A Study of the Capacity for Different Element Spacing on Compact MIMO Platforms

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Abstract—The multiple-input multiple output (MIMO) technique is an enabling technology for meeting the capacity demands in future wireless communication systems. For military tactical communication use, one of the interesting frequency bands is around 300 MHz. The frequency is low enough to provide good coverage, e.g., in urban peer-to-peer scenarios where line-of-sight (LOS) rarely exist. However, the size of the MIMO antennas arrays increases with the wavelength. At 300 MHz, an element separation of about 0.5 m is often used. For larger platforms, such as vehicles, arrays with quite many elements are feasible. However, on small platforms (handheld devices being one extreme), the number of possible elements can be very restricted. In this work, the impact of reducing the antenna element separation is studied in terms of reduced capacity for a circular antenna array in a $4 \times 4$ (four transmit antennas and four receive antennas) and a $2 \times 2$ MIMO system. This is investigated by using a double-directional channel description derived from measurements in a suburban area. It is shown that the degradation is graceful. For example, for a $2 \times 2$ system, the penalty of using a radius of 0.125 m is only about 10% compared to the capacity for a radius of 0.25 m. This opens up for MIMO antenna configurations with spatially separated antenna elements on very compact platforms. For these results, it is assumed that the mutual coupling between the antenna elements is taken care of by other techniques and is therefore not treated in this work.

Index Terms—MIMO systems, antenna arrays, information rates, element spacing

I. INTRODUCTION

To meet the foreseen needs concerning capacity in tactical communication systems, multiple-input multiple output (MIMO) technique is necessary, especially for systems intended to operate in an urban environment. The urban environment is characterized by severe multipath propagation due to buildings and other possible reflectors, which will deteriorate the transmission with consequences such as capacity reduction, introduction of errors in the information, and reduction of communication range. The demand for high capacity in communication systems has been one of the reasons behind the rapid development of MIMO techniques. The use of more than one antenna at the transmitter and the receiver makes it possible to increase the capacity compared to a single-input single-output (SISO) system, where only one transmit and receive antenna are used [1]. The MIMO technique is already implemented in several civilian standards for wireless networks, such as IEEE 802.16 (WiMAX) and IEEE 802.11n (Wi-Fi). Wireless routers using MIMO have been available at the market for some years. Also the long-term evolution (LTE) of the 3G standard includes MIMO.

One of the interesting frequency bands for military use of MIMO is around 300 MHz. There are many advantages with this frequency band. For example, these frequencies allow relatively long communication ranges and the due to the large wavelength waves can easily diffract around obstacles [2]; which property is especially important in peer-to-peer networks, where line-of-sight (LOS) exists only rarely. However, the size of the antenna arrays increases with the wavelength. To achieve large capacity gains with MIMO systems (by spatial multiplexing), the received signal in the different antennas should be as uncorrelated as possible. In an ideal environment, where the scattering is horizontally uniformly distributed, this is obtained for an element spacing of about $\lambda/2$ (where $\lambda$ is the wavelength), when the mutual element coupling is neglected [3]. However, in a real environment a uniform distribution of scatters might not be fulfilled, why the element spacing needs to be larger. At 300 MHz, an element separation of $\lambda/2$ implies a separation of 0.5 m between the antenna elements. For larger platforms, such as vehicles, arrays with quite many elements are feasible. However, on small platforms (handheld devices being one extreme), the number of possible spatially separated antenna elements can be very restricted; double-polarized antenna elements are of cause very interesting in this context. Therefore, it would be interesting to study the performance reduction if a smaller antenna element separation is used. If there is only a minor performance reduction, the use of MIMO antennas at 300 MHz might be functional even for handheld use.

In this work, the resulting capacity for different antenna separations of a circular antenna array is studied with $4 \times 4$ (4 transmit (Tx) and 4 receive (Rx) antennas) and $2 \times 2$ antennas. The aim is to study the performance degradation due to smaller antenna platforms. By bringing the elements closer to each other, the antenna will experience mutual coupling between the elements. However, it is shown in [4], that the impact on the capacity of strong mutual coupling can substantially be improved by implementing a good matching network. For this reason, mutual antenna coupling is not considered in this work, i.e., only the impact of the multipath propagation and the correlation due to this are considered. Furthermore, the potential interference-suppressing capabilities might also be affected by the antenna element spacing, but is not studied in this work. The resulting capacity of the $4 \times 4$ and $2 \times 2$ systems for different radius is then compared to the capacity of a SISO.
system. The purpose of this work is to study the price paid for an antenna array that is made more compact than what is optimal.

This work is based on earlier work [5], [6] and channel measurements performed with the RUSK channel sounder owned by Lund University, Sweden. In [5], the capacity is calculated directly from the measured data for a $7 \times 7$ system and in [6], SAGE estimates [7] are derived and used for further investigations. The measurements are performed in a suburban area in Linköping. The Tx and the Rx antennas were mounted on top of cars, why the results are derived for a peer-to-peer channel.

The paper is organized as follows. Section II gives a brief description of the measurements and calculations the work in this paper is based on. In Sec. III, the derived capacities for different antenna configurations and different element spacing are presented. Finally, the paper is summarized in Sec. IV.

II. PRELIMINARIES

A. Brief Description of the Measurement System

The measurements were performed with the RUSK Lund channel sounder [8], which sequentially measured the transfer function between all combinations of Tx and Rx antenna elements within a short time. In our measurements, two identically vertically polarized antenna arrays were used at the transmitter and the receiver. Each antenna consists of a seven element uniform circular dipole antenna (UCDA), with one additional center element located in an elevated position with respect to the UCDA. Additionally, a cylindrical reflector is placed in the center of the UCDA giving each element a directional radiation pattern. The Tx and Rx antennas were mounted approximately 1.8 m and 2.1 m over ground, respectively. The measurements were performed at a center frequency of 285 MHz and with a signal bandwidth of 20 MHz.

B. Measurement Routes

The measurements were performed in a suburban area, where the Tx was stationary [5], [6], [8]. The Rx was driven along four routes denoted as R1, R2, R3 and R4, see Fig. 1. The distance between the Tx and Rx was between 200 and 450 m. LOS between Tx and Rx was possible in parts of the routes. In particular the first half of Route 1, and parts of Route 2 are characterized by LOS. On the other hand, the last part of Route 1, and parts of Route 4 are obstructed by buildings. The trees within the areas marked A, C and E are a mixture of coniferous and deciduous trees. As the measurements were performed in the end of March, the trees were not in leaf.

C. Pre-Processing of Measurement Data

The channel transfer function of every antenna combination was measured in a so called snapshot every 1.8 ms. Then, the measurement system collects four consecutive snapshots every 0.12 m of the channel transfer function over the frequency band, divided in 257 frequency subchannels. Every four consecutive snapshots is averaged to a so-called block, see Fig. 2. This information forms a channel transfer matrix $H_m$, of the $m$th data block, with the elements $h_{n_1,n_2}^{m}$, which denotes the transfer function between transmit antenna $n_2$ and receive antenna $n_1$.

D. Double-Directional Channel Description

Based on a signal model, which contains double directional information, the channel parameters can be extracted from the measured transfer matrix $H_m$. In accordance with this model, the channel transfer function can be described as a finite sum of $L$ multipath components (MPCs) [9] as

\[
h_{n_1,n_2}(k, \alpha_l, \tau_l, \phi_{1,Tx}^{T}, \phi_{1,Rx}^{R}) = \sum_{l=1}^{L} \alpha_l e^{j2\pi\Delta f \tau_l} G_{Tx}(n_2, \phi_{1,Tx}) G_{Rx}(n_1, \phi_{1,Rx}), \tag{1}
\]

where $\Delta f$ denotes the spacing between the frequency sub-channels and $\alpha_l, \tau_l, \phi_{1,Tx}^{T}$, and $\phi_{1,Rx}^{R}$ are the complex amplitude, delay, angle of departure (AOD), and angle of arrival (AOA), respectively, of the $l$th MPC. Furthermore, $k, n_1, n_2, G_{Tx}$, and $G_{Rx}$ are the frequency subchannel index, receive element index, transmit element index, receive antenna response and transmit antenna response, respectively. All angles are measured counter-clockwise and are given in a local coordinate system, where 0 deg corresponds to the heading of each
vehicle. The parameters $\alpha, \gamma, \phi_{1}^{Tx}$, and $\phi_{1}^{Rx}$ are estimated by the SAGE algorithm, which is a high resolution maximum likelihood (ML) algorithm [7]. The estimation is derived for 200 multipath components, which in [6] has been shown to catch the major behavior of the impulse response of these routes. In this method, the antenna properties, obtained from the calibration measurements, is considered. The estimates are calculated for each meter along all routes. Thus, the channel properties has been separated from the antenna properties [9] and new transfer matrices are calculated for the antenna constellations considered in this work.

E. Capacity Calculations

For a transmitter without channel state information, the narrowband MIMO capacity is given by [1]

$$C_{\text{MIMO}} = \log_{2} \det \left( I + \frac{\rho}{n_{Tx}} HH^{*} \right),$$

(2)

where $\det(\cdot)$ is the determinant operator, $(\cdot)^{*}$ the Hermitian transpose, $n_{Tx}$ the number of Tx antennas, $\rho$ the signal-to-noise ratio (SNR), and $I$ is the identity matrix. $H$ is normalized such that the expected value of its squared Frobenius norm $E[\|H\|_{F}^2] = n_{Rx}n_{Tx}$, where $n_{Rx}$ is the number of Rx antennas.

Based on the double-directional channel description, new $H_{m}$ are computed from (2) corresponding to circular Tx and Rx antennas with one, two and four elements with different spacing between the antenna elements, see Fig. 3. For the Tx, the computations are made by using

$$G_{\text{Tx}}(n_{2}, \phi_{1}^{Tx}) = e^{j \frac{2\pi f_{0}r}{c} \cos(\phi_{1}^{Tx} - \phi_{n_{2}})},$$

(3)

where $f_{0}, c$, and $r$ are the center frequency, the propagation velocity, and array radius, respectively. The angle $\phi_{n_{2}}$ describes the angle to array element $n_{2}$, see Fig. 3. The array is oriented such that the antenna element 1 is towards the driving direction. For the Rx, $G_{\text{Rx}}(n_{1}, \phi_{1}^{Rx})$ is defined in a similar manner. The circular antenna constellation with four antennas and the radius $r$ will have the separation $\sqrt{2r}$ between the closest antenna elements. For the antenna constellation with two antenna elements, the separation between the elements is $2r$. The constellation with two elements corresponds to a uniformly linear antenna (ULA). In this derivation, the propagation effect due to a certain antenna constellation and element separation are considered. The derived $H_{m}$ for a $4 \times 4$, a $2 \times 2$, and a $1 \times 1$ system with the antenna radius $r \in \{0.125, 0.25, 0.5, 1.0\}$ m are used for capacity calculations.

III. RESULTS

Based on the extracted channel parameters from the measured channel matrices, the MIMO and SISO mean capacities for the investigated antenna constellation are calculated for a fixed SNR of 20 dB. That is, the capacities analyzed in the following are normalized capacities and not the actual capacities. The capacities are averaged over the 257 frequencies observed and calculated for every eighth $H_{m}$, i.e., for $H_{1}, H_{5}, H_{17}, \ldots$, which results in a derived capacity every meter. It is assumed that the mutual coupling between the antenna elements is compensated for by e.g. a matching network, and is neglected. In Fig. 4–7, the mean capacity is presented as a function of the driving distance along the four measured routes R1–R4.
The capacities shown in Fig. 4–7, for the $4 \times 4$ antenna constellation with radius 1, 0.5, and 0.25 m, follow the shape of the capacity curves for the same routes reported in [5]. In [5], the capacity was calculated from the measured transfer matrix for a $7 \times 7$ MIMO system with fixed mean SNR of 20 dB.

**B. $2 \times 2$ Antenna System**

Compared to the capacity calculated in [5] and the capacity shown in Fig. 4–7 for $4 \times 4$ with the radius 1, 0.5, and 0.25 m, the capacities calculated for the $2 \times 2$ antenna systems do not fluctuate much. The long-term average follows the one for the $4 \times 4$ system although the level is lower and the deviations are not as large as for $4 \times 4$. As for the $4 \times 4$ systems, the capacity decreases as the radius decreases. However, the capacity for the radius 1 m is not substantially larger than for the radius 0.5 m, which was the case for the $4 \times 4$ configuration. The capacity for a $2 \times 2$ antenna system in an i.i.d. Rayleigh fading channel is about 11.3 bits/s/Hz. In the figures, it can be seen that a capacity of 11.3 bits/s/Hz is achieved for the largest radius for parts of the routes.

**C. Different Values of the Radius**

In an ideal environment, where the scattering is horizontally uniformly distributed, the correlation between the antenna elements is minimized for an element spacing of about $\lambda/2$, when the mutual element coupling is neglected [3]. However, for real channels when the scatterers are not uniformly distributed, the antenna separation needs to be larger to achieve uncorrelated antenna elements.

For 300 MHz, the antenna separation between two antenna elements in a $2 \times 2$ system should be at least $\lambda/2 = 0.5$ m, which yields a radius of at least 0.25 m. In Fig. 4–7, it can be seen that the capacity increases as the radius becomes larger than 0.25 m. This indicates that the correlation is reduced even more as the radius becomes larger than 0.25 m. For a $4 \times 4$ antenna constellation, a separation of 0.5 m between the closest antenna elements will result in a radius of about $0.5/\sqrt{2} \approx 0.35$ m. In Fig. 8, we can see the calculated capacity for different values of the radius at the first position on route 4. Here, a $4 \times 4$ antenna constellation is studied for different average SNR. The capacity not strictly monotonically increases as the radius increases, although the general conclusion is that larger radius implies less correlation and potentially higher capacity. The fact that the capacity does not monotonically increases, explains the small difference between the radius 0.25 and 0.5 m and the large difference between radius 0.5 and 1 m in Fig. 4–7. In Fig. 8, we can see that local maxima approximately appear at $0.5n/\sqrt{2}$, $n > 0$, which corresponds to a multiple of $\lambda/2$ between the closest elements. Furthermore, the figure shows the same behaviour for different values of the average SNR. However, the deviations increase as SNR increases. In Fig. 9, the corresponding parameters are studied for a $2 \times 2$ antenna constellation for the same position. In this figure, local maxima of the capacity are not as obvious as for the $4 \times 4$ system. For the $2 \times 2$ system, a local maximum

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**Fig. 6.** Calculated capacity along route 3 for a $4 \times 4$, $2 \times 2$, and for a $1 \times 1$ antenna system with the same direction 1, 0.5, 0.25, and 0.125 m.

**Fig. 7.** Calculated capacity along route 4 for a $4 \times 4$, $2 \times 2$, and for a $1 \times 1$ antenna system with the same direction 1, 0.5, 0.25, and 0.125 m.

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### A. $4 \times 4$ Antenna System

It can be observed that the shape of the capacity curves as a function of the distance for the $4 \times 4$ antenna constellation with a radius of 1, 0.5, and 0.25 m follow each other very well. Except from the rapid fluctuations, the long-term average capacity agrees very well for different radii. The major difference is that the curves for the different radii differ in the absolute value. As a comparison, an independent identically distributed (i.i.d.) Rayleigh fading channel will have a capacity of about 22.1 bits/s/Hz, which is not achieved for the routes studied. Largest radius yields highest capacity due to lower correlation between the antenna elements. The largest radius also shows slightly larger variations in capacity. The capacity calculated for the radius 0.125 m has least variations. It varies around a mean value of 11 bit/s/Hz for routes 1, 2, and 4, and 12 bit/s/Hz for route 3.
about 10%. For the average SNR. The capacity is calculated for different values of the radius and a radius of 0.125 m for the 2\times 2 antenna configuration. Compared to a radius \(0.25\) m, the reduction for a radius of 0.125 m is about 20-25%. The capacity for a radius of 0.125 m for a 4\times 4 system is about the double of a SISO system. For a 2\times 2 with the same radius, the gain in capacity is about 1.5 times a SISO system.

Fig. 8. Calculated capacity at the first position of route 4 for a 4\times 4 antenna constellation. The capacity is calculated for different values of the radius and average SNR.

It can be stated that the antenna element separation can be lower than the suggested \(\lambda/2\) for the routes studied. For the routes studied, even a radius of 0.125 m can be used for a 4\times 4 and a 2\times 2 antenna configuration, and still yield a major improvement compared to a SISO system. A radius of 0.125 m corresponds to an element separation of 0.18 m (\(\approx \lambda/5.5\)) for the 4\times 4 system and an element separation of 0.25 m (\(\approx \lambda/4\)) for the 2\times 2 system. This makes it possible to use such an antenna configuration on handheld equipment.

IV. CONCLUSION

In this work the effects of reducing the antenna array element separation was studied in terms of the reduced capacity for a MIMO system under realistic channel conditions. For frequencies around 300 MHz, the generally used separation of at least \(\lambda/2\) implies an element separation of at least 0.5 m. For compact platforms, e.g., handheld equipment, this can be very challenging. We found that the degradation is graceful when the element separation decreases below \(\lambda/2\). For example, for the 2\times 2 system, the penalty of using a radius of 0.125 m (separation of 0.25 m) is only about 10% compared to the capacity for a radius of 0.25 m. In the same way, there is a capacity enhancement for larger element separations. For a radius of 0.125 m, the capacity for the 4\times 4 system is about twice of the capacity for a SISO system, and for the 2\times 2 system the capacity is about 1.5 times of a SISO system. Throughout this work, it was assumed that the mutual coupling can be taken care of and was therefore neglected.

REFERENCES