n)} |h_{0k}^{(n)}|^2 + \text{SNR}_{jk}\alpha_{jk}^{(n)} |h_{jk}^{(n)}|^2 \right) \right\}.$$

Note that, since indicator variables can only take integer values, the problem is still a combinatorial optimization problem. As in the previous section, our strategy to solve this problem is to relax the corresponding constraint and allow each stream to be transmitted both directly as well as cooperatively through multiple relays. Thus, indicator variables of each individual subcarriers can take any rational value on the convex hull of the original discrete set. Consequently, the resulting solution from the relaxed problem is an upper bound to the min-rate of the original subcarrier-based problem (UBSB) formulated in (9)-(13). Furthermore, $\rho_{jk}^{(n)}$ can now be interpreted as a fraction of time that $s_k$ transmits over its $n^{th}$ subcarrier directly ($j = 0$) and cooperatively ($j \in \mathcal{J}$).

$R_k$ consists of three different terms: S-D, S-R, and S-R-D rates. One can simply show that none of them is jointly concave in the set of variables. Using the approach of [10], we set

$$\rho_{jk}^{(n)} \alpha_{jk}^{(n)} = r_{jk}^{(n)}, j \in \mathcal{J}_+, \quad \rho_{jk}^{(n)} \alpha_{jk}^{(n)} = p_{jk}^{(n)}, j \in \mathcal{J}.$$

It is worth noting that this is a key difference from the work in [15], [21] which did not take the coupling constraint between time-sharing coefficients and power into account. Thus, the new optimization problem in terms of $(\rho, r, p)$ can be formulated as

$$\max_{(\rho, r, p)} \min_k R_k$$

s.t. $C_1 : \rho_{jk}^{(n)} \geq r_{jk}^{(n)} \geq 0, \forall k, n, j \in \mathcal{J}_+$, (16)

$$C_2 : \rho_{jk}^{(n)} \geq p_{jk}^{(n)} \geq 0, \forall k, n, j \in \mathcal{J},$$ (17)

$$C_3 : \sum_{\mathcal{K}} \sum_{N_k} p_{jk}^{(n)} \leq 1, \forall j \in \mathcal{J},$$ (18)

$$C_4 : \sum_{\mathcal{J}_+} \sum_{N_k} r_{jk}^{(n)} = 1, \forall k,$$ (19)

$$C_5 : \sum_{\mathcal{J}_+} \rho_{jk}^{(n)} = 1, \forall k, n.$$ (20)
\( R_k \) is therefore rewritten as

\[
R_k = \sum_{N_k} \rho_{0k}^{(n)} \log_2 \left( 1 + \frac{\text{SNR}_{0k} r_{0k}^{(n)} |h_{0k}^{(n)}|^2}{\rho_{0k}^{(n)}} \right) + \\
\sum_{\mathcal{J}} \sum_{N_k} \rho_{jk}^{(n)} \min \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{\text{SNR}_{jk} r_{jk}^{(n)} |h_{jk}^{(n)}|^2}{\rho_{jk}^{(n)}} \right), \frac{1}{2} \log_2 \left( 1 + \frac{\text{SNR}_{jk} r_{jk}^{(n)} |h_{jk}^{(n)}|^2}{\rho_{jk}^{(n)}} + \frac{\text{SNR}_{jk} p_{jk}^{(n)} |h_{jk}^{(n)}|^2}{\rho_{jk}^{(n)}} \right) \right\}.
\]

**Theorem 1:** The objective function in (15) is jointly concave in \( \rho, r, \) and \( p. \)

**Proof:** The S-D and S-R rates are in the form of \( f(x, y) = x \log (1 + y/x) \) and the rate of the compound S-R-D channel is in the form of \( g(x, y, z) = x \log (1 + y/x + z/x). \) In addition, \( x, y, \) and \( z \) are non-negative variables. One can show that the Hessian of \( f \) is

\[
\nabla^2 f = \frac{1}{(1 + y/x)^2} \begin{bmatrix}
-y^2/x^3 & y/x^2 \\
(y/x^2) & -1/x
\end{bmatrix}.
\]

The determinant of \( \nabla^2 f, \) the product of the eigenvalues, is zero. The trace of the \( \nabla^2 f, \) the sum of the eigenvalues, is a negative value, which certifies that the \( \nabla^2 f \leq 0, \) i.e., the Hessian evaluated within the optimization region is a negative semi-definite matrix. Now, let us follow the same strategy to show that the third term is also jointly concave in the set of consisting variables. Therefore

\[
\nabla^2 g = \frac{1}{(1 + y/x + z/x)^2} \begin{bmatrix}
-(y + z)^2/x^3 & (y + z)/x^2 & (y + z)/x^2 \\
(y + z)/x^2 & -1/x & -1/x \\
(y + z)/x^2 & -1/x & -1/x
\end{bmatrix}.
\]

Similar to the previous case, the determinant and trace of the \( \nabla^2 g \) are, respectively, zero and negative. Moreover, \( \nabla^2 g \) is a rank one matrix, i.e., it has one non-positive eigenvalue and two zero ones. Thus, \( \nabla^2 g \) is a negative semi-definite matrix which proves that the rate of the S-R-D channel is jointly concave in \((\rho, r, p).\) It is also known that a point-wise minimum and the non-negative summation of a set of concave functions are also concave functions [20]. Hence, the underlying objective function is jointly concave in \((\rho, r, p).\)

Although the objective function is jointly concave, it is not differentiable. By rewriting it in the
epigraph form, the final optimization problem can be stated as follows

\[
\max_{\{t, \zeta, \rho, r, p\}} t
\]

s.t. \( C_1: \) \( (16) - (20) \)
\[
C_2: \sum_{J_k} \sum_{N_k} \zeta^{(n)}_{jk} \geq t, \forall k;
\]
\[
C_3: \rho^{(n)}_{0k} \mathcal{C} \left( \frac{\text{SNR}_{0k} r^{(n)}_{0k} |h^{(n)}_{0k}|^2}{\rho^{(n)}_{0k}} \right) \geq \zeta^{(n)}_{0k}, \forall k, n;
\]
\[
C_4: \frac{\rho^{(n)}_{jk}}{2} \mathcal{C} \left( \frac{\text{SNR}_{kj} r^{(n)}_{jk} |h^{(n)}_{kj}|^2}{\rho^{(n)}_{jk}} \right) \geq \zeta^{(n)}_{jk}, \forall k, n, j \in J;
\]
\[
C_5: \frac{\rho^{(n)}_{jk}}{2} \mathcal{C} \left( \frac{\text{SNR}_{0k} r^{(n)}_{0k} |h^{(n)}_{0k}|^2 + \text{SNR}_{kj} r^{(n)}_{kj} |h^{(n)}_{kj}|^2}{\rho^{(n)}_{jk}} \right) \geq \zeta^{(n)}_{jk}, \forall k, n, j \in J,
\]

where \( \mathcal{C}(x) = \log_2(1 + x) \). The modified optimization problem is a standard convex optimization problem which can be solved using well established and efficient iterative algorithms [20].

2) A Heuristic Algorithm and a Lower Bound: The upper bound derived in the previous section approximates, but does not meet the selection constraint. As in Section III-A2, our approach to imposing selection is to assign to each subcarrier the transmission strategy and the relay that provides the maximum achievable rate. The selection constraint is enforced as

\[
R_k^{(n)} = \max \left\{ I_{s_k d_k}^{(n)}, \min \{ I_{s_k r_m}, I_{s_k r_m d_k}^{(n)} \} \right\}, \quad m = \arg \max_j \min \left\{ I_{s_k r_j}, I_{s_k r_j d_k}^{(n)} \right\}.
\]

Since this solution satisfies all constraints of the original problem in (9)-(13), this heuristic scheme provides a lower bound (LBSB). Moreover, the power freed up by the selection step can be reused by waterfilling over other source nodes which are helped by each individual relays. However, as we show in Section V the performance gap is not noticeable; thus, there is no need to apply a second round of waterfilling. Finally, it is worth emphasizing that if direct transmission were optimal, the power allocated at all relays would be zero, i.e., the approach is adaptive across relay strategies.

B. Block-Based Resource Allocation with Imperfect S-R Channels

This section deals with the selection and power allocation at the level of an entire OFDM block in the general case of imperfect S-D, S-R and R-D channels. As in the previous section, the solution to this problem optimizes the transmission strategy for each individual source node.
The achievable rate of each source node across the whole OFDM block is
\[
R_k = \max \left\{ \sum_{N_k} I_{s_kd_k}, \max_j \left\{ \sum_{N_k} I_{s_kr_j}, \sum_{N_k} I_{s_kr_jd_k} \right\} \right\},
\]
which states that each block of OFDM can be transmitted either directly or via the relay node which supports a higher data rate. The formal optimization problem therefore is
\[
\max_{\alpha} \min_k R_k \\
\text{s.t. } C_1 : \sum_{N_k} \alpha_{j_1k} \times \sum_{N_k} \alpha_{j_2k} = 0, \forall k, n, \{j_1, j_2\} \in J, \\
C_2 : \alpha_{jk}^{(n)} \geq 0, \forall k, n, j \in J_+, \\
C_3 : \sum_{N_k} \sum_{N_k} \alpha_{jk}^{(n)} \leq 1, \forall j \in J, \quad C_4 : \sum_{N_k} \alpha_{0k}^{(n)} = 1, \forall k.
\]
\(C_1\) states that each OFDM block can be helped by at most one relay node. Other constraints are similar to those of the original subcarrier-based scheme formulated in Section IV-A.

1) An Approximate Solution and Upper Bound: The block-level optimization problem is again combinatorial with exponential complexity. Thus, again, we introduce \(K(J + 1)\) time-sharing coefficients to the objective function and rewrite the achievable rate of source \(k\) as
\[
R_k = \rho_{0k} \sum_{N_k} \log_2 \left(1 + \text{SNR}_{0k} \alpha_{0k}^{(n)} |h_{0k}^{(n)}|^2 \right) + \\
\sum_{J} \rho_{jk} \min \left\{ \frac{1}{2} \sum_{N_k} \log_2 \left(1 + \text{SNR}_{jk} \alpha_{0k}^{(n)} |h_{jk}^{(n)}|^2 \right), \sum_{N_k} \frac{1}{2} \log_2 \left(1 + \text{SNR}_{0k} \alpha_{0k}^{(n)} |h_{0k}^{(n)}|^2 + \text{SNR}_{jk} \alpha_{jk}^{(n)} |h_{jk}^{(n)}|^2 \right) \right\}.
\]
Following the same approach as the previous section, we relax the selection constraint and set
\[
\rho_{jk} \alpha_{0k}^{(n)} = r_{jk}^{(n)}, \quad j \in J_+, \quad \rho_{jk} \alpha_{jk}^{(n)} = p_{jk}^{(n)}, \quad j \in J.
\]
Using Theorem 1, it is straightforward to prove that the resulting optimization problem is jointly concave in \((\rho, r, p)\). Finally, by rewriting the objective function in epigraph form, the standard convex optimization problem can be formulated. Since we relaxed the selection constraint, this solution provides an upper bound to the minimum rate of the original block-based relaying (UBBB). The approach developed in the Section IV-A therefore provides the basis for block-based optimization as well.

2) A Heuristic Algorithm and a Lower Bound: Having generalized our approach in Section IV-A2 to the block-based relaying, the lower bound to the block-based scheme (LBBB) can be applied by choosing the best relay for individual source nodes.
3) Decentralized Resource Allocation: The optimization problem and solution detailed so far is to jointly select the transmission strategy, the relay node, and to allocate power to each source in the network. This solution requires a central resource allocation unit which has the full CSI of all channels. The required transmission and coordination overhead will likely make this impractical. In this section, we develop a simplified three-stage decentralized scheme, wherein, similar to [III-B2], at the first stage each source selects its best relay independently

\[ r_k = r_m, \quad m = \arg \max_j \left\{ \min_{N_k} \{ \sum_{I_s} I_{s_k r_j}, \sum_{I_d} I_{s_k r_j d_k} \} \right\}. \]

Second, the transmission strategy is chosen by comparing the rates of the direct and relaying transmissions. Given that each individual source node has already selected its transmission strategy, at most \( J \) waterfilling problems need to be solved to maximize the minimum rate across source nodes. Furthermore, if a source node has selected to transmit directly, power is distributed based on the S-D channel state. In Section [V] we will show that, in fact, the performance of the distributed scheme closely tracks that of the optimum algorithm.

V. SIMULATION RESULTS AND DISCUSSION

This section presents simulation result for the proposed relay selection and resource allocation schemes described in Sections [III] and [IV]. We consider two different network geometries. In the first scenario, all inter-node channels are modeled as independent and identically distributed (i.i.d.) random variables. The second network setup is a more realistic scenario wherein nodes are randomly distributed. The communication channels are modeled using the COST-231 channel model recommended by the IEEE 802.16j working group [22]. Therefore, inter-node channels have uneven average power. The chosen parameters for the COST-231 are given in Table [I].

A. Resource Allocation with Perfect S-R Channels in I.I.D. Scenario

Our first example implements relay selection and resource allocation for a mesh network with \( K = 3 \) or \( K = 4 \) access points and 32 subcarriers. The S-R channels are assumed perfect; the average SNR of all S-D channels is set to 5dB. Figure [2] plots the minimum achievable rate across the \( K \) source nodes for different values of the R-D SNRs. As can be seen from the figure, the upper and lower bounds are indistinguishable. In this setup, at most one of the subcarriers of
each source node can be helped by both relays. Since the number of subcarriers, $N$, is generally much larger than the number of relays, $J$, selection is close-to-optimal.

Given the additional flexibility of subcarrier-based cooperation schemes, both UBSB and LBSB outperform block-based schemes. Moreover, although the optimal block-based relaying scheme is computationally much more complex than the decentralized scheme, the performance benefit is negligible. Enforcing direct transmission has the worst performance, validating the fact that cooperation transmission can boost network performance under the max-min metric.

B. Resource Allocation with Perfect S-R Channels in Distributed Scenario

In this example, nodes are geographically distributed and inter-node channels are modeled using the COST-231 channel model. We generate the random node locations over an square area of $200m \times 200m$. Source and destination nodes are located on the edges of the square area while relays are randomly distributed inside it. The transmitted power of each potential node is also fixed to $[26, 28, 30, 32, 34]$ dBm. The variance of the log-normal fading is set to $10.6dB$. In this experiment, for each set of locations, independent channel realizations are simulated and results averaged over both node locations and channel realizations.

Figure 3 plots the max-min achievable rate across all APs and compares the performance of various resource allocation schemes. From the figure, the performance gap between UBSB and LBSB is, again, negligible. This proves that the heuristic method to find the solution of the original convex optimization problem is almost exact. However, it worth emphasizing that in both Figs. 2 and 3 there is a difference, albeit minuscule, between the UBSB and LBSB performance; selection, is an approximate, not optimal solution.

Figure 3 also compares the performance of block-based schemes. Simple relay selection closely tracks the optimal relay selection method, but with significantly less complexity. This result indicates that the simple relay selection scheme can be implemented in decentralized manner without significant performance loss.

Figure 4 illustrates the importance of node locations on the performance of different resource allocation schemes. This example simulates a single source-destination pair with two relay nodes. The S-D distance is fixed to $0.2\sqrt{2}$ km. Relays are located on both sides of S-D path. Clearly one wants to use the relay close to the destination; however, note that this may impact on the assumption that the relay can always decode. Simulation results show that relaying schemes
outperform direct transmission whenever relays are located between the source and destination nodes. While the upper bound on subcarrier-based selection outperforms block-based selection, the performance loss for this more practical approach is surprisingly small.

C. Resource Allocation with Imperfect S-R Channels in I.I.D. Scenario

With imperfect S-R channels, we now use the comprehensive resource allocation and relay assignment schemes developed in Section IV. Figures 5 ($K = 3$) and 6 ($K = 4$) plot the achievable minimum rate across source nodes for various values of R-D SNRs. The SNR of the S-R and S-D channels are, respectively, set to 10dB and 5dB. Both figures show that at high SNRs, subcarrier-based methods outperform other resource allocation schemes. This is expected since subcarrier-based methods exploit the frequency diversity across relays provided by the assumption that individual subcarriers can be transmitted independently. However, at low SNRs, the UBBB outperforms the LBSB scheme and the decentralized selection scheme outperforms the centralized LBBB. This can be explained by recognizing the fact that our heuristic method to impose selection on individual flows does not use all available power at the relay nodes. If we apply a second round of power allocation at the relays, power freed up from enforcing selection can be distributed amongst all other source nodes which are assigned to those relays and a tighter lower bound will result.

D. Resource Allocation with Imperfect S-R Channels in Distributed Scenario

Figure 7 plots the minimum rate across users versus the maximum available power of the sources and relays when the nodes are geographically distributed and channels are simulated using the COST-231 model. Although the decentralized scheme uses only local CSI, it has a close-to-optimal performance. This method also decreases the computation and coordination burden of the network. Again, since LBBB does not use the total available power, it is probable that its achievable rate is less than that of the decentralized scheme.

VI. CONCLUSION

This paper developed subcarrier and block-based relaying schemes in the context of selection-based OFDM networks in order to maximize the minimum rate across sources. The first part of this paper investigated the resource allocation problem by assuming that all relay nodes can
decode the received signals and that the source allocates power equally across subcarriers. Since
the resulting mixed integer programming problem is computationally complex, we relaxed the
selection constraint and formulated the convex optimization problem that provides a tight upper
bound. We showed that selection is violated in a maximum of \( J - 1 \) out of \( N \) subcarriers.
This in turn leads to a heuristic solution to the original problem and a tight lower bound. We
then considered block-based relaying for multi-source networks. Two cooperation schemes with
different computation complexities are proposed. Simulation results showed that the simple relay
selection scheme offers computational benefits compare to the optimal relay selection scheme
while resulting in negligible performance loss.

The second part of this paper considers the resource allocation problem, while unlike previous
works, taking the S-D, S-R, and R-D channels into account. We formulated the underlying
problem with a selection constraint on each subcarrier/block which ensures that not only the
destination node but also the potential relay can fully decode the received information. Introdu-
cing the time-sharing factors into the objective functions of both subcarrier-based and block-
based schemes and relaxing the selection constraint lead to upper bounds to the solution of the
seemingly difficult original problems. The solution to the relaxed problem simultaneously solves
transmission strategy selection (DF v/s no relaying), relay assignment, and power allocation
problem. Imposing the selection constraint on each flow provides a heuristic solution to the
original problems and lower bounds for both relaying schemes. We also proposed a simple,
decentralized, relaying scheme and the required guidelines to select the best transmission strategy
and relay for sources. Compared to the subcarrier-based and block-based resource allocation
schemes, this method significantly decreases the required computational complexity while it has
a close-to-optimal performance under a realistic (COST-231) channel model.

REFERENCES


Fig. 1. Cooperative OFDM-based multi-source multi-destination mesh network with $K = 3$ and $J = 2$.

### TABLE I

PARAMETER VALUES IN COST-231

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP Height</td>
<td>15m</td>
<td>Frequency</td>
<td>3.5 GHz</td>
</tr>
<tr>
<td>Building Spacing</td>
<td>50m</td>
<td>Rooftop Height</td>
<td>30m</td>
</tr>
<tr>
<td>Destination Height</td>
<td>15m</td>
<td>Road Orientation</td>
<td>90 deg.</td>
</tr>
<tr>
<td>Street Width</td>
<td>12m</td>
<td>Noise PSD</td>
<td>-174 dBm</td>
</tr>
</tbody>
</table>

Fig. 2. Achievable min rate of different resource allocation strategies in “i.i.d channel” scenario with $J = 2$ and $N = 32$. 
Fig. 3. Achievable min rate of different resource allocation strategies in “distributive” scenario with $J = 2$ and $N = 16$.

Fig. 4. Source transmission rate in a single source-destination pair network with $J = 2$ and $N = 16$. 
Fig. 5. Max-min rate across all source nodes of different resource allocation strategies in the “i.i.d. channel” scenario with $K = 3$, $J = 2$, and $N = 8$.

Fig. 6. Max-min rate across all source nodes of different resource allocation strategies in the “i.i.d. channel” scenario with $K = 4$, $J = 2$, and $N = 8$. 
Fig. 7. Max-min rate across all source nodes of different resource allocation strategies in the “distributive” scenario with $K = 3$, $J = 2$, and $N = 8$. 