A Novel PN Complementary Pair for Synchronization and Channel Estimation

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SUMMARY In this paper, a novel pseudo-random noise complementary pair (PNCP) is proposed and adopted as the guard intervals in the time-domain synchronous OFDM (TDS-OFDM) system. The proposed PNCP has nearly ideal aperiodic auto-correlation property and inherits the differential property of the PN sequence. Simulations demonstrate the proposed TDS-OFDM system padded with PNCP could achieve better performance in both synchronization and channel estimation than the conventional TDS-OFDM system.

key words: TDS-OFDM, PN complementary pair, aperiodic auto-correlation, synchronization, channel estimation

1. Introduction

Time-Domain Synchronous Orthogonal Frequency Division Multiplexing (TDS-OFDM) is one of the key technologies for Chinese Digital Television/terrestrial Multimedia Broadcasting (DTMB) standard [1]. Unlike other OFDM techniques, TDS-OFDM inserts the pseudo-noise (PN) sequences as the guard intervals instead of cyclic prefix (CP) or zero padding (ZP). The PN sequences could also be used as training for both synchronization and channel estimation [2–5].

A distinguished advantage of the PN sequence lies in its differential property [3], which could be exploited for primary synchronization in the presence of large carrier frequency offset (CFO), when the conventional cross correlation algorithm [2] fails to work. With the differential operation of the received data, the phase rotation due to CFO is eliminated while the training sequence retains good auto-correlation property.

The δ-like auto-correlation property of the PN sequence could also be adopted for channel estimation in the time domain [4]. However, the aperiodic auto-correlation of the PN sequence is far from ideal. The power of the side-lobes accounts for considerable amount of the total power of the correlation, which causes high estimation errors. The Golay complementary pair (GCP) is proposed to be a pair of sequences with the sum of their auto-correlations equal to the δ function [6]. However, the GCP does not have the similar differential property as the PN sequence and hence could not support synchronization under large CFO, which greatly limits its application.

In this paper, we construct a novel complementary pair from the PN sequences, according to the construction rule of GCP. The proposed PN complementary pair (PNCP) inherits the differential property of the PN sequence and the side-lobes in the aperiodic auto-correlation of the PNCP could be reduced to the minimum level through a transversal search of all the PN sequences. The proposed PNCP could be exploited as the training sequences in the TDS-OFDM system to improve the performance of synchronization and channel estimation.

2. PN Complementary Pair

The PN complementary pair \((a, b)\) is defined as

\[
a = [p_1, p_2], \quad b = [p_1, -p_2] \tag{1}
\]

where \(p_1, p_2\) are two PN sequences with equal length, and \([p_1, p_2]\) denotes the serial concatenation of \(p_1\) and \(p_2\). The aperiodic auto-correlation of the PNCP is defined by

\[
R_c(n) = (R_{aa}(n) + R_{bb}(n))/2 \tag{2}
\]

where \(R_{aa}\) and \(R_{bb}\) are the aperiodic auto-correlations of \(a\) and \(b\), respectively.

The correlation property could be measured by its merit factor, which is defined to be the ratio of the power of the correlation peak to the power of the side-lobes [7],

\[
F = |R_c(0)|^2 / \sum_{n \neq 0} |R_c(n)|^2 \tag{3}
\]

The optimal PNCP with the largest merit factor could be searched through all the available PN sequences. Take the length-254 PNCP (PNCP254) as an example, the PNCP254 is composed by two length-127 PN sequences via equation (1). The aperiodic auto-correlation of the optimal PNCP254 is shown in Fig. 1(b). Meanwhile, the correlations of the corresponding length-255 PN sequence (PN255) and length-256 GCP (GCP256) are also plotted in Fig. 1(a) and (c), respectively. Compared to the PN sequence, the side-lobes
The proposed PNCP are significantly reduced. The search results for the optimal PN255, PN511 (length-511 PN sequence), PN595 (a truncation of length-1024 PN sequence), and the optimal PNCP254, PNCP510 (composed by two length-255 PN sequences), PNCP594 (composed by two truncated length-297 PN sequences) are listed in Table 1, respectively.

**Table 1** Search Results for the Optimal PNCP

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Polynomial</th>
<th>Initial Phase</th>
<th>Merit Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>PN255</td>
<td>(x^4 + x^3 + x^2 )</td>
<td>00111100</td>
<td>3.6601</td>
</tr>
<tr>
<td>PNCP254</td>
<td>(p_1)</td>
<td>(x^6 + x^2)</td>
<td>0111100</td>
</tr>
<tr>
<td></td>
<td>(p_2)</td>
<td>(x^2 + x^5)</td>
<td>1101001</td>
</tr>
<tr>
<td>PN511</td>
<td>(x^6 + x^3 + x^2 + x + 1)</td>
<td>000101010</td>
<td>14.8663</td>
</tr>
<tr>
<td>PNCP510</td>
<td>(p_1)</td>
<td>(x^8 + x^4 + x + 1)</td>
<td>00011001</td>
</tr>
<tr>
<td></td>
<td>(p_2)</td>
<td>(x^2 + x^5 + x^2 + 1)</td>
<td>10011001</td>
</tr>
<tr>
<td>PN595</td>
<td>(x^8 + x^4 + x^2 + 1)</td>
<td>0010011101</td>
<td>1.9543</td>
</tr>
<tr>
<td>PNCP594</td>
<td>(p_1)</td>
<td>(x^8 + x^4 + x^2 + 1)</td>
<td>01101001</td>
</tr>
<tr>
<td></td>
<td>(p_2)</td>
<td>(x^2 + x^5 + x^2 + 1)</td>
<td>10110001</td>
</tr>
</tbody>
</table>

The proposed PNCP inherits the differential property of the PN sequence. The differential operation of \(a\) and \(b\) are defined as

\[
\tilde{a}_D(n) = a(n) \cdot a^*((n - D) \mod L), \quad 0 \leq n < L \quad (4)
\]

\[
\tilde{b}_D(n) = \begin{cases} 
  b(n) \cdot b^*((n - D) \mod L), & 0 \leq n < L/2 \\
  -b(n) \cdot b^*(n - D), & L/2 \leq n < L 
\end{cases} \quad (5)
\]

where \(L\) is the length of PNCP, \(D\) is the differential interval, \(*\) and \(mod\) denotes the complex conjugation and the modular operation, respectively. It should be noted that the second half of \(b_D(n)\) is phase-reversed of the original differential sequence. When \(D\) is much less than \(L\), the pair \((\tilde{a}_D, \tilde{b}_D)\) could be approximated to

\[
\tilde{a}_D \approx [\tilde{p}_{1D}, \tilde{p}_{2D}], \quad \tilde{b}_D \approx [\tilde{p}_{1D}, -\tilde{p}_{2D}] \quad (6)
\]

where \(\tilde{p}_{1D}\) and \(\tilde{p}_{2D}\) are the \(D\)-spaced differential sequences of \(p_1\) and \(p_2\), respectively.

The differential correlation for the PNCP is defined as

\[
R_d(n) = \frac{1}{2} \left( \sum_{i=0}^{L-1-n} \tilde{a}_D(i) \tilde{a}_D^*(i) + \sum_{i=0}^{L-1-n} \tilde{b}_D(i) \tilde{b}_D^*(i) \right) \quad (7)
\]

Since \(\tilde{p}_{1D}\) and \(\tilde{p}_{2D}\) are both PN sequences, the pair \((\tilde{a}_D, \tilde{b}_D)\) could be regarded as a new PNCP. Therefore, good correlation property of \(R_d(n)\) is expected.

Fig. 2 (a), (b), and (c) depict the differential correlations of PN255, PNCP254 and GCP256 when \(D = 1\), respectively. The differential correlations of the PN sequence and PNCP are still of large merit factors while the result of GCP degrades severely.

With a cascaded construction from (1), we can further define the multi-level PN complementary pair,

\[
a^{(k)} = [a^{(k-1)}, b^{(k-1)}], \quad b^{(k)} = [a^{(k-1)}, -b^{(k-1)}] \quad (8)
\]

where \((a^{(k)}, b^{(k)})\) is the \(k\)-th level PNCP, and the 1st level PNCP \((a^{(1)}, b^{(1)})\) is defined by (1). Meanwhile, The multi-level PNCP retains good auto-correlation and differential correlation properties.

### 3. Synchronization and Channel Estimation Based on PNCP

In this section, we exploit the proposed PNCP to pad the guard intervals of the TDS-OFDM system. The frame structure is shown in Fig. 3. The paired sequences \(a\) and \(b\) are padded into the guard intervals between OFDM blocks alternately.

Considering the transmitted signal \(s(n)\) as shown in Fig. 3, the received signal can be represented as

\[
r(n) = (s(n - \theta) \cdot e^{j n \omega_c}) \otimes h(n) + \nu(n) \quad (9)
\]

where \(\theta, \omega_c, \) and \(h(n)\) are the unknown symbol arrival time, normalized CFO, channel impulse response (CIR) and additive white Gaussian noise (AWGN), respectively. The operator \(\otimes\) denotes linear convolution.

In presence of large CFO, the differential correlation between the received signal and the local PNCP is applied for synchronization, which is given by
Since the complexity of the conventional length-L PNCP for example, from (1) and (13), the cross correlation with the local PNCP. Therefore, the hardware implementation is achieved through the aperiodic differential/cross correlation due to CFO is reduced to a fixed phase offset. 

The aperiodic auto-correlation of the PNCP is restricted in $(-\pi, \pi / D)$, the estimation range is $(-\pi/2, \pi/2)$.

After frame synchronization and CFO compensation, the received PNCP can be extracted for channel estimation. Correlate the compensated data $r(n)$ with the local PNCP,

$$R_{c,rx}(n) = \sum_{i=0}^{L-1} r(i + n) \cdot a^*(i) + \sum_{i=0}^{L-1} r(i + n + N) \cdot b^*(i)$$

where $a(n)$ and $b(n)$ are GCP instead. The length of the guard intervals is chosen to be 420, which is composed of PN255, the 1st level PNCP and 16 local PNCPs, respectively.

Since the aperiodic auto-correlation of the PNCP is approximate to the $\delta$ function, the result in (13) gives a fine estimation of the CIR.

From the above analysis, both the primary synchronization and channel estimation could be accomplished through the aperiodic differential/cross correlations with the local PNCP. Therefore, the hardware complexity of the receiver is directly related to the implementation of the correlator. Take the 1st level PNCP for example, from (1) and (13), the cross correlation algorithm can be decomposed by

$$R_{c,rx}(n) = \sum_{i=0}^{L/2-1} p_1(i) \cdot (r(i + n) + r(i + n + N)) + \sum_{i=0}^{L/2-1} p_2(i) \cdot (r(i + n + L/2) - r(i + n + L/2 + N))$$

From (14), the cross correlation can be obtained through two sub-correlators of $p_1$ and $p_2$. The structure of the cross correlator for PNCP is shown in Fig. 4. Since the complexity of the conventional length-$L/2$ PN correlator is $O(L/2)$, the complexity of the 1st level length-$L$ PNCP is $O(L)$, which is equal to the length-$L$ PN correlator.

The differential correlator for PNCP can be derived from (1) and (10) with the similar method. The main differences between the differential correlator and the cross correlator in Fig. 4 lie in that the input $r(n)$ should be differentially decoded first, and the coefficients of both sub-correlators are $\tilde{p}_1$ and $\tilde{p}_2$ instead of $p_1$ and $p_2$, respectively.

With the same decomposition method in (14), we can derive the correlator for the multi-level PNCP, which is also composed of the two sub-correlators. The complexity of the correlator for the $k$-th level PNCP with length $L$ is $O(L/2^{k-1})$.

4. Simulation Results

In this section, computer simulations are performed to evaluate the proposed synchronization and channel estimation method for the PNCP-padded TDS-OFDM system. The simulations of PN sequence and GCP are also given for comparison. The signal model for GCP padded TDS-OFDM is the same as Fig 4, where $(a, b)$ are GCP instead. The length of the guard intervals is chosen to be 420, which is composed of PN255, the 1st level PNCP254 and GCP256 with their cyclic extensions, respectively.

In the synchronization process, the differential correlation discussed in Section III is adopted for PNCP. The $D$-spaced differential correlation algorithm in [3] is adopted for the conventional PN-padded TDS-OFDM system. Since the GCP does not have the differential property, the cross correlation in [2] is applied for joint timing and frequency synchronization.

Fig. 5 and Fig. 6 depict the Mean Square Error (MSE) for CFO estimation under AWGN channel versus different CFOs and different SNRs, respectively. It can be observed from Fig 5 that the proposed PNCP could achieve as large estimation range as the PN sequence while the GCP only works within a limited range of less than 20 kHz. Fig. 6 shows that the proposed CFO estimation method based on PNCP improves about 1.5dB compared to the differential PN estimator in [3]. As the interval $D$ increases, the estimation accuracy is improved while the estimation range...
Fig. 5  MSE of CFO estimation versus different CFOs under AWGN channel with SNR=3dB.

Fig. 6  MSE for CFO estimation versus SNR under AWGN channel.

Fig. 7  MSE for channel estimation under Brazil-B channel.

is decreased. It should be noted that the estimation performance of PNCP degrades severely from the theoretical Cramer-Rao Bound (CRB) [8] when $D$ is large, because the formula (6) is an approximation.

The channel estimation results under the DTV field test model Brazil-B [1] for the above three systems are shown in Fig. 7. The GCP achieves the best estimation performance for its ideal auto-correlation property. The proposed channel estimation method based on PNCP improves nearly 2dB under Brazil-B channel compared to the conventional TDS-OFDM when the MSE is required to be no more than $3 \times 10^{-1}$.

5. Conclusion

In this paper, we propose a novel PN complementary pair (PNCP), which retains the differential property of the PN sequence and its aperiodic auto-correlation is nearly ideal. We adopt the proposed PNCP to pad the guard intervals of the TDS-OFDM system. Simulations demonstrate that the PNCP-padded TDS-OFDM could achieve better performance in both synchronization and channel estimation with no hardware complexity increase. In conclusion, the proposed PNCP provides a tradeoff between the PN sequence and Golay complementary pair and inherits the advantages of them.

References