An Adaptive Rate Assignment Strategy for CDMA2000 IS-856 Subject to RAB Delay

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Abstract—In this paper, the problem of resource allocation in IS-856 uplink in the presence of time-delay is studied. A set of nonlinear adaptive controllers are designed to stabilize the wireless network and use the system resources efficiently. The controllers obtained are then modified properly to retain network stability and performance in the presence of time-delay. Simulation results are presented to show the effectiveness of the proposed approach.

I. INTRODUCTION

CDMA2000 1xEV-DO (IS-856) is designed for high-performance wireless networks to provide high-speed service at low cost. IS-856 uses the same bandwidth as traditional CDMA2000, but provides higher throughput [9]; this translates to very high data rate on downlink channel. On the other hand, on the uplink channel, IS-856 is the first CDMA system that uses closed-loop interference control [9]. Despite recent progress in this area, new data applications require higher efficiency on uplink channel [1], [9]. This is due mainly to the need for higher data rate in such applications, compared to traditional ones.

To efficiently utilize the bandwidth on uplink, a distributed feedback-based resource allocation algorithm is proposed in [11], which uses transition (state) probabilities to set the transmission rates of the users on uplink channel. In that algorithm, each user sets its transmission rate locally by monitoring a common single bit and acting upon it, accordingly (note that it is normally desirable to assign as many bits as possible for data, and as few bits as possible for control signal). This algorithm was adopted by IS-856 standard [1], but due to its performance deficiency, a token bucket mechanism was later adopted in IS-856 Rev A [2]. The resource allocation problem was solved in [2], [3], [4] using a framework analogous to the one presented in [7]. In the above works, distributed resource allocation is formulated as a utility maximization problem. In other words, the rate allocation problem in the above-mentioned papers is studied in the context of auctioning algorithms subject to existing constraints in the physical system in order to allocate the available resources to all users in a fair manner.

As mentioned earlier, the control signal transmitted from the base station to the users is a single bit called the reverse activity bit (RAB). This bit is set to +1 by the base station if the existing interference is below a certain reference level, meaning that the network resources are under-utilized. If, on the other hand, the interference level in the network is above a certain threshold, then the RAB transmitted to the users is set to 0. In the context of control, this binary comparison introduces nonlinearity in the closed-loop system through the control operation. A novel framework was proposed in [5] to tackle the underlying problem using a control theoretic view. The information transmitted through the RAB was modeled in the forward path of the closed-loop system, and a simple lead compensator was designed for the network. However, the results in [5] may not be accurate enough in practice, due to the simplifying assumptions in the network model. To address this shortcoming, a dynamic adaptive control strategy is introduced in [6], and network stability is analyzed using the Lyapunov method. The algorithm proposed in [6] suffers mainly from two shortcomings. First, the signaling between base station and users is not minimal. Furthermore, the effect of delay in the control loop is not taken into account in the stability analysis.

In this paper, an adaptive control strategy is presented for IS-856. Stability analysis for the network is performed by using Lyapunov technique. Moreover, minimal communication (for control signal) is assumed between base station and users. The proposed control strategy guarantees the overall performance of the network in the presence of delay. Certain conditions on controller parameters are derived to guarantee the stability and boundedness of system state as well as controller gains. Furthermore, error bounds on the system output and desired system performance for each AT and the overall network are obtained.

The rest of the paper is organized as follows. In Section II, IS-856 uplink is modeled and the structure of the control loop is introduced. The proposed rate control strategy is presented in detail in Section III. The stability of the system is also
studied in this section and upper bounds on the output error are derived subsequently, in the presence of time delay in the communication link. Simulation results are provided in Section IV to demonstrate the efficacy of the results obtained. Finally, concluding remarks are summarized in Section V.

II. PROBLEM FORMULATION

Definition 1: Access terminals (AT) are the mobile components of the network that provide data to users. Access network (AN), on the other hand, is the fixed element of the network, including the base station which provides data for ATs. A CDMA channel in an AN along with the corresponding covered area is called a sector [1], [2].

Let the number of ATs in a sector of IS-856 at time \( t \) be denoted by \( n(t) \). Assume that the base station receives the same pilot power \( (P_{\text{pilot}}) \) from all ATs [2], [3], [5]. In IS-856, ATs can transmit data with a rate \( R_i(t) \) which belongs to a finite set \( \Gamma \) of feasible rates. There is also assumed to be a one-to-one mapping \( F(\cdot) \) between these feasible rates and the ratio of the data power to the pilot power \( T2P \). This mapping is represented for the \( i \)-th AT as follows

\[
T2P(t) = F(R_i(t))
\]  
(1)

The mapping \( F(\cdot) \) is provided in [1], and is depicted in Fig. 1 by asterisks. Note that in the theoretical developments in the next section, a continuous approximation of this mapping (in the whole range of feasible rates) will be required. A second-order polynomial is also depicted in Fig. 1 as a smooth approximation of this mapping, which will be used later in simulations.

On the other hand, at the AN side of the network, the raise over thermal (ROT) at any time \( t \) is defined as [2]

\[
Z(t) = 10 \log_{10} \left( \frac{I(t) + N_0W}{N_0W} \right)
\]  
(2)

where \( N_0W \) is the total background thermal noise, \( W \) is the system bandwidth and \( N_0 \) is the spectral density of noise. Moreover, \( I(t) \) is the in-sector interference resulted by ATs in the network

\[
I(t) = \sum_{i=1}^{n(t)} T2P_i(t) P_{\text{pilot}}
\]  
(3)

It is to be noted that a large ROT is a sign of high interference in the network, and a small ROT is a sign of under-utilization of network resources. It is desired to regulate the ROT to a sufficiently small neighborhood of a threshold given by \( Z_{th} \). Define

\[
y(t) = \sum_{i=1}^{n(t)} F(R_i(t))
\]  
(4)

which is, in fact, the feedback from ATs to the corresponding AN. It can be easily verified that in order to regulate ROT to the threshold value \( Z_{th} \), the above should be regulated to a reference value \( Z_r \), which can be found from (1) and (3) as

\[
Z_r = \frac{N_0W}{P_{\text{pilot}}} \left( 10^{Z_{th}/10} - 1 \right)
\]  
(5)

Now, the RAB \((U(t))\) can be defined as the error signal produced by applying a two-level comparator to \( y(t) \) and the reference value \( Z_r \). This operation is formulated below

\[
e(t) = Z_r - \sum_{i=1}^{n(t)} F(R_i(t))
\]  
(6)

\[
U(t) = \text{sgn}(e(t))
\]  
(7)

This means that when the total power of the ATs in a sector is less than a desired value, then \( U(t) = 1 \); otherwise, \( U(t) = -1 \), and ATs increase or decrease their rates accordingly. As a result, upon receiving the RAB signal from AN, the ATs increase or decrease their rates properly in order to achieve the regulation objective. It is desired to design a controller for each AT in order to adjust its rate in such a way that the feedback signal \( y(t) \) converges to a satisfactorily small neighborhood of the desired value. The network configuration of a cell along with the corresponding controllers is shown in Fig. 2.

III. MAIN RESULTS

In this section, a nonlinear adaptive controller is designed for each AT, and the stability of the network is analyzed for the resultant closed-loop system. As pointed out earlier, each AT receives a one-bit control signal from AN, and adjusts its rate accordingly. Note that ATs have no information about the number of online users in the cell and their corresponding rates.
A. Controller Design for Nominal System

Let $a$ be a strictly positive number, and $N$ be the maximum number of ATs in a cell. Consider the control structure in Fig. 3, which consists of two simple first-order transfer functions and two time varying coefficients $k_{i1}(t)$, $k_{i2}(t)$ for the $i$-th AT, $i \in \{1, \ldots, N\}$. The state space representation for the closed-loop system can be written as follows

$$\begin{bmatrix}
\dot{x}_{11}(t) \\
\dot{x}_{12}(t) \\
\vdots \\
\dot{x}_{N1}(t) \\
\dot{x}_{N2}(t)
\end{bmatrix} =
\begin{bmatrix}
-a & 0 & & & 0 & 0 \\
-k_{12}(t) & -a & & & 0 & 0 \\
& & \ddots & & & \vdots \\
0 & 0 & & -a & 0 & 0 \\
0 & 0 & & \cdots & k_{N2}(t) & -a
\end{bmatrix}
\begin{bmatrix}
x_{11}(t) \\
x_{12}(t) \\
\vdots \\
x_{N1}(t) \\
x_{N2}(t)
\end{bmatrix}
+ \begin{bmatrix}
0 \\
U(t)
\end{bmatrix}
\begin{bmatrix}
ax_{11} + k_{11}(t)U(t) \\
ax_{12} \\
\vdots \\
ax_{N1} + k_{N1}(t)U(t) \\
ax_{N2}
\end{bmatrix}
$$

(8a)

$$y(t) = \frac{I(t)}{I_{\text{pilot}}}$$

(8c)

The objective is to ensure the convergence of the state vector $[x_{11}(t) \ x_{12}(t) \ \cdots \ x_{N1}(t) \ x_{N2}(t)]^T$ to a nominal state $[\bar{x}_{11}(t) \ \bar{x}_{12}(t) \ \cdots \ \bar{x}_{N1}(t) \ \bar{x}_{N2}(t)]^T$ which is, in fact, an equilibrium point for the network (which corresponds to the perfect regulation of ROT). This nominal state is chosen only based on the common single-bit signal (7) received from AN as follows

$$\dot{x}_{i2}(t) = U(t)$$

for all $i \in \{1, \ldots, N\}$. For simplicity and with no loss of generality, assume that

$$\dot{x}_{i1}(t) = 0$$

(10)

(notice that the above assumption does not limit the scope of the results). The dynamic evolution of the time-varying coefficients $k_{i1}(t)$ and $k_{i2}(t)$ is given by

$$\dot{k}_{i1}(t) = -ak_{i1}(t) - U(t)x_{i1}(t)$$

(11a)

$$\dot{k}_{i2}(t) = -ak_{i2}(t) - x_{i1}(t)x_{i2}(t) + x_{i1}(t)x_{i2}(t)$$

(11b)

for any $i \in \{1, \ldots, N\}$ (it will be shown later in Theorem 1 that the stability of the network is guaranteed under these adaptive coefficients). Define the new state vector $Y(t)$ as follows

$$Y(t) =
\begin{bmatrix}
y_{11}(t) \\
y_{12}(t) \\
\vdots \\
y_{N1}(t) \\
y_{N2}(t)
\end{bmatrix} =
\begin{bmatrix}
x_{11}(t) - \bar{x}_{11}(t) \\
x_{12}(t) - \bar{x}_{12}(t) \\
\vdots \\
x_{N1}(t) - \bar{x}_{N1}(t) \\
x_{N2}(t) - \bar{x}_{N2}(t)
\end{bmatrix}
$$

(12)

Define also the matrices $A(t)$ and $B(t)$ as

$$A(t) =
\begin{bmatrix}
-a & 0 & \cdots & 0 & 0 \\
-k_{12}(t) & -a & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & -a & 0 \\
0 & 0 & \cdots & k_{N2}(t) & -a
\end{bmatrix}
$$

(13)

$$B(t) =
\begin{bmatrix}
k_{11}(t)U(t) \\
\vdots \\
k_{N1}(t)U(t) \end{bmatrix}
$$

(14)

The state-space representation (8) in the new coordinates can be obtained as follows

$$\dot{Y}(t) = A(t)Y(t) + B(t)$$

(15)

It can be easily verified that the origin is the equilibrium point in the new coordinates. Note that using (12), the adaptive gains $k_{i1}(t)$ and $k_{i2}(t)$ can be rewritten as follows

$$\dot{k}_{i1}(t) = -ak_{i1}(t) - U(t)y_{i1}(t)$$

(16a)

$$\dot{k}_{i2}(t) = -ak_{i2}(t) - y_{i1}(t)y_{i2}(t)$$

(16b)

for all $i \in \{1, \ldots, N\}$. The following theorem states that the state of the system converges, and eventually remains in a certain neighborhood of the equilibrium.

**Theorem 1:** The state trajectories of system (15) along with the time-varying coefficients (16) are globally ultimately stable in the sense of Lyapunov.

**Proof:** Choose the following positive-definite Lyapunov function

$$V(t) = \frac{1}{2}y^T(t)Y(t) + \frac{1}{2} \sum_{i=1}^{N} [k_{i1}(t)^2 + k_{i2}(t)^2]$$

(17)

The derivative of (17) is given by

$$\dot{V}(t) = \frac{1}{2} (\dot{Y}^T(t)Y(t) + Y^T(t)\dot{Y}(t))$$

$$+ \sum_{i=1}^{N} \dot{k}_{i1}(t)k_{i1}(t) + \sum_{i=1}^{N} \dot{k}_{i2}(t)k_{i2}(t)$$

(18)
Substituting (15) in (18) yields
\[
\dot{V}(t) = -\alpha \sum_{i=1}^{N} y_{1i}(t)^2 - \alpha \sum_{i=1}^{N} y_{2i}(t)^2 + \sum_{i=1}^{N} k_{1i}(t)U(y_{1i}(t)) + \sum_{i=1}^{N} k_{2i}(t)y_{1i}(t) + \sum_{i=1}^{N} \delta k_{1i}(t)k_{2i}(t)
\]

Now, using (16) one will arrive at the following relation
\[
\dot{V}(t) = -\alpha \sum_{i=1}^{N} y_{1i}(t)^2 - \alpha \sum_{i=1}^{N} k_{1i}(t)^2 T - \sum_{i=1}^{N} U(t)y_{1i}(t) + \sum_{i=1}^{N} \delta k_{1i}(t)k_{2i}(t)
\]

Considering (20), and in light of Lyapunov theorem [12], one can conclude that the overall system (the communication network and the control gains) is globally ultimately stable. This completes the proof.

Corollary 1: The maximum bound on \( y_{2i}(t) \) is given by
\[
|y_{2i}(t)| \leq \frac{\sqrt{N}+1}{2a}
\]

where \( i \in \{1, \ldots, N\} \). Furthermore, the bound on the overall performance of the system (15) is
\[
\sum_{i=1}^{N} |y_{2i}(t)| \leq \frac{N}{a}
\]

Proof: For stability of the system (15), the derivative of the Lyapunov function as given by (20) has to be negative. In other words, to find the region of attraction, the trajectories \( y_{1i}(t), y_{2i}(t), k_{1i}(t) \) and \( k_{2i}(t) \) must be such that the following inequality holds
\[
\dot{V} \leq 0
\]

for all \( i \in \{1, \ldots, N\} \). In this region of attraction, the maximum error magnitude for the \( i \)-th AT \( \left(y_{2i}(t)\right) \) is resulted when \( y_{1i}(t) = 0, k_{1i}(t) = 0, k_{2i}(t) = 0 \), for all \( i \in \{1, \ldots, N\} \), and \( y_{2i} = 0 \), for all \( i \neq i \), i.e., \( |y_{2i}(t)| \leq \frac{\sqrt{N}+1}{2a} \).

In addition, to find the bound on the overall performance \( \sum_{i=1}^{N} |y_{2i}(t)| \), set \( y_{1i}(t) = 0, k_{1i}(t) = 0, k_{2i}(t) = 0 \), for all \( i \in \{1, \ldots, N\} \). Then, using (23) one arrives at the following inequality
\[
\sum_{i=1}^{N} \left( \sqrt{\alpha} y_{2i}(t) - \frac{U(t)}{2\sqrt{\alpha}} \right)^2 \leq \frac{U(t)^2}{4a} N
\]

This bound can be simplified as
\[
\sum_{i=1}^{N} |y_{2i}(t)| \leq \frac{N}{a}
\]

Remark 1: According to Corollary 1, the upper bound on the discrepancy between the nominal rate \( \dot{x}_{2i} \) and the actual AT rate \( x_{2i} \) for each AT is equal to \( \frac{\sqrt{N}+1}{2a} \). This implies that the upper bound on said discrepancy is inversely proportional to the control parameter \( a \).

**B. Controller Design in Presence of Time-Delay**

It is now desired to investigate the impact of transmission delay on stability, and revisit the controller design proposed in the preceding subsection accordingly. Let the time-delay in the forward path be denoted by \( \tau \), as shown in Fig. 4. In other words, AN transmits the signal \( U(t) \) to ATs, and the ATs receive it after some delay. In terms of control operation, the input to the adaptive controller in this case is \( \hat{U}(t-\tau) \).

The state space representation of the system with time delay is given by
\[
\begin{bmatrix}
\dot{x}_{1i}(t) \\
\dot{x}_{12}(t) \\
\vdots \\
\dot{x}_{N1}(t) \\
\end{bmatrix} =
\begin{bmatrix}
-a & 0 & \cdots & 0 \\
k_{12}(t) & -a & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -a \\
\end{bmatrix}
\begin{bmatrix}
x_{1i}(t) \\
x_{12}(t) \\
\vdots \\
x_{N1}(t) \\
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
\begin{bmatrix}
k_{2i}(t) \\
k_{21}(t) \\
\vdots \\
k_{2N}(t) \\
\end{bmatrix}
\]

and
\[
\hat{\dot{x}}_{12}(t) = \hat{U}(t-\tau)
\]

Similar to the preceding subsection, the objective here is to ensure that \( x_{1i}(t) \) and \( x_{12}(t) \) are regulated to a satisfactorily small neighborhood of the ideal nominal values 0 and \( \hat{x}_{12}(t) \), respectively, where \( \hat{x}_{12}(t) = u(t) \). It is to be noted that the controller cannot generate the control signal \( U(t) \) at the AT. Instead, the controller will be using \( U(t-\tau) \) in order to generate the control signal \( \hat{x}_{12}(t) \).

By defining
\[
\begin{align}
\gamma_{1i}(t) &= x_{1i}(t) - \hat{x}_{1i}(t) \\
\gamma_{12}(t) &= x_{12}(t) - \hat{x}_{12}(t)
\end{align}
\]
the state space representation (26) in the new coordination can be rewritten as follows
\[
\begin{bmatrix}
\dot{\gamma}_{1i}(t) \\
\dot{\gamma}_{12}(t) \\
\end{bmatrix} =
\begin{bmatrix}
-k_{1i}(t)U(t-\tau) \\
\alpha(\hat{x}_{12}(t) - \hat{x}_{21}(t)) - U(t) \\
\vdots \\
\alpha(\hat{x}_{N2}(t) - \hat{x}_{N1}(t)) - U(t) \\
\end{bmatrix}
\]

where
\[
\dot{B}(t) =
\begin{bmatrix}
k_{1i}(t)U(t-\tau) \\
\alpha(\hat{x}_{12}(t) - \hat{x}_{21}(t)) - U(t) \\
\vdots \\
\alpha(\hat{x}_{N2}(t) - \hat{x}_{N1}(t)) - U(t) \\
\end{bmatrix}
\]
Using (28), the differential equations governing the adaptive gains can be written analogously to (16) to obtain
\[ \dot{k}_1(t) = -ak_1(t) - U(t - \tau)y_1(t) \quad (31a) \]
\[ \dot{k}_2(t) = -ak_2(t) - y_1(t)y_2(t)\dot{x}_2(t) - \dot{\bar{x}}_2(t) \quad (31b) \]
for all \( i \in \{1, \ldots, N\} \).

The following theorem provides a sufficient condition for the stability of the system (29) in terms of the control parameter \( a \).

**Theorem 2:** The state of the system (29) and the time-varying coefficients (31) are globally ultimately stable in the sense of Lyapunov if \( a > \tau \).

**Proof:** Choose the same Lyapunov function as (17). Differentiating (17) along the state trajectory of the system (29) yields
\[ V(t) = -a \sum_{i=1}^{N} \dot{y}_i(t)(\dot{y}_i(t) + k_i(t)U(t - \tau)y_1(t)) \]
\[ + \sum_{i=1}^{N} k_i(t)y_1(t)y_2(t) + a \sum_{i=1}^{N} y_2(t)(\dot{x}_2 - \dot{\bar{x}}_2) \]
\[ = -\sum_{i=1}^{N} U(t)y_2(t) + \sum_{i=1}^{N} k_i(t)y_1(t) + \sum_{i=1}^{N} k_i(t)k_i(t) \]

(32)

Now, substituting (31) in (32), one will arrive at
\[ V(t) = -a \sum_{i=1}^{N} \dot{y}_i(t)(\dot{y}_i(t) + k_i(t) + k_i(t)) \]
\[ + a \sum_{i=1}^{N} y_2(t)(\dot{x}_2 - \dot{\bar{x}}_2) - \sum_{i=1}^{N} U(t)y_2(t) \quad (33) \]
\[ + \sum_{i=1}^{N} k_i(t)y_1(t)(\dot{x}_2 - \dot{\bar{x}}_2) \quad \text{Note that} \]
\[ |\dot{x}_2 - \dot{\bar{x}}_2| = \left| \int_{0}^{t_0} U(t - \tau) - \int_{0}^{t_0} U(t) \right| \leq 2\tau \]
for all \( t_0 > 0 \). Therefore, the derivative of the Lyapunov function in (33) can be rewritten as follows
\[ V(t) \leq -a \sum_{i=1}^{N} \dot{y}_i(t)(\dot{y}_i(t) + k_i(t) + k_i(t)) \]
\[ + 2a \sum_{i=1}^{N} |y_2(t)|^2 \tau + 2a \sum_{i=1}^{N} |k_i(t)y_1(t)| - \sum_{i=1}^{N} U(t)y_2(t) \]
\[ = -a \sum_{i=1}^{N} \dot{k}_i(t) - \sum_{i=1}^{N} \left( \sqrt{a}|y_2(t)| - \frac{2a - U(t)}{2\sqrt{a}} \right)^2 \]
\[ - \sum_{i=1}^{N} \left( \sqrt{a}|y_1(t)| - \frac{|k_i(t)|\tau}{\sqrt{a}} \right)^2 \]
\[ - \sum_{i=1}^{N} k_i^2(t) \left( a - \frac{\tau^2}{a} \right) + N \left( \frac{2a - U(t)}{4a} \right)^2 \]

(34)

Since \( a > \tau \), it can be concluded from Lyapunov theorem [10], [12] that the system is stable and all state variables and control parameters remain bounded. This completes the proof.

The following corollary gives an upper bound for each state variable and for the overall performance of the system.

**Corollary 2:** The maximum bound on each \( y_2(t) \) is given by
\[ |y_2(t)| \leq \frac{(2a\tau + 1)}{-2a} \left( 1 + \sqrt{N} \right) \quad (35) \]
In addition, the bound on the sum of state variables is
\[ \sum_{i=1}^{N} |y_2(t)| \leq 2\tau N + \frac{N}{a} \quad (36) \]

**Proof:** The proof is similar to the one given for Corollary 1, and is omitted due to space restrictions.

**Remark 2:** Considering the stability condition \( a > \tau \) in Theorem 2, the maximum bounds on the state variables and the overall performance of the system can be obtained as follows
\[ \max_{i=1}^{N} |y_2(t)| = \left( \frac{\tau + 1}{2\tau} \right) \left( 1 + \sqrt{N} \right) \quad (37) \]
\[ \max_{i=1}^{N} \sum_{i=1}^{N} |y_2(t)| = 2\tau N + \frac{N}{\tau} \quad (38) \]

Remark 3: The results obtained show that choosing a large value for the control parameter \( a \) would be advantageous in terms of the maximum bound on the output (as can be seen from (21), (22), (35) and (36)) and also in terms of convergence time (as can be seen from (20)). However, a large \( a \) can have implications in terms of the implementation of the system in practice. A large \( a \) would increase the control bandwidth, which in turn increases its sensitivity to noise. This introduces a trade-off in the choice of the coefficient \( a \) in the control law.

**IV. SIMULATION RESULTS**

In this section, a single cell environment is considered with one AN and several online ATs. The reference signal \( Z_r \) is set to 171.23, which corresponds to the system capacity of 600 kbps [9]. The delay in the network structure is assumed to be constant and equal to 13.36 msec, which is equivalent to 8 time slots. Simulations are repeated for two different values of \( a \), to examine the effect of this control parameter on system response. It is to be noted that the probabilistic rate selection scheme is adopted in all simulations [3]. More precisely, if the output of the controller of the \( i \)-th AT at time \( t_0 \) is between two feasible rates \( R_1 \) and \( R_2 \), i.e.
\[ R_1 < s_{2i}(t_0) < R_2 \quad (39) \]
then this AT transmits data at the rate \( R_1 \), with the probability \( p_1 \) given by
\[ p_1 = \frac{F(R_2) - F(s_{2i}(t_0))}{F(R_2) - F(R_1)} \quad (40) \]
or at the feasible rate \( R_2 \), with the probability \( p_2 \) given below
\[ p_2 = \frac{F(R_2)}{F(R_2) - F(R_1)} \quad (41) \]
Note that \( \tilde{F}(\cdot) \) in (40) and (41) is, in fact, a proper approximation of the mapping \( F(\cdot) \) from the feasible rates to feasible \( T2P \). In the simulations, the second-order polynomial depicted in Fig. 1 is used as the approximate mapping \( \tilde{F}(\cdot) \).

The network output \( y(t) \) is depicted in Fig. 5 for two different values of \( a \): 0.01 and 10. Note that according to Theorem 2 a sufficient condition for the stability of the system is that \( a \) is greater than the magnitude of the delay, i.e. 0.01336. Figs. 5(b) confirm the results of Theorem 2, as the overall system is stable for \( a = 10 \), and the output is regulated around the reference \( Z_r \). Figure 5(a), on the other hand, shows that the system output is not regulated for \( a = 0.01 \). Note that the number of available users is not known \textit{a priori}, and the results obtained in this paper are independent of \( n \), as pointed out earlier. Nevertheless, the value of \( n \) used in the above simulations at any point in time is shown in Fig. 5(c). The moving average of the output for different values of \( a \) considered above is shown in Fig. 6.

Remark 4: It can be observed from the network output in Fig. 5(a) that choosing a small value for \( a \) (with respect to \( \tau \)) degrades the output performance. Note that Theorem 2 provides a sufficient condition only. In other words, the system may still be stable for the values of \( a \) smaller than \( \tau \). On the other hand, while the Lyapunov stability analysis in this paper is based on a smooth approximation of the mapping \( F(\cdot) \), in reality a finite set of distinct rates are used, and in particular \( T2P \) can not exceed a certain value. This, in fact, limits the magnitude of the output oscillation, and prevents it from diverging as time increases. This means that while small values of \( a \) might not destabilize the network in practice, the resultant performance would be very poor.

V. CONCLUSIONS

In this work, a set of adaptive controllers are designed to stabilize the IS-856 uplink in the presence of time-delay in forward path. One bit control signal is assumed to be transmitted from the base station to the users. A sufficient condition for the stability of the network and control parameters in presence of input delay in the network is provided using Lyapunov stability analysis. An upper bound for the error between the real and nominal outputs is obtained. Simulations demonstrate the effectiveness of the results in a real-world setting.

REFERENCES