Joint Synchronization Using Cyclic Property

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Abstract—It is well known that the cyclic property of transmitted signal can be used to achieve joint frame and carrier frequency synchronization, while the uncompensated sampling frequency offset (SFO) would still be a problem. Based on the observation that the impact of SFO on the received cyclic block and its replica is equivalent to a relative time shift, a joint synchronization scheme solely based on cyclic property is proposed to achieve frame, sampling frequency and carrier frequency synchronization simultaneously. A two-Dimensional Sliding-Window Auto-Correlator (2D-SWAC) is presented to implement the joint synchronization with low complexity. The proposed scheme is suitable for any transmission system with similar cyclic property. Meanwhile, it is robust to large sampling and carrier frequency offset and strong channel frequency selectivity, as demonstrated by the analysis and simulations.

I. INTRODUCTION

Synchronization is an essential task in wireless digital communications. There are three typical synchronizations at the receiver of widely used block transmission systems, i.e. frame or block synchronization, sampling frequency synchronization, and carrier frequency synchronization [1]. The idea of joint synchronization performed only in time domain is desirable, since it is suitable for both single-carrier (SC) and multi-carrier (MC) block transmission systems with or without frequency domain pilots. Methods based on this idea are mainly divided into two types. The first type uses dedicated training sequence and the corresponding methods can be derived directly from the ML estimation, as shown in [2], [3]. Methods of the second type [4]–[6] resort to the cyclic property, which is usually possessed by the transmitted signal of block transmission systems, where each guard interval padded with a cyclic prefix (CP) or a fixed known sequence can be regarded as a cyclic block (CB). The synchronization scheme derived for OFDM systems in [4] was extended to SC systems with repeated training sequence in [5], and was improved to fit for time domain windowed OFDM systems in [6].

All of the above synchronization schemes can jointly achieve frame and carrier frequency synchronization, while they all assume perfect sampling frequency synchronization between the sampling clocks at transmitter and receiver. The sample timing at the receiver drifts linearly with time if the sampling frequency offset (SFO) is left uncompensated, which is detrimental for both SC and MC systems [7]. For OFDM systems, the influence of SFO is twofold. Firstly data subcarriers are rotated and attenuated. Secondly inter-carrier interference approximately proportional to the square of subcarrier index and the square of SFO is introduced [8]. When the number of subcarriers is large, the uncompensated SFO dramatically degrades the receiver’s performance, as pointed in [9]. Sampling frequency synchronization methods for OFDM systems proposed in the literature are mainly based on frequency domain pilots, e.g. [9], [10]. Nevertheless, the joint frame and sampling frequency synchronization method proposed in [11] is carried out in time domain, which is applicable to block transmission systems with fixed training sequence as guard intervals.

This study is devoted to developing a time domain joint frame, sampling frequency and carrier frequency synchronization scheme solely based on cyclic property. In Section II, it is investigated how the cyclic property can be used to achieve sampling frequency estimation. Based on the fact that the SFO induces a relative time shift on the received CB and its replica, we obtain the estimate of SFO from estimating this time shift through cross-correlation. Inspired by the method in [4], we then convert the cumbersome cross-correlation into a two-dimensional Sliding-Window Auto-Correlation (2D-SWAC) of the received signal, in Section III. With the two-dimensional correlation result of the 2D-SWAC being exploited, frame, sampling frequency and carrier frequency synchronization are achieved simultaneously. In addition, a low complexity structure to efficiently implement the 2D-SWAC is proposed. Finally, simulations are provided to demonstrate the effectiveness of the proposed scheme in Section IV.

II. SFO ESTIMATION USING CYCLIC PROPERTY

Take a block transmission system with CP padded guard interval as an example. Each guard interval can be regarded as a CB. The transmitted signal $x(t)$ is shown in Fig. 1, where the shaded areas represent the CB and its replica with the time interval of $T_F$; $t_0$ and $T_C$ are the start time and duration of the CB, respectively. For $t \in [t_0, t_0 + T_C)$, $x(t) = x(t + T_F)$, which means the cyclic property.

For ease of analysis, we assume data outside the CB and its replica being zero and the absence of noise. The received signal $r(t)$ can then be expressed as

$$r(t) = \{x(t) * h(t)\} e^{j\Omega t},$$

(1)

where $\Omega$ denotes the carrier frequency offset (CFO), $h(t)$ denotes the channel impulse response (CIR), and $*$ stands for linear convolution. Therefore, for $t \in [t_0, t_0 + T_C)$,

$$r(t) = r(t + T_F) e^{-j\Omega T_F},$$

(2)
which means the received signal still possesses the cyclic property under delay spread channel with carrier frequency offset, except for an extra phase rotation due to the CFO. 

Suppose the A/D at the receiver runs at the fixed sampling period of \( T' \), which is typical for all-digital receiver. The corresponding sampling period at the transmitter is assumed to be \( T \), which is \( 1/M \) of the critical sampling period, and the oversampling factor \( M \) is typically 4 or 2. The SFO \( \beta \) is defined as

\[
\beta \triangleq (T - T')/T'.
\]

Let \( N \) be the number of sample intervals between the CB and its replica in the transmitted sequence. \( T_F \) can be rewritten as

\[
T_F = NT' \triangleq NT' + mT' + \mu T',
\]

where \( m \) is restricted to be an integer, while \( \mu \) is restricted within \([-0.5, 0.5)\). Let \( r_1[k] \triangleq r(t_0 + \tau + kT') \), \( r_2[k] \triangleq r(t_0 + \tau + kT' + NT') \), \( k = 0, 1, \ldots, K - 1 \) be the sampled version of the received CB and its replica, where \( r(t_0 + \tau + kT' + NT') \) stands for the sampling phase offset (SPO). According to (2) and (4),

\[
r_1[k] = r(t_0 + \tau + kT' + NT' + mT' + \mu T')e^{-j2\pi f T_F}.
\]

Comparing \( r_2[k] \) with (5), the following equation is obtained,

\[
r_2[k] = \{r_1[k]e^{j2\pi f r_T} \} * h_1(-mT' - \mu T'),
\]

where \( h_1(-\varepsilon) \) denotes the impulse response of the ideal interpolator with the delay of \( \varepsilon \) [12, pp.238]. Equation (6) reveals that the only difference between the sampled received CB and its replica is a phase rotation of \( \Omega T_F \), caused by the CFO, and a relative time shift consisting of the integer part \( m \) and the fractional part \( \mu \), originated from the SFO. Recalling (3) and (4) gives

\[
\theta \triangleq m + \mu = \beta N,
\]

implying that the estimate of the SFO can be obtained through the estimation of the relative time shift denoted as \( \theta \).

To estimate \( \theta \), let’s consider the cross-correlation between \( r_1[k] \) and \( r_2[k] \), which is given by

\[
R_{21}[l] = r_1^*[-l] * r_2[l] = r_1^*[-l] * r_1[l] * h_1(-\theta T')e^{j2\pi f T_F}.
\]

The magnitude of \( R_{21}[l] \) satisfies

\[
|R_{21}[l]| = |G[l]*h_1(-\theta T')|,
\]

where \( G[l] \triangleq r_1^*[-l] * r_1[l] \) is the auto-correlation of the received CB. Since \( |G[l]| \) is symmetric, the asymmetry of \( |R_{21}[l]| \) is only determined by \( \theta \). Consequently, \( \theta \) can be estimated through the asymmetry of \( |R_{21}[l]| \) and the offset of its peak position.

The asymmetry of \( |R_{21}[l]| \) is measured as

\[
\]

Normalizing \( d[l_m] \) yields the estimate of \( \mu \),

\[
\hat{\mu} = \begin{cases} 
\frac{d[l_m]}{d[l_m] - d[l_{m-1}]}, & \text{if } |R_{21}[l_{m-1}]| \geq |R_{21}[l_m]| \\
\frac{d[l_m] - d[l_{m-1}]}{d[l_{m-1}]}, & \text{if } |R_{21}[l_{m-1}]| < |R_{21}[l_m]| 
\end{cases},
\]

where \( l_m \) denotes the maximum position of \( |R_{21}[l]| \). Since \( l_m \) is 0 when SFO is zero, it directly gives the estimate of \( m \),

\[
\hat{m} = l_m.
\]

From (7), the estimate of the SFO is obtained as

\[
\hat{\beta} = \hat{\theta}/N = (\hat{m} + \hat{\mu})/N.
\]

The proposed approach estimating \( \theta \) is similar to the one estimating the timing offset of the received signal in [11], where cross-correlation between the received signal and locally-generated training sequence was used. As a counterpart, the proposed approach measures the asymmetry of the cross-correlation magnitude more precisely; also, it is much more robust to CFO and channel frequency selectivity, since the symmetry of \( |G[l]| \) is not affected by \( \Omega \) or \( h(t) \).

III. PROPOSED JOINT SYNCHRONIZATION SCHEME

The cross-correlation between \( r_1[k] \) and \( r_2[k] \) can be calculated as

\[
R_{21}[l] = \sum_{k=0}^{K-1} r_1^*[k]r_2[k+l],
\]

where \( K \) is the length of \( r_1[k] \). Based on the definition of \( r_1[k] \) and \( r_2[k] \), (14) can be turned into the auto-correlation of the received signal with the sampling period of \( T' \),

\[
R_{21}[l] = \sum_{k=0}^{K-1} r^*[t_0 + \tau + kT']r(t_0 + \tau + (k + N + l)T').
\]

In order to perform the auto-correlation in (15), \( t_0 \) has to be known, i.e. the frame synchronization is a prerequisite. In the light of the joint synchronization scheme proposed in [4], we extend (15) into a novel two-dimensional Sliding-Window Auto-Correlation (2D-SWAC), defined as

\[
R[n, l] = \sum_{k=0}^{K-1} r_{n+k}^*r_{n+k+N+l},
\]

where \( r_n \triangleq r(nT') \) denotes the sampled received signal, \( n \in [0, \infty) \), and \( l \in [-D, D] \). In (16), \( N + l \) can be viewed as the interval between two \( K \)-length rectangular windows sliding with the start position of \( n \). The sampled received signal selected by the two sliding windows is correlated with the lag of \( l \). By definition, \( R[n, l] \) is a two-dimensional function of the discrete time index \( n \) and the correlation lag \( l \). By exploiting the two-dimensional correlation result of 2D-SWAC, we propose the following joint synchronization scheme.
A. Frame Synchronization

Let $n_0$ be the discrete time index closest to the start time of the received CB, i.e., $n_0 T' = t_0 + \tau$, hence
\[ R[n_0, l] = R_{21}[l]. \]  
(17)
The frame synchronization can be accomplished via the estimation of $n_0$. By searching the two-dimensional maximum of the energy-normalized 2D-SWAC result
\[ C[n, l] \triangleq |R[n, l]| - P[n, l], \]  
the 2D-peak position is obtained as
\[ [n_p, l_p] = \arg \max_{n,l} \{|C[n, l]| \}. \]  
(19)
$P[n, l]$ in (18) is an energy normalization term defined as
\[ P[n, l] = \frac{1}{2} \sum_{k=0}^{K-1} |r_n+k|^2 + |r_{n+k+N+l}|^2, \]  
(20)
which is also adopted by the coarse timing synchronization metric in [10], in order to avoid large variations of peak magnitudes of $C[n, l]$ when the content of CBs are not fixed. The 2D-peak position on the discrete time dimension then gives the estimate of $n_0$,
\[ \hat{n}_0 = n_p. \]  
(21)
It is noteworthy that the highest accuracy of the frame synchronization is within $t_0 \pm 0.5T'$, because the SPO $\tau$ cannot be resolved, which is an intrinsic limitation of frame synchronization using cyclic property [13].

B. SFO Estimation

Now that we get the estimation of $n_0$, according to (17) the SFO estimation approach proposed in Section II can be used. The 2D-peak position on the correlation lag dimension gives the estimate of the integer part of $\theta$,
\[ \hat{m} = l_p. \]  
(22)
Since $P[n, l]$ little affects the asymmetry of $C[n_p, l]$, the fractional part of $\theta$ can be estimated with $C[n_p, l]$ on the bilateral of the 2D-peak,
\[ \hat{\mu} = \begin{cases} \frac{d[l_p]}{d[l_p - 1] - d[l_p]}, & \text{if } R[n_p, l_p - 1] \geq R[n_p, l_p + 1] \\ \frac{d[l_p]}{d[l_p] - d[l_p + 1]}, & \text{if } R[n_p, l_p - 1] < R[n_p, l_p + 1] \end{cases}, \]  
(23)
where
\[ d[l] = C[n_p, l+1] - C[n_p, l-1] + \frac{1}{4}(C[n_p, l+2] - C[n_p, l-2]). \]  
(24)
The estimate of the SFO is thus obtained as
\[ \hat{\Omega} = (\hat{m} + \hat{\mu})/N. \]  
(25)
The estimation range of the SFO is mainly determined by the estimation range of $m$, which is determined by the range of $l$. Therefore, the parameter $D$ in (16) can be chosen in accordance with the demanded estimation range of the SFO. For instance, if $m$ varies between $\pm 1$, $D$ should be set to 4 to calculate $\mu$, whereas the two-dimensional peak search in (19) on the correlation lag domain can be conducted only within $[-1, 1]$ instead of $[-D, D]$.

C. CFO Estimation

From (8) we know
\[ \arg \{ R_{21}[l] \} = \arg \{ G[l] * h_1(-\theta T') + \Omega T_F \} \]  
(26)
where $[\cdot]_{2\pi}$ denotes modulo $2\pi$ within $[-\pi, \pi]$. Notice that the received signal is oversampled by the factor $M$, thus the angle of $G[l]$ adjacent to its peak can be approximated as
\[ \arg \{ G[l] \} \approx \Omega T', \quad l \in [-M, M], \]  
(27)
while $\arg \{ G[0] \}$ is exactly zero. Consequently, the angles adjacent to $R[n_p, l_p]$ are approximated to be
\[ \arg \{ R[n_p, l] \} = \Omega T'(l - l_p) - \Omega T'\mu + \Omega T_F \} \]  
(28)
where $l \in [l_p - M, l_p + M]$ Based on these observations, the CFO can be estimated coarsely as
\[ \hat{\Omega}_C = \arg \{ R^* [n_p, l_p - 1] R[n_p, l_p + 1] \} / (2T'), \]  
(29)
and precisely as
\[ \hat{\Omega}_P = \arg \{ R[n_p, l_p] \} / (T_F - \mu T'). \]  
(30)
Since $T_F$ is usually large compared with $T'$, the precise estimation is far more accurate than the coarse estimation. On the contrary, the estimation range of $\hat{\Omega}_P$ is constrained within $\pm \pi / ((N + M) T')$, which is much smaller than the coarse estimation and thereby, the coarse and precise CFO estimation can be utilized in the carrier frequency acquisition mode and tracking mode respectively.

The above joint synchronization scheme is not directly applicable to a delay spread channel environment, as the analysis in Section II ignores the inter-block interference (IBI) caused by data outside the CB and its replica. However, if it is possible to estimate the delay spread $\gamma$ in advance, the summation range of 2D-SWAC can be adjusted into the IBI-free region where the relationship in (2) is still valid. Now (16) becomes
\[ R[n, l] = \sum_{k=0}^{K-1} r_{n+k}^* r_{n+k+N+l}, \]  
(31)
where $k_0 T' > t_0 + \gamma$. In the case the whole CB and its replica are contaminated by IBI, the proposed scheme could still work relying on proper averaging or loop filtering.

As for hardware implementation, the calculation of 2D-SWAC can be realized in a parallel structure with $2D + 1$ branches, where the $i$th branch calculates $R[n, l_i]$ and $C[n, l_i]$, as illustrated in Fig. 2. To reduce complexity, the total $2D + 1$ delay-lines with $(N + l_i)$-depth for each branch can be replaced by one $(N + D)$-depth delay-line with $2D + 1$ taps shared by all the branches. Moreover, the moving-sum unit in each branch can be implemented in a low complexity form with a $K$-depth delay-line, a subtractor and an accumulator, as shown in Fig. 3. With these simplifications the overall complexity remains low even though the whole parallel structure is used.
IV. SIMULATIONS & PERFORMANCE EVALUATION

We test the performance of the proposed joint synchronization scheme in two Digital Terrestrial Television Broadcasting (DTTB) systems, i.e. DVB-T [14] and DTMB [15]. DTMB is the newly announced Chinese DTTB standard, featuring of using PN sequence as guard intervals between MC or SC data blocks. We choose the fixed-PN420 mode for DTMB, where the same 420-symbol PN sequences are padded between 3780-symbol data blocks with the symbol rate of 7.56 Msymbol/s. For DVB-T, we choose the 8k mode with 1/8-length GI.

Fig. 4 shows the energy-normalized 2D-SWAC result $C[n, l]$ of the 4-times oversampled DTMB received signal under AWGN channel, where $N$ is set to $4200 \cdot 4$ and $l$ varies from $-8$ to $8$. $C[n, l_p]$ and $C[n_p, l]$ are shown separately in Fig. 5, where the SFO is set to 0 ppm and 40 ppm for comparison. From Fig. 5 we can see that when the 40 ppm SFO exists, the 2D-peak position on the correlation lag dimension is shifted by one sample, meanwhile, $C[n_p, l]$ is no longer symmetric.

The performance of the frame synchronization is evaluated through the variance of $\Delta \theta$ versus SNR per sample as displayed in Fig. 6 for both DVB-T and DTMB, with 2-times oversampling. The SFO and CFO are set to 80 ppm and 100 kHz to demonstrate the robustness of the frame synchronization to large SFO and CFO. Brazil B channel model [15] is adopted as a delay spread channel, which consists of 6 paths with the longest delay spread of 12.7 μs. From Fig. 6 we see that the frame synchronization performance is quite satisfactory under AWGN channel but degraded under delay spread environment. Fortunately, the performance under delay spread channel can be significantly improved though averaging over several consecutive frames, as shown by dash-dotted lines in Fig. 6. Notice that although the length of CP in the chosen DVB-T system is more than twice the length of PN sequence in DTMB, the PN sequence possesses better auto-correlation property, hence the two systems show similar frame synchronization performance.

With respect to SFO estimation, we choose the estimation range of $\theta$ to be ±1.5 for both 2-times oversampled DVB-T and 4-times oversampled DTMB, so that the SFO estimation range for both systems are nearly ±90 ppm. For the delay spread channel, we use the IBI-free region of the CB and its replica to conduct the estimation. The estimation MSEs are shown separately in Fig. 7. We see that for the 2-times oversampled DVB-T, the SFO estimation already works well under the SNR per sample of 5 dB, with the standard deviation less than 2.5 ppm. The fluctuation of MSE with SFO indicates the estimation of the fractional relative time shift is biased in certain ranges of $\mu$. However the estimation is unbiased around zero SFO, which enables the convergence of iteration or loop filtering. Furthermore, the biasness can be greatly reduced with the increase of the oversample rate, as shown at the bottom picture of Fig. 7 for 4-times oversampled DTMB. For both systems, the MSE under delay spread channel is slightly higher than AWGN channel, because the length of the IBI-free region is only 8/9 of the CP in DVB-T and 3/4 of the PN sequence in DTMB.

Simulations of the coarse and precise CFO estimation are carried out on DTMB and DVB-T respectively, where the SFO is set to 80 ppm. The top picture of Fig. 8 illustrates the coarse CFO estimation results which are averaged over 100 consecutive frames. The averaged estimation result is acceptable for the carrier frequency acquisition mode, in which the usually large initial CFO has to be coarsely estimated.
and compensated into a smaller range. The MSEs of the precise CFO estimation with different SNR per sample values are shown in the bottom picture of Fig. 8, where the MSE under delay spread channel is slightly degraded due to the same reason as in the SFO estimation. The accuracy of the precise CFO estimation is good enough for carrier frequency tracking mode, in which the small and varying residual CFO is estimated and tracked.

V. CONCLUSION

This paper presents a novel time domain joint synchronization scheme solely based on cyclic property. By exploiting the two-dimensional correlation result of 2D-SWAC, frame synchronization, SFO and CFO estimation are achieved simultaneously. DVB-T and DTMB systems are adopted to evaluate the performance of the proposed scheme under both AWGN and delay spread channel. Simulations show that the proposed scheme is robust to large SFO and CFO as well as channel frequency selectivity, and it works well with 2-times oversampling at the receiver. The proposed joint synchronization scheme is suitable for all transmission systems with cyclic property.

REFERENCES