Iterative NR Decoding and Channel Estimation for TDS-OFDM System

Qiuliang Xie, Kewu Peng, Fang Yang and Zhixing Yang
Tsinghua National Laboratory for Information Science and Technology (TNList)
Department of Electronic Engineering, Tsinghua University, Beijing 100084, China
Email: xql06@mails.tsinghua.edu.cn

Abstract—To improve the overall performance of time-domain synchronous (TDS) orthogonal frequency division multiplexing (OFDM) systems, novel iterative decoding and channel estimation algorithm with Nordstrom-Robinson (NR) code is proposed in this paper. Compared to the non-iterative case, the system performance can be significantly improved by the proposed iterative algorithm at the cost of a little complexity and latency increase. Simulation results show that, for example, with maximum Doppler spread of 25 Hz under two typical multipath channels, at bit error rate (BER) of $10^{-4}$, the signal to noise ratio (SNR) performance of the proposed iterative algorithm is only 0.2 to 0.3 dB away from the ideal case, and about 1 dB or more better performance can be significantly improved by the proposed iterative algorithm with Nordstrom-Robinson code.

I. INTRODUCTION

To combat with inter-symbol interference (ISI) caused by multi-path propagation, orthogonal frequency division multiplexing (OFDM) is one of the most promising techniques and has found a wide utilization in broadband wireless systems, such as the wireless local area network (WLAN), digital audio/video broadcasting, etc. All above systems insert cyclic-prefix (CP) as guard interval (GI) between adjacent OFDM symbols. Besides CP-OFDM, an alternative technique called time-domain synchronous OFDM (TDS-OFDM, also known as pseudorandom postfix (PRP)-OFDM) [1] has been proposed in recent years where CP is replaced by known training sequence (TS) as GI [2]. Digital television/terrestrial multimedia broadcasting (DTMB) [3], announced as the Chinese digital television terrestrial broadcasting (DTTB) standard, employs TDS-OFDM as one of the core techniques to achieve both fast and reliable synchronization and channel estimation.

In TDS-OFDM systems, OFDM symbols and TSs must be separated at the receiver side and usually the TSs are used to perform channel estimation. However, OFDM symbols and TSs will interfere with each other due to multipath propagation in broadband wireless channels. As a result, channel estimation becomes a big challenge in TDS-OFDM systems. Several techniques were proposed to solve this challenge [4]–[8], among which Tang [6] proposed a decision-aided method where the OFDM symbol is partially hard decided to help suppress its interference on TS, then the TS with less interference can be obtained for better channel estimation, compared to those without partial decision. Yang [8] extended this partial decision idea and proposed a decision-directed channel estimation (DDCE) method for TDS-OFDM systems where the data sub-carriers after partial decision are also used for channel estimation. Both methods mentioned above work well over static or slow time-varying frequency-selective fading channels. However, for fast-varying frequency-selective channel with large maximum Doppler spread, the system performance is greatly suffered.

On the other hand, DDCE combined with channel decoding, also named iterative decoding and channel estimation (IDCE), has been drawing intensive and extensive attentions nowadays, since significant performance gain can be provided compared to non-iterative cases [9], [10]. IDCE with limit-approaching codes, such as Turbo codes and low-density parity check (LDPC) codes, has attracted much interest in recent years [11]–[14]. However, such schemes suffer from high implementation complexity and long time delay, especially when long time-interleaving is employed, which is typical in DTTB systems. A possible solution may be the serial concatenation of a small/short inner code for IDCE with a powerful outer code, e.g., a rate-1/2 4-state recursive systematic convolutional code as the inner code for IDCE is serially concatenated with a rate-1/2 LDPC outer code in [15]. However, convolutional codes will cause error propagation and block codes should be more suitable for this iterative case. Therefore, Nordstrom-Robinson (NR) code is chosen as the inner code for IDCE combined with an LDPC outer code in this paper. The most relevant work to ours was done by Muck etc. [16], where iterative decoding and interference cancelation were combined for PRP-OFDM systems, and convolutional codes were employed.

NR code, invented by Nordstrom and Robinson [17], is an optimal non-linear code that maximizes the minimum Hamming distance up to 6 with coding length 16 and rate-1/2. NR code concatenated with 4 quadrature-amplitude modulation (4QAM) mapping has been employed by DTMB as one working mode [3]. For more information about NR code please see literature [18] and references therein.

The rest of this paper is organized as follows. TDS-OFDM system model and the channel estimation method with partial hard decision [6] are introduced in Section II. In Section III, IDCE with NR soft decoding feedback is proposed for TDS-OFDM systems. Simulation results are presented in Section IV.
to demonstrate the performance gain, and finally conclusions are drawn in Section V.

II. SYSTEM MODEL AND CHANNEL ESTIMATION

A. System Model for TDS-OFDM

The baseband model of a typical LDPC-coded TDS-OFDM transceiver is shown in Fig. 1. At the transmitter side, after channel coding, interleaving and symbol mapping, \{S_k^{(i)}\}_{k=0}^{N_d-1} are OFDM modulated to \{s_n^{(i)}\}_{n=0}^{N_i-1}, and then TSs are inserted as GI. At the receiver side with perfect synchronization assumed, TSs are used for channel estimation usually. However, multipath propagation will cause mutual-interferences between TSs and OFDM symbols. To achieve accurate channel estimation based on TS, the interferences of OFDM symbols on TSs should be cancelled or suppressed. Nevertheless, both OFDM symbols and the channel impulse response (CIR) are unknown, which presents a big challenge for channel estimation in TDS-OFDM systems. To reduce the implementation complexity as well as to achieve accurate CIR initialization, a special slot structure of TDS-OFDM system is proposed in [8] and reused in this paper, as shown in Fig. 2. Each slot consists of \( F \) signal frames and a slot head (SH), which is the repetition of the TS in the first signal frame. Identical TSs from frame to frame are assumed in this paper. The SH overhead does not cause much spectrum efficiency penalty, e.g., such penalty is only about 1\% with \( F = 10 \) for TS420 mode in DTMB system, where the length of TS is 420 and the OFDM size is 3780.

In this paper, channels are assumed quasi-static and modeled as an \( L \)-th order finite impulse response (FIR) filter, i.e., the CIR \( \{h_n^{(i)}\}_{n=0}^{L-1} \) is static within at least each signal frame period. The TS length \( N_t \) is assumed no shorter than the channel length \( L (N_t \geq L) \) such that inter block interference between adjacent OFDM symbols can be fully avoided. As the CIR length \( L \) is unknown to the receiver, \( L = N_t \) is assumed for the worst case. The linear convolution between OFDM symbol \( \{s_n^{(i)}\}_{n=0}^{N_d-1} \) and CIR \( \{h_n^{(i)}\}_{n=0}^{L-1} \) is written as

\[
x_n^{(i)} = s_n^{(i)} * h_n^{(i)}, \quad 0 \leq n < N_d + N_t - 1
\]

where * denotes linear convolution operation. The linear convolution of TS \( \{c_n\}_{n=0}^{N_t-1} \) and CIR is given by

\[
y_n^{(i)} = c_n * h_n^{(i)}, \quad 0 \leq n < 2N_t - 1.
\]

Then, the \( i \)th received signal frame can be expressed as

\[
r_n^{(i)} = \begin{cases} 
  x_{n+N_d}^{(i-1)} + y_n^{(i)} + w_n^{(i)}, & 0 \leq n < N_i \\
  x_{n-N_i}^{(i-1)} + y_n^{(i)} + w_n^{(i)}, & N_i \leq n < 2N_t \\
  x_{n-N_i}^{(i-1)} + w_n^{(i)}, & 2N_t \leq n < N_t + N_d
  \end{cases}
\]

where \( x_{n+N_d}^{(i-1)} \) is the interference of OFDM symbol in \( (i-1) \)-th frame on the TS in \( i \)th frame, and \( \{y_n^{(i)}\}_{n=2N_t}^{2N_i} \) is the interference of TS on OFDM symbol in \( i \)th frame.

B. Channel Estimation

As shown in (3), OFDM symbols and TSs interfere with each other at the receiver side. We denote

\[
\hat{x}_n^{(i)} = s_n^{(i)} \otimes h_n^{(i)}, \quad 0 \leq n < N_d
\]

and

\[
\hat{y}_n^{(i)} = c_n \otimes h_n^{(i)}, \quad 0 \leq n < N_t
\]

as the circular convolution of \( s_n^{(i)} \) and \( c_n \) with CIR \( h_n^{(i)} \), respectively. Then, as shown in Fig. 3, the following equation holds that

\[
r_n^{(i)} = r_n^{(i)} + \hat{r}_n^{(i-1)} = \left[x_n^{(i-1)} + x_{n+N_t}^{(i-1)}\right] + \left[y_n^{(i)} + y_{n+N_t}^{(i)}\right] + \left[w_n^{(i)} + w_{n+N_t}^{(i-1)}\right] \approx \hat{x}_n^{(i-1)} + \hat{y}_n^{(i)} + w_n^{(i)} + w_{n+N_t}^{(i-1)}, \quad 0 \leq n < N_t
\]
with the assumption of slow-varying CIR ($h_n^{(i-1)} \approx h_n^{(i)}$) and identical TS in each frame, under which we have $y_{n+N_i}^{(i)} \approx y_{n+N_i}^{(i-1)}$. When the CIR estimation of the $(i-1)$th frame $\hat{h}_{n}^{(i-1)}$ is obtained, $\hat{y}_{n}^{(i-1)} \approx c_n \otimes \hat{h}_{n}^{(i-1)}$ can be removed from (6) and we approximately get

$$
\begin{align*}
\left\{ \hat{x}_{n}^{(i-1)} + w_{n+N_i}^{(i-1)} + w_{n}^{(i)} , \quad 0 \leq n < N_t \\
\hat{x}_{n}^{(i-1)} + w_{n+N_i}^{(i-1)} , \quad N_t \leq n < N_d
\end{align*}
$$
(7)

for OFDM demodulation [6], by which mean $s_{n}^{(i-1)}$ can also be obtained. Then, $\hat{x}_{n}^{(i-1)}$ in (6) can be derived according to (4) and be subtracted from it and finally $\hat{y}_{n}^{(i)} + w_{n}^{(i)} + w_{n}^{(i-1)}$ for CIR estimation of $i$th frame is approximately obtained. The CIR can be estimated via least square (LS) algorithm given by

$$
\hat{h}_{n}^{(i)} = F_{N_t}^{-1}(F_{N_s}(\hat{h}_{n}^{(i)} - \hat{x}_{n}^{(i-1)})/F_{N_s}(c_n)), \quad 0 \leq n < N_t
$$
(8)

where $F_{N_s}(-)$ and $F_{N_t}^{-1}(-)$ denote $N_t$-point discrete Fourier transform (DFT) and inverse discrete Fourier transform (IDFT), respectively. These four steps of cancelling CIR interference on OFDM symbol, OFDM demodulation, cancelling OFDM symbol interference on TS, and CIR estimation will move on from frame to frame.

By taking a careful look at the CIR estimation procedure shown above, we conclude that this algorithm is sensitive to the initial CIR and not suitable for fast varying channel based on the following two observations:

1) Interference cancellation between OFDM symbols and TSs depends on the initial CIR. If the initial estimation is not accurate enough, such interference cancellation will not be effective and consequently cause great performance degradation.

2) During the interference cancellation step, both approximations $y_{n+N_i}^{(i-1)} \approx y_{n+N_i}^{(i)}$ and $\hat{y}_{n}^{(i-1)} \approx \hat{y}_{n}^{(i)}$ are based on the assumption that $h_{n}^{(i)} \approx \hat{h}_{n}^{(i-1)}$. Therefore, if the channel varies too fast, the system performance will also suffer greatly.

III. ITERATIVE NR DECODING AND CHANNEL ESTIMATION

The initial CIR problem can be solved by adding a SH for each slot as shown in Fig. 2. Nevertheless, the time-varying problem remains a big challenge. In this section, iterative NR decoding and channel estimation is discussed in detail, which can significantly improve the system performance, especially for time-varying channels. Details of the proposed IDCE procedure goes as follows.

A. CIR initialization and noise power estimation

For the first frame in each slot, since the TS is circular protected by SH, the initial CIR can be obtained easily as

$$
\hat{h}_{n}^{(0)} = F_{N_t}^{-1}[F_{N_s}(r_n^{(0)})/F_{N_s}(c_n)], \quad 0 \leq n < N_t
$$
(9)

However, since $c_n$ may not be flat in frequency domain (or DFT domain, to be exactly) which will cause noise amplification in LS algorithm, and based on the observation that usually the CIR is a sparse vector, noise-suppression work, such as clipping in frequency domain and forcing small paths to be zero in time domain, need to be done. After these operations, such CIR estimation is more accurate and the variance of the AWGN can be estimated as

$$
\hat{\sigma}^2_w = \frac{1}{N_t} \sum_{n=0}^{N_t-1} \| F_{N_s}(r_n^{(0)}) - F_{N_s}(c_n)F_{N_s}(\hat{h}_{n}^{(0)}) \|^2.
$$
(10)

The power of the noise is assumed constant even in time-varying channels, hence $\hat{\sigma}^2_w$ is only estimated at the first frame for each slot.

B. TS subtraction and OFDM symbol reconstruction

Then, with the slow time-varying assumption $h_{n}^{(i+1)} \approx h_{n}^{(i)}$, $\hat{y}_{n}^{(i+1)}$ can also be obtained according to (5) and subtracted from (6). In this manner,

$$
\begin{align*}
\hat{x}_{n}^{(i)} &\approx \hat{x}_{n}^{(i-1)} + w_{n+N_i}^{(i)} + w_{n}^{(i+1)}, \quad 0 \leq n < N_t \\
\hat{x}_{n}^{(i)} &\approx \hat{x}_{n}^{(i-1)} + w_{n+N_i}^{(i)}, \quad N_t \leq n < N_d
\end{align*}
$$
(11)

can be derived for OFDM demodulation of $i$th frame.

C. OFDM demodulation and soft NR decoding

After being reconstructed according to (11), the OFDM symbol $\{c_n\}_{n=0}^{N_s-1}$ should be demapped with the help of CIR estimation of $i$th frame. In this paper, we only consider 4QAM constellation with Gray mapping. By denoting the channel frequency response (CFR), the $N_d$-point DFT of CIR, as

$$
H_k^{(i)} = F_{N_s}(\hat{h}_{n}^{(i)}), \quad 0 \leq k < N_d
$$
(12)

the two bits corresponding to one 4QAM symbol in the $k$th subcarrier of $i$th frame can be demapped as [19]

$$
L_{k,0}^{(i)} = \frac{4}{\hat{\sigma}^2_w} \mathcal{A}\{ Z_k^{(i)} H_k^{(i)*}\}
$$
(13)

and

$$
L_{k,1}^{(i)} = \frac{4}{\hat{\sigma}^2_w} \mathcal{J}\{ Z_k^{(i)} H_k^{(i)*}\},
$$
(14)

where $\mathcal{A}\{\cdot\}$ and $\mathcal{J}\{\cdot\}$ denote conjugation and the operations of picking up the real and imaginary parts, respectively, and $Z_k^{(i)} = F_{N_s}(c_n^{(i)})$ is the frequency representation of the reconstructed OFDM symbol.

The log-likelihood ratio (LLR) values derived from the demapper according to (13) and (14) are then passed to the NR decoder after being de-interleaved ($\Pi_2^{-1}$). The decoding algorithm employed in this paper is called log-sum approximation (LSA) algorithm given by [18]

$$
\hat{L}(b_j) = \frac{1}{2} \max_{c \in S_j^{(i)}} \gamma(c) - \max_{c \in S_j^{(i-1)}} \gamma(c),
$$
(15)

where $\gamma(c) = \sum_{m=0}^{15} c_n L(b_m)$ is the cross correlation between the demapping output and NR codeword $c$, and $S_j^{(p)}$ denotes the codeword subset with the $j$th bit $p \in \{-1, 1\}$. 

978-1-4244-4148-8/09/$25.00 ©2009
This full paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.
D. Soft remapping and CFR refinement

Although no proof has been provided that the following remapping method is optimal, it has been demonstrated working well through empirical experiments for the case of BPSK that [9]

\[ \hat{b}_j = E\{b_j | \hat{L}(b_j)\} = \tanh\left(\frac{\hat{L}(b_j)}{2}\right). \]  

(16)

This method can be easily extended to 4QAM remapping where (16) is used directly by both the real and imaginary parts. The remapped 4QAM symbols will be used to refine the CFR estimation, where those located in the area \( \Omega \) in Fig. 1 are believed reliable and are rounded to the nearest 4QAM points. For subcarriers with these reliable 4QAM symbols, their CFRs can be refined as

\[ \hat{H}^{(i)}_k = Z^{(i)}_k / \hat{Z}^{(i)}_k. \]  

(17)

For other subcarriers, we simply let \( \hat{H}^{(i)}_k = \tilde{H}^{(i)}_k \).

Operations in this subsection can be operated in an iterative manner with OFDM demodulation and NR decoding shown in subsection III-C. Otherwise, \( \{Z^{(i)}_k\}_{k=0}^{N_d-1} \) with the help of \( \{\hat{H}^{(i)}_k\}_{k=0}^{N_d-1} \) are demapped, de-interleaved, NR decoded and passed to the outer LDPC decoder.

E. OFDM subtraction and CIR updating based on TS

After NR decoding and 4QAM remapping, the transmitted OFDM symbol and CIR of \( i \)th frame can be regarded as \( \hat{s}^{(i)}_n \approx F^{-1}N_d(\hat{Z}^{(i)}_k) \) and \( \hat{h}^{(i)}_n \approx F^{-1}N_d(\hat{H}^{(i)}_k) \), respectively. According to (4) and (6), the interference-free TS can be derived as

\[ t^{(i+1)}_n = t^{(i)}_n + F^{-1}N_d(\hat{Z}^{(i)}_k \hat{H}^{(i)}_k). \]  

(18)

\[ = t^{(i+1)}_n + w^{(i+1)}_n + w^{(i)}_n + e^{(i+1)}_n, 0 \leq n < N_t \]

where \( e^{(i+1)}_n \) denotes errors caused by non-accurate CIR, non-ideal OFDM decision and the influence of channel varying. Then CIR of the \( (i+1) \)th frame can be estimated as

\[ \hat{h}^{(i+1)}_n = F^{-1}N_d(F_{N_t}(t^{(i+1)}_n)/F_{N_t}(c_n)), 0 \leq n < N_t. \]  

(19)

Since the CIR of \( i \)th frame \( \hat{h}^{(i)}_n \) has been refined by the OFDM symbol, which is adjacent to the TS of \( (i+1) \)th frame, \( \hat{h}^{(i+1)}_n \) can be smoothly averaged as

\[ \hat{h}^{(i+1)}_n \leftarrow (1 - \alpha) \hat{h}^{(i)}_n + \alpha \hat{h}^{(i+1)}_n, \]

(20)

where \( \alpha \in (0, 1] \) is the smoothing factor. Generally speaking, if \( \hat{h}^{(i)}_n \) is accurate, which can be measured by the number \( N_d \) of 4QAM symbols used for CFR refinement in (17), \( \alpha \) should be small. Otherwise, \( \alpha \) should be large. On the other side, as CFR derived from TS is always useful, \( \alpha \) should be larger than 0. An empirical method to determine \( \alpha \) can be expressed as, for example, \( \alpha = 1 - N_s / (2N_d) \).

When the CIR of the \( (i+1) \)-th frame is obtained, the TS subtraction and OFDM symbol reconstruction of \( (i+1) \)-th frame could move on, until all the frames in this slot have been processed.

### IV. Simulation Results

Simulation results are presented in this section to evaluate the performance of proposed IDCE algorithm. The major parameters are listed in Table I, and two typical broadcasting multipath channels are shown in Table II, where Channel I consists of a violent long delay echo for single frequency network environment, and Channel II is Brazilian channel model A from field test.

Three main parameters for performance evaluation have been investigated, including normalized mean square error (MSE) and bit error rate (BER) with and without outer LDPC code. The system performance using the proposed iterative algorithm is compared to the non-iterative case [6] and the ideal case with perfect channel estimation. The TS power is set twice that of OFDM symbols for better channel estimation, under which condition the non-iterative case benefits a lot compared to that with equal power.

The normalized MSE given by \( E\{||H - H_{\text{ideal}}||^2/||H_{\text{ideal}}||^2\} \), is shown in Fig. 4 and Fig. 5 for the iterative and non-iterative cases under Channel I and II, respectively. It can be observed that the accuracy of the channel estimation can be significantly improved by the iterative algorithm in all conditions compared to the non-iterative case. Especially, in fast-varying channels with large maximum Doppler spread, the CFR accuracy almost keeps the same with the increasing signal-to-noise ratio (SNR) in the non-iterative case, but it can be significantly improved by the proposed iterative algorithm.

The decoding performances without outer LDPC code are shown in Fig. 6 and Fig. 7 for channel I and channel II, respectively. As shown in these two figures, when there is no Doppler spread that \( f_d = 0 \) Hz, BERs of both iterative...
and non-iterative cases are indistinguishable from the ideal one. This is reasonable, because the CFR estimation is very accurate when $f_d = 0$, as shown in Fig. 4 and 5 that their normalized MSEs are both less than $-16$ dB at the SNR region we are usually interested in. For Doppler spread channels, since the iterative algorithm can significantly improve the CFR accuracy, the decoding performance can also be greatly improved, compared to the non-iterative case.

A non-regular (7488,6096) LDPC code in DTMB is used for the test of serial concatenation case. With 4QAM constellation, 3744 OFDM subcarriers (the left 36 subcarriers are for transmission parameter symbol (TPS) in DTMB [3]) can carry 7488 bits. Considering of the rate-1/2 NR code, two signal frame can carry one LDPC codeword, and therefore one slot can carry 5 LDPC codewords since it contains $F = 10$ frames. 7488 $\times$ 150 bits after LDPC encoding are block interleaved before passing them through the NR encoder, i.e., the size of interleaver $\Pi_1$ shown in Fig. 1 is 30 slots. The LDPC decoder does not participate in IDCE, but a maximum of 30 iterations is used with normalized MIN-SUM algorithm [20]. The LDPC-coded system performances are shown in Fig. 8 and Fig. 9 for channel I and channel II, respectively. As shown in these two figures, decoding performance can be significantly improved with the help of the outer LDPC code. Similar to Fig. 6 and Fig. 7, the SNR vs BER performances of the ideal, IDCE and non-iterative cases are indistinguishable over no Doppler spread channels where $f_d = 0$. In Doppler spread channels, better BER performance can be gained via the proposed IDCE algorithm. Furthermore, when the channel is fast-varying with large maximum Doppler spread such as $f_d = 50$ Hz, the non-iterative scheme even cannot work, while the IDCE scheme works well with only about 1 to 1.5 dB SNR loss from the ideal case, at BER of $10^{-4}$. This phenomenon is also reasonable, because CFR in the non-iterative case is far from accurate in fast-varying channels which mainly limits the decoding performance.
Therefore, it can be concluded that such iterative system is suitable for practical implementation.

Furthermore, this paper only deals with 4QAM. Other high order constellations are still under consideration, such as 16QAM, 64QAM, etc.

V. CONCLUSION

Novel iterative NR decoding and channel estimation algorithm is proposed in this paper for TDS-OFDM system. With only one iteration, this algorithm results in significant improvement relative to the non-iterative approach, especially in time-varying channels with large maximum Doppler spread. Compared to the non-iterative case at BER of $10^{-4}$, for slow fading case $f_d = 25$ Hz, 1 dB or more SNR gain is achieved with only about 0.2 to 0.3 dB gap from the ideal case, for both channel I and channel II. For fast fading case $f_d = 50$ Hz, the non-iterative system fails to work while the iterative one still works well with only about 1 to 1.5 dB gap from the ideal case.

On the other hand, compared to the non-iterative case, the implementation complexity and latency increase is controlled in the iterative one, since only one iteration is employed, and the iterative algorithm is performed within one signal frame. Therefore, it can be concluded that such iterative system is suitable for practical implementation.

REFERENCES