Particle Swarm Optimization for Single Phase PWM Inverters

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Abstract—This article discusses a design procedure of DC-AC inverter. The DC-AC inverter is to produce a sinusoidal AC voltage with adjustable amplitude and frequency. Pulse-width modulation (abbr. PWM) is one of the most used techniques in static inverters. For the PWM, the switching angle is most important, and the switching angle controls the efficiency of DC-AC inversion. For this reason, the design of the optimal switching angle vector is very important. In this article, we obtain such switching angle vector by particle swarm optimization system (abbr. PSO). Our simulation results indicate that the proposed design procedure gives high efficiency inversion.

Index Terms—PSO, PWM, DC-AC inverter, multi-objective optimization problem, elimination band

I. INTRODUCTION

Recently, DC-AC inverters are attracting attention. For example, photovoltaic system generates a DC power, to use the DC power must be inverted an AC power. The DC-AC inverter is to produce a sinusoidal ac voltage with adjustable amplitude and frequency. For such purposes, various inversion procedures have been proposed. Pulse-width modulation (abbr. PWM) is a commonly used technique for controlling power to an electrical device, and it is one of the most used techniques in static inverters. In this article, we focus on a PWM DC-AC inverter.

The efficiency of the PWM DC-AC inversion is depended on the switching angles. For this reason, the design of the optimal switching angles is a very important problem. One of the most popular solutions to such problem is sinusoidal PWM operation as shown in Fig. 1[1]. The sinusoidal PWM procedure with the sinusoidal modulating wave and the triangular carrier wave are employed for sinusoidal waves in AC sides. Comparing with these two waveforms, the switching angles of the output voltage are obtained. The upper figure of Fig. 1 shows the objective sinusoidal waveform and the triangular carrier waveform. The switching angles of the output pulse voltage of the inverter are determined by the frequency of the triangular wave. If the frequency of the triangular wave is high, the switching times of the output pulse are increased. In general, a pulse waveform contains many harmonic components. However, the output pulse wave of the sinusoidal PWM does not include low frequency harmonic components which are near the fundamental component of a desired sinusoidal waveform.

The lower figure of Fig. 1 shows the PWM output voltage of the inverter. Figure 2 shows the power spectrum of the corresponding PWM output voltage which is shown in Fig. 1. The ideal spectra shaping of the PWM DC-AC inverter typically means the creation of a “dead band” between wanted and unwanted spectra components. In the case of Fig.1, the switching times is 13, thus, the power spectrum has a “dead band” between the fundamental frequency and the 13-th harmonic frequency as shown in Fig. 2. The upper bound of the dead band is determined by the frequency of the triangular carrier wave. The sinusoidal PWM operation is very useful. However, in order to increase the interval of the dead band, the switching times must be increased. Also, the design of switching angle is very complicated and it requires large amount of computation[1]. Therefore, many heuristic design procedures have been proposed. Genetic algorithms are recently applied to this problem[2]. Also, ant colony optimization procedure is applied to this problem, too[3]. In this article, we apply a particle swarm optimization system[4]-[9] to the design of switching angles of the PWM DC-AC inverter without changing the switching times.

II. PWM

The objective single phase PWM inverter circuit is shown in Fig. 3. Since this operation has a symmetric property, we consider only the half-cycle period, loss of generality. For generality, the output voltage is assumed to have 2k pulses per half-cycle, with the switching angle symmetrical to \( \pi/2 \). We assume that \( k \) is an odd number, and \( \theta_k \) denotes the switching
Fig. 1. Scheme of conventional sinusoidal PWM inverter. The upper figure shows the objective sinusoidal waveform and the triangular carrier waveform. The lower figure shows the obtain output PWM waveform.

Fig. 3. Single phase PWM inverter circuit

angle. In this case, the output voltage $V_o$ can be expressed using Fourier series as

$$V_o = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t). \quad (1)$$

We assume this output voltage has symmetric property, therefore, the coefficients $A_n$ and $A_0$ are zero. Thus, the preceding equation reduces to

$$V_o = \sum_{n=1}^{\infty} B_n \sin(n\omega t) \quad (2)$$

The value of $B_n$ is calculated as

$$B_n(\theta_1, \theta_2, \ldots, \theta_{k-1}, \theta_k) = \frac{4V_{dc}}{n\pi} \left[\cos(\omega t)\right]_{\theta_1, \theta_3, \ldots, \theta_k} \quad (3)$$

The fundamental component $B_1$ is given by

$$B_1(\theta_1, \theta_2, \ldots, \theta_{k-1}, \theta_k) = \frac{4V_{dc}}{\pi} \left[\cos(\omega t)\right]_{\theta_1, \theta_3, \ldots, \theta_k} \quad (4)$$

where $V_{dc}$ denotes a pulse height of the output voltage.

By using Eqs. (3) and (4), we define the following two evaluation functions [11].

$$F_i(\theta) = \sqrt{\frac{\sum_{n=2}^{K} B_n(\theta)^2}{B_1(\theta)}} \quad (5)$$
\[ F_p(\theta) = 1 - \frac{\sum_{n=1}^{K} B_n(\theta)^2}{V_{dc}P_d} \]  

where \( K \) denotes the valuation range, \( 0 < Pd < 1 \) denotes a desired effective power rate, and \( \theta \equiv (\theta_1, \theta_2, \ldots, \theta_{k-1}, \theta_k) \) is a switching angle vector.

\( F_1(\theta) \) means the total harmonic distortion (abbr. THD) that evaluates the fundamental component of the output. If this function becomes zero, it corresponds to the output of only the fundamental component. \( F_p(\theta) \) evaluates the effective power of the output. If this function becomes zero, the effective power rate is close to the desired value \( Pd \).

This problem has two independent evaluation functions. Thus, this problem can be regarded as a kind of multi-objective optimization problems. The integrated evaluation function \( F(\theta) \) is

\[ F(\theta) = \alpha F_1(\theta) + (1 - \alpha) F_p(\theta). \]  

where, \( \alpha \) is a mixture ratio parameter.

In this article, we will solve such multi-objective optimization problem by using PSO.

III. PSO

Searching for an optimal value of a given evaluation function of various problems is very important in engineering fields. In order to solve such optimization problems efficiently, various heuristic optimization algorithms have been proposed. Particle swarm optimization (abbr. PSO), which was originally proposed by J. Kennedy and R. Eberhart [4],[5], is one of such heuristic algorithms.

The original PSO is described as

\[ v_{ij}^{t+1} = w v_{ij}^t + c_1 r_1 (pbest_{ij}^t - x_{ij}^t) + c_2 r_2 (gbest_t - x_{ij}^t) \]  

\[ x_{ij}^{t+1} = x_{ij}^t + v_{ij}^{t+1} \]  

where \( w \geq 0 \) is an inertia weight coefficient, \( c_1 \geq 0 \), and \( c_2 \geq 0 \) are acceleration coefficients, and \( r_1 \in [0, 1] \), and \( r_2 \in [0, 1] \) are two separately generated uniformly distributed random numbers in the range \([0, 1]\). \( x_{ij}^t \in \mathbb{R}^N \) denotes the location of the \( j \)-th particle on the \( t \)-th iteration in the \( N \)-dimensional solution space, and \( v_{ij}^t \in \mathbb{R}^N \) denotes a velocity vector of the \( j \)-th particle on the \( t \)-th iteration.

\( pbest_{ij}^t \in \mathbb{R}^N \) means the location that gives the best value of the evaluation function of the \( j \)-th particle on the \( t \)-th iteration. In this article, \( pbest_{ij}^t \) is called as a personal best location information of \( j \)-th particle on \( t \)-th iteration. \( pbest_{ij}^t \) can be given as

\[ pbest_{ij}^t = \min_{\tau} f(x_{ij}^\tau), \quad \tau \leq t \]  

\( gbest^t \in \mathbb{R}^N \) means the location which gives the best value of the evaluation function on the \( t \)-th iteration. \( gbest^t \in \mathbb{R}^N \) can be given as

\[ gbest^t = \min_j pbest_{ij}^t = \min_j f(x_{ij}^\tau), \quad \tau \leq t \]  

The particles in the swarm fly through the \( N \)-dimensional solution space according with Eqs. (8) and (9). Each particle shares information of a current optimal value of the evaluation function and its corresponding location of the best particle. Also, each particle memorizes its record of the best evaluation value and its best location. On the basis of such information, the moving direction and velocity are calculated by Eq. (8). Namely, all particles will move toward a coordinate that gives the current best value of the evaluation function.

In order to apply the above PSO to the design procedure of the switching angles of PWM DC-AC inverters, we give the following specification to the PSO. Each dimension of the particle \( x_{ij} \) corresponds to switching angles of the DC-AC inverters. We define \( M \) denotes the number of switching angles per 1/4 period. For the PSO system which is described by Eqs. (8) and (9), we bring in the following rules. First, we apply the limitation of the velocity as

\[ v_{ij}^t = \begin{cases} V_{max}, & \text{for } v_{ij}^t > V_{max} \\ V_{min}, & \text{for } v_{ij}^t < V_{min} \\ v_{ij}^t, & \text{otherwise} \end{cases} \]  

The above operation is used to constrain the range of switching angles.

Next, we assume that each dimension has an order, because each dimension variable corresponds to the switching angle. The order is

\[ 0 \leq x_{1j} \leq x_{2j} \leq \cdots \leq x_{kj} \leq \pi/2. \]  

To satisfy the above order, we introduce an adjustment parameter \( V_{limit} \) to the velocity as the follow.

\[ x_{ij}^t = x_{i-1,j}^t + V_{limit}, \text{ for } x_{ij}^t > x_{i-1,j}^t \]  

Also, if the switching angle exceeds the defined domain which is \( 0 \leq x_{ij}^t \leq \pi/2 \), the variable is modified as follows.

\[ x_{ij}^t = \begin{cases} 0, & \text{for } x_{ij}^t < 0 \\ \pi/2, & \text{for } x_{ij}^t > \pi/2 \\ x_{ij}^t, & \text{otherwise} \end{cases} \]  

The above constrains are applied to Eqs. (8) and (9).

IV. SIMULATION

In order to confirm the performance of our proposed procedure, we carry out numerical simulations. In numerical simulations, we fix the parameters as shown in Table I. First, we confirm the influence of the mixture parameter \( \alpha \). The simulation results are shown in Figs. 4, 5, and 6. We suppose that the system can perfectly eliminate the desired higher frequency components from the output waveform. In our simulations, we consider that the output current waveform contains only lower frequencies which are less than 9-th components. In these figures, (a) illustrates the output...
\( \alpha = 0.0 \) (it evaluates only \( F_p(\theta) \)), \( F_i(\theta) = 8.76 \times 10^{-01}, F_p(\theta) = 1.69 \times 10^{-07} \)

\( \alpha = 0.5, F_i(\theta) = 1.68 \times 10^{-02}, F_p(\theta) = 4.49 \times 10^{-03} \)

\( \alpha = 1.0 \) (it evaluates only \( F_i(\theta) \)), \( F_i(\theta) = 6.73 \times 10^{-03}, F_p(\theta) = 1.61 \times 10^{-01} \)

<table>
<thead>
<tr>
<th>Simulation Parameters</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>The trial number</td>
<td>50</td>
</tr>
<tr>
<td>The number of particles</td>
<td>10</td>
</tr>
<tr>
<td>The maximum iteration number</td>
<td>1000</td>
</tr>
<tr>
<td>The number of switchings among quarter period ( N )</td>
<td>11</td>
</tr>
<tr>
<td>Maximum velocity ( V_{\text{max}} )</td>
<td>( \pi/100 )</td>
</tr>
<tr>
<td>Minimum velocity ( V_{\text{min}} )</td>
<td>(-\pi/100 )</td>
</tr>
<tr>
<td>Criterion velocity ( V_{\text{limit}} )</td>
<td>( \pi/1000 )</td>
</tr>
<tr>
<td>( K )</td>
<td>9</td>
</tr>
<tr>
<td>( P_d )</td>
<td>0.707</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.0, 0.5, 1.0</td>
</tr>
<tr>
<td>The inertia weight coefficient ( w )</td>
<td>( w = 0.7298 )</td>
</tr>
<tr>
<td>The acceleration coefficient ( c_1, c_2 )</td>
<td>( c_1 = c_2 = 1.49 )</td>
</tr>
</tbody>
</table>
In the case of $\alpha = 0.5$, the harmonic components become low. Thus, the output current waveform is similar to the desired sinusoidal waveform. The evaluation values are $F_1(\theta) = 1.68e-02$ and $F_p(\theta) = 4.49e-03$.

If we evaluate only $F_1(\theta)$, the output waveform almost does not contain the harmonic components. However, it does not evaluate $F_p(\theta)$, the power of the fundamental component becomes low. In this case, the evaluation values are $F_1(\theta) = 6.73e-03$ and $F_p(\theta) = 1.61e-01$.

Figure 7 shows $\alpha$ dependence of the evaluation values. Based on the results, we estimate the evaluation function gives the optimal value around 0.5. However, the theoretical analysis of the optimal evaluation value is not sufficient. This point is one of our future problems. Next, we confirm the characteristic of the band elimination ability. The initial switching angles are given by the conventional sinusoidal PWM DC-AC inverter procedure in the case where the switching times is nine in 1/4 period. Figure 8 shows the optimization process of the proposed design procedure. The horizontal axis denotes the iteration of proposed algorithm, and the vertical axis means the value of the evaluation function $F(\theta)$. The mixture parameter $\alpha$ is set as $\alpha = 0.5$, and the switching times are fixed as $N = 9$. The other parameters are the same as the previous simulation. Each curve corresponds to the width of each desired elimination band. Namely, the interval of the desired elimination band is between the fundamental component and $k$-th component. In the case of $K = 9$, the evaluation value is not improved, since the sinusoidal PWM control procedure gives an optimal switching angle for nine switching. In the cases of $K = 19$ and $K = 29$, however, the evaluation values are improved. This result indicates our proposed design procedure can improve the performance comparing with the conventional sinusoidal PWM control procedure.

V. CONCLUSIONS

In this article, we proposed the design procedure by using the PSO system. Our proposed design procedure consists of two kinds evaluation functions, one is based on the total harmonic distortion, the other one is based on the average power rate of the output. Thus, this problem can be regarded as a kind of multi-objective optimization problems. For such multi-objective optimization problems, the synthesis of some evaluation functions is very important. We proposed one example of such synthesis procedure, and we obtain the optimal value of such function by our numerical simulations. Also, we confirmed the performance of our proposed procedure can be improved over the conventional sinusoidal PWM procedure.

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REFERENCES