Abstract

Increased reliance upon outsourcing has made the issue of vendor selection even more critical to the success of the modern manufacturing organization. The usual performance measure on which selection is based has been the distribution of the vendor’s delivery lead-time (LT), often as characterized by the mean and variance. In this paper, we show that the distribution of demand per unit time (DPUT) must also be considered if an optimal decision is to be made.

We consider a standard fixed quantity-reorder point \((Q, r)\) inventory policy. We first show that the optimal policy and the resulting costs remain invariant to changes in the mean lead-time demand (LTD) when the variance of LTD is fixed. A change in the mean LTD is completely (linearly) absorbed by the reorder point \(r\), while the safety stock is unaffected.

Intuitively, we would expect that the vendor with the shortest expected lead-time would represent the best choice. However, this expectation is not always correct and we then provide a simple rule for vendor selection based upon the coefficient of variation (CV) for the distribution of DPUT. In particular, when the CV is small (large), it pays to choose a vendor whose lead-time variance (mean) is small. A series of numerical examples illustrates the main theoretical points.

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1. Introduction

As competitive pressures increase, organizations have increasingly restructured their operations by focusing on their core strategically important activities and outsourcing their peripheral operations. Not only has the frequency and volume of outsourcing transactions increased, their terms and bases have also matured radically...
Consequently, these changes have essentially redefined outsourcing as a strategically important activity, as recognized by Hahn et al. (1986) among others. As outsourcing has increased, so has the importance of vendor selection decisions. In this selection process, delivery performance remains an important criterion. A prime determinant of delivery performance for vendors is their lead-time, defined as the time it takes from the moment an order is placed until it arrives. The lead-time is often characterized by its mean and standard deviation. As different vendors have different combinations of these two parameters, choosing among available alternatives in such cases becomes a critical concern. The motivations for this paper are to demonstrate that examination of the lead-time alone is insufficient. The decision-maker must consider the overall distribution of demand during the lead-time. In particular, the correct analysis requires a trade-off between the characteristics of the distributions describing lead-time and demand per unit time. We develop a simple criterion that enables such a trade-off decision to be made.

Vendor selection is important as a way to reduce the supply risk; that is, cost overruns that arise because of unreliable outsourcing. The multiple vendor option has often been proposed in the related literature as a means to reduce such risks (Ammer, 1980; Herron, 1983). Effectively, this alternative reduces service level uncertainty but involves dual or multiple order sourcing. If multiple vendors are to be used, the selection problem does not go away, but becomes much more complex as we must choose both the vendors and the relative amounts to be ordered from each. Indeed, as may be inferred from the analysis in Ramasesh et al. (1993), the best single vendor may not figure in the final solution. Two vendors with very different characteristics might serve to dominate a single vendor whose performance is better than either one when considered separately. Such complex structures are of considerable interest if dual sourcing is considered, but that is not the focus of this paper. Rather, we assume that the decision has been made to use sole sourcing, whether for strategic or other purposes.

We analyze the effect of lead-time on the vendor selection problem using a standard \((Q, r)\) inventory model. In a continuous review system, an order for \(Q\) units of the item is placed with the supplier when the inventory level reaches the reorder point \(r\). The system is subjected to a continuous random demand by its customers. We assume that unsatisfied demand may be backordered, since we envision a continuing relationship between the vendor and the user. The \((Q, r)\) inventory policy, first developed by Hadley and Whitin (1963) is the most commonly used model due to its good performance and ease of implementation. The optimal order quantity, \(Q\) and reorder point, \(r\) are selected to minimize the average annual variable cost. We assume that the system is analyzed in such a way as to provide optimal performance for each vendor and that the choice among vendors is then based upon cost minimization.

The paper is organized as follows. The next section discusses the factors affecting lead-time demand and the following section presents some analytical results from a new perspective. These results are then used to provide a decision rule for vendor choice that takes account of both the lead-time and demand per unit time distributions. This rule is then illustrated by a numerical example. Concluding remarks are presented in the last section.

2. The factors affecting lead-time demands

In 1982, Brown published the text *Advanced Service Parts Inventory Control*. Publication was on a restricted basis and, as a consequence, some of the points made therein are sometimes overlooked. In particular, Brown (1982, p. 257) introduces the notion of a freeze period as follows:

“The meaning of the freeze period is like a conventional lead-time. However, it is not usually observed as the interval of time taken to deliver parts in response to an order. It should be negotiated between the user and the supplier. The implication is that the user has reasonable freedom to change the requirements outside the freeze period. The supplier has an obligation to deliver on time to cover requirements that were stated originally outside the freeze period.”
Brown is clearly thinking in terms of a fixed lead-time, whereas our interest focuses on a more general stochastic lead-time. Nevertheless, the notion of the freeze period is useful in two ways. First of all, in a continuing relationship between buyer and supplier, the freeze period may be much less than the conventional lead-time, allowing for reduced safety stocks. Second, Brown makes the point that the freeze period needs to be compared to the need date, the time at which the inventory is required. With a stochastic lead-time \((L)\) and a stochastic demand per unit time \((D)\), we cannot guarantee deliveries. However, we can observe that it is the relative pattern of freeze period to need date that is important, rather than a naïve assessment based purely on the expected length of the lead-time. From the viewpoint of implementation in a continuing buyer–supplier relationship, it is the freeze period that is important, as the buyer may be assumed to have the flexibility to change the timing of an order until the moment at which it has to be locked in. From the viewpoint of the analysis in this paper, all that matters is that the lead-time distribution is suitably specified, but the appropriate choice is important conceptually.

2.1. The underlying model

The overall lead-time demand is affected by:

(i) The demand per unit time, also known as the Demand rate, \((D)\) which is customer or market driven and therefore, is beyond the realm of management’s control of the system considered in the analysis.

(ii) The lead-time, \((L)\) which is quoted by the vendor and hence, an input to the decision making process in the current context.

The lead-time demand (LTD) is defined as the product of the lead-time and the demand per unit time (annual), or LTD = \(D \times L\). Let \(X\) denote LTD with mean \(\mu\) and standard deviation \(\sigma\) and define the original distribution of \(X\) to be \(F_X(x|\mu, \sigma)\). We may then define the standardized random variable \(Z = (X - \mu)/\sigma\), where \(Z\) is assumed to have the standard distribution function \(F_Z(z)\), independent of \(\mu\) and \(\sigma\). This assumption is more general than the assumption of normality as it corresponds to the location-scale family of distributions (see Johnson et al., 1994, p. 12). For example, this class includes the Laplace, logistic and extreme value distributions but it does not include the lognormal distribution. In essence, we are assuming that all distributions have the same shape after transforming to \(Z\). We also assume that \(L\) and \(D\) are independent random variables. When the lead-time has a large mean and a small standard deviation, we may think of LTD as the sum of unit demands over many time periods and the distribution of LTD will be approximately normal, from the central limit theorem. However, for slow moving items, or when the lead-time is more variable, the normal approximation will be inadequate, and more general results are needed (c.f. Silver et al., 1998, pp. 318–325).

The purpose of this section is to fill that gap.

Given the definition of LTD, we have the following standard results (see, for example, Drake, 1967). The average demand during the lead-time is

\[
E(\text{LTD}) = \mu = E(D)E(L),
\]

with standard deviation

\[
\text{SD}(\text{LTD}) = \sigma = \sqrt{E(L)\text{Var}(D) + E(D)^2\text{Var}(L)},
\]

where \(E(L)\), and \(\text{Var}(L)\) are the mean and variance of lead-time, \(E(D)\) and \(\text{Var}(D)\), are the mean and variance of the demand per unit time, respectively. The standard assumptions implicit in these results are that the demands in successive unit time periods are independent and identically distributed. Since demand per unit time and lead-time are stochastic, we assume that buffer or safety stocks are built into the inventory system as an insurance against the demand and lead-time uncertainty.

To obtain the average annual costs, let

\[
\delta \quad \text{average demand per unit time in units per year (taken to be a finite constant)} \quad (\delta = E(D)),
\]

\[
A \quad \text{ordering cost in dollars per order},
\]

\[
I \quad \text{average annual cost of carrying inventory (i.e., holding cost)},
\]
\[ B \quad \text{backorder cost per unit backordered}, \]
\[ f(\cdot) \quad \text{probability density function for the LTD}, \]
with the average annual cost \( K \) given by the standard \((Q,r)\) model
\[ K = A \frac{\delta}{Q} + I \left( \frac{Q}{2} + r - \mu \right) + B \frac{\delta}{Q} \left[ \sigma \int_{(r-\mu)/\sigma}^{\infty} uf(u) du - (r - \mu) G \left( \frac{r - \mu}{\sigma} \right) \right], \]
(2)
where \( G(\cdot) = 1 - F(\cdot) \), and \( F(\cdot) \) is the distribution function corresponding to \( f(\cdot) \); the term in square brackets represents the expected amount backordered. Eq. (2) requires the location-scale property described earlier, otherwise \( G \left( \frac{r - \mu}{\sigma} \right) \) will not be location and scale invariant. The model assumes that there is never more than one order outstanding, and that the reorder point is positive. He et al. (1998) examined the question of order crossover in depth; when crossovers are allowed, the total cost is somewhat reduced, but the optimal parameter settings are only very slightly affected unless the coefficient of variation of the lead-time is large. Thus, the issue of order crossover is unlikely to affect the results.

Hadley and Whitin (1963) present a numerical (iterative) procedure for optimally solving Eq. (2). It turns out that the optimal solution \((Q^*, r^*)\) is always unique. The proof simply follows from the fact that \( K \) is convex. Thus, we compute the expected cost per cycle and then multiply by the average number of cycles per year, to approximate the average annual cost. The exact solution would require that we compute the state probabilities first, and then use them to obtain the average annual cost. The resulting equations, however, are complex (Hadley and Whitin, 1963). The exact approach is rarely if ever used in practice and the approach described in Eqs. (1) and (2) is the standard formulation used throughout the literature.

3. Some comments

In light of the above, we can now provide a few insightful, though often overlooked, results for the \((Q,r)\) inventory policy model. As noted above, we assume that LTD follows a distribution of form \( F(z) \), \( z = (x - \mu)/\sigma \), where \((\mu, \sigma)\) denote the mean and standard deviation, respectively. Results 1 and 2 are well known when the LTD is normally distributed but as we now show, they hold quite generally. Result 3 is straightforward analytically, but the implications are important. That is, we arrive at an index involving the distributions of both \( L \) and \( D \), which allows us to identify the preferred vendor. The necessary proofs are presented in Appendix A.

**Result 1.** For a fixed value of the standard deviation \( \sigma \) and independent of changes in the mean \( \mu \) of the LTD, the following hold:

(a) the optimal reorder level \( r^* \) changes linearly with \( \mu \),
(b) the optimal order quantity \( Q^* \) remains fixed,
(c) the optimal cost \( K^* \) remains fixed.

**Corollary to Result 1.** All results from Result 1 also apply for a \((Q, \mu)\) inventory policy with lost sales (i.e., instead of backorders).

**Result 2.** The optimal cost \( K^* \) increases as the standard deviation of the lead-time demand \( \sigma \) increases, regardless of the mean lead time demand, \( \mu \).

**Comment:** The implication of Results 1 and 2 is that, when \( A, B, I \) and \( \delta \) are held fixed, the cost is a function only of \( \sigma \). From Eq. (1b), we know that \( \sigma \) depends upon means and variances of both the lead-time and the demand per unit time. This observation leads to the construction on a simple index, derived from \( \sigma \), as in the following result.

**Result 3.** In a two-vendor problem, the standard deviation in lead-time demand \( \sigma \) would be the same for both vendors if the condition given below is satisfied. Furthermore, if this condition holds we would be indifferent toward the vendor implied lead-time demand risk.

The condition is \( C_1 = C_2 \), where
\[ C_i = E_i [CV]^2 + V_i, \quad i = 1, 2, \] (3)
where \( E_i = E_i(L) \) and \( V_i = \text{Var}(L) \) are the mean lead-time and variance in lead-time quoted by
vendor \(i\), where \(i = 1, 2\) and \(CV = \left[\text{Var}(D)\right]^{1/2}/E(D)\) is the coefficient of variation of the distribution for demand per unit time. We refer to the expression, \(C_i\), as the Comparative Index for vendor \(i\).

**Corollary to Result 3.** If the Comparative Index for vendors 1 is less than that for vendor 2, the expected cost for vendor 1 is lower than that for vendor 2, and vice versa. Likewise, for \(k\) vendors, we simply choose vendor \(j\) such that \(C_j = \min_i[C_i]\). Since \(CV\) is pre-specified, examination of the \(\{C_i\}\) produces a piece-wise linear function defining the preferred vendor for a given range of \(CV\).

The above analysis has exclusively focused on the vendor lead-times. However, the results are equally applicable to other analyses such as evaluation of available transportation alternatives. In this case, lead-time refers to the lead-times offered by the different modes of transportation instead of those quoted by different vendors.

### 3.1. Discussion

The simple assessment of vendor performance based only on lead-time distributions requires knowledge of \(E_i\) and \(V_i\) for each vendor, along with the corresponding lead-time distribution. Further, if the comparisons using means and variances are to be valid, these distributions must have the same general form. In order to use the Comparative Index, the only additional information required is knowledge of the manufacturer’s own demand per unit time distribution, an essential ingredient to efficient inventory management. Therefore, although the information required appears to be greater than that needed for the simpler but erroneous rule, the actual requirements correspond to the standard inputs to \((Q,r)\) models. We now illustrate these findings numerically to show how the choice among vendors may be effected.

### 4. A numerical example

An assembly unit requires a part with an average demand of 100 units per month and a standard deviation of 25 units per month. Each part costs $75 and needs to be outsourced. Thus, vendor selection is of critical importance. The cost of placing an order is $4000. The inventory carrying charge for the automobile unit is 20% per unit, per annum, resulting in an inventory carrying cost of $15. Backorder cost is estimated to be $2000. Before proceeding further it is important to note that:

- The optimal policy is based upon the distribution of total demand. Thus, in the context of choosing among vendors whose lead-time distributions yield the same \(F(z)\), the optimal values of \(Q, S\) and \(K\) will be the same.
- Following from the first point, vendor selection will not depend upon \(A, B\) or \(I\) so that the particular settings of these parameters are not relevant to the vendor selection problem. They are included solely to provide familiar total cost figures for comparison.
- The optimal policy is selected on the basis of cost minimization, as given by Eq. (2). We could analyze the decision by examining the probability of stock-out. Using Results 1 and 2, the decision comes down to a comparison of the standard deviations of the lead-time demand for each vendor. In turn, we arrive back at the Comparative Index. Since this analysis does not provide an explicit choice of \(Q\), we stay with cost minimization, but stress that the same vendor selection decision would be reached.

We suppose that the buyer has to choose from a set of four potential vendors. They are approximately equivalent in all dimensions (e.g., quality, cost, cooperation, assistance). However, they have quoted different lead-time figures as given in Table 1, panel A. From the last two columns, we see that vendor 4 is dominated by vendor 1 and need not be considered further. For the other three, the mean lead-time demand levels are different, but the standard deviations are very similar. It is intuitive to expect that vendor 3 with the lower mean lead-time demand figure would provide the best option. In panel B of Table 1, we present the optimal order quantity \(Q^*\), the reorder point \(r^*\). the
safety stock $S^*$ and the optimal cost $K^*$, assuming the distribution of total demand to be normal. We note that $S^*$ is determined from the minimum cost solution, and not from any pre-specified service level. From these results, we see that the company should be indifferent between the vendors 1 and 2. Further, vendor 3 is only very slightly inferior to both 1 and 2.

It is evident from these figures that slight changes in the distribution of demand per unit time ($D$) could affect the ordering among these three vendors, although vendor 4 is always out of the picture. Simple analysis using Eq. (3) provides the range of values of CV for which each vendor is best; see panel C of Table 1. Further, when the CV is low vendor 2 will be preferred, as shown in Table 1, panel D. Finally, as seen from Table 1, panel E, vendor 3 will be preferred whenever CV is high. In addition to dropping vendor 4, we lose very little by dropping vendor 1 from the list of candidates, particularly as the estimates of the parameters are unlikely to be highly accurate.

The simple rule implied by this example is as follows. Examine CV, the coefficient of variation of the demand per unit time:

- when CV is small, choose a vendor whose lead-time has a small variance;
- when CV is large, choose a vendor whose lead-time has a small mean.

### Table 1
Results for numerical example, showing the comparisons among four vendors

<table>
<thead>
<tr>
<th>Vendor</th>
<th>$E(L)$, in months</th>
<th>Var($L$), in (months)$^2$</th>
<th>$\mu = E(LTD)$</th>
<th>$\sigma = SD(LTD)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Lead-time characteristics for different vendors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.1250</td>
<td>200</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0</td>
<td>400</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.1976</td>
<td>100</td>
<td>51</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.1250</td>
<td>300</td>
<td>56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$Q^*$</th>
<th>$R^*$</th>
<th>$S^*$</th>
<th>$K^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>816</td>
<td>328</td>
<td>128</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>816</td>
<td>528</td>
<td>128</td>
</tr>
<tr>
<td>3</td>
<td>51</td>
<td>816</td>
<td>231</td>
<td>131</td>
</tr>
<tr>
<td>4</td>
<td>56</td>
<td>817</td>
<td>444</td>
<td>144</td>
</tr>
</tbody>
</table>

Panel B: Optimal settings and minimum costs when $SD(D) = 25$

| Panel C: Preferred vendor, as indicated by the value of CV, the coefficient of variation of the demand per unit time, when $E(D) = 100$ and $SD(D) = 25$ |
| \(\sigma\) | 0.25 | 0.268 |
| 1 | 0 | 0.25 |
| 3 | 0.268 | infinity |

| Panel D: Optimal settings and minimum costs when $SD(D) = 10$ |
| \(\sigma\) | 38 | 812 | 298 | 98 | 13,650 |
| 2 | 20 | 807 | 451 | 51 | 12,867 |
| 3 | 46 | 815 | 217 | 117 | 13,975 |
| 4 | 39 | 813 | 401 | 101 | 13,706 |

| Panel E: Optimal settings and minimum costs when $SD(D) = 40$ |
| \(\sigma\) | 67 | 822 | 371 | 171 | 14,890 |
| 2 | 80 | 826 | 605 | 205 | 15,464 |
| 3 | 60 | 819 | 254 | 154 | 14,591 |
| 4 | 78 | 825 | 500 | 200 | 15,368 |
5. Conclusions

The growing strategic importance of outsourcing activities has necessitated the development of new approaches to decision-making in vendor selection. The paper presents a simple decision rule for choosing vendors based on their lead-time specifications. The lead-time effects are studied in the context of standard \((Q, r)\) inventory policy. It is shown that the coefficient of variation of the demand per unit time \((D)\) may be used in conjunction with the vendors’ specifications (as measured by the mean and variance of the lead-time) to produce a Comparative Index, as defined by Eq. (3). Vendors may then be evaluated using this index. In particular, our approach leads to the selection of the vendor who minimizes operating costs. By contrast, selecting the vendor with the smallest mean lead-time may be inefficient.

Appendix A

Proof of Result 1. Defining safety stock, \(S\) as \(S = r - \mu\); then by substituting in Eq. (2) we obtain

\[
K = A\frac{\delta}{Q} + I\left(\frac{Q}{2} + S\right) + B\frac{\delta}{Q}\left[\int_{S/\sigma}^{\infty} uf(u)\,du - SG(S/\sigma)\right]. \tag{A.1}
\]

The function in (A.1) is independent of \(\mu\), implying that the optimal values for \(Q^*, K^*\) and \(S^*\) are independent of \(\mu\). Also, since \(S^* = r^* - \mu\), and \(S^*\) is independent of \(\mu\), it follows that \(r^*\) varies linearly with \(\mu\). \(\square\)

Proof of Corollary to Result 1. Following the analysis of Hadley and Whitin (1963) the only difference between the backorder and lost sales model is in evaluating the safety stock expression. If \(B^*\) represents the cost per unit lost sale, we also include the additional inventory cost due to unrealized sales. After the appropriate changes, the resulting average annual variable cost \((K)\) becomes

\[
K = A\frac{\delta}{Q} + I\left(\frac{Q}{2} + S\right) + \left(I + B\frac{\delta}{Q}\right)\left[\int_{S/\sigma}^{\infty} uf(u)\,du - SG(S/\sigma)\right]. \tag{A.2}
\]

The proof of Result 1 and the other results extends to the new formulation. \(\square\)

Proof of Result 2. Rewriting \(S = z\sigma\) and replacing in (A.1) we obtain

\[
K = A\frac{\delta}{Q} + I\left(\frac{Q}{2} + z\sigma\right) + B\frac{\delta}{\sigma}\int_{z}^{\infty} uf(u)\,du - zG(z). \tag{A.3}
\]

Examination of Eq. (A.3) reveals that the optimal choice of \(z, z^*\) say, does not depend upon \(\sigma\), so that \(z^*\) is a function of \(Q\) and the parameters other than \(\sigma\), which are taken as fixed. Thus, we may rewrite the cost function in the generic form

\[K(Q, z, \sigma) = K^*(Q, \sigma).\]

Given that the last term captures the expected number of backorders it is always non-negative. Therefore, expression (A.3) increases linearly with increases and is independent of \(\mu\). \(\square\)

Proof of Result 3 and Corollary. Let \(E = E(D)\) and \(V = \text{Var}(D)\). Then, from Eq. (1b)

\[
\begin{align*}
\sigma_1 &= \sigma_2 \\
&\iff E_1V + E_2V_1 = E_2V + E_2V_2 \\
&\iff E_1(CV)^2 + V_1 = E_2(CV)^2 + V_2,
\end{align*}
\]

or \(C_1 = C_2\).

The corollary then follows directly. \(\square\)

References