A Novel Design of Unitary Space-Time Constellations

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Abstract—In this paper, we propose a novel method for creating constellations of unitary space-time signals for noncoherent multiple-antenna communication link, where neither the transmitter nor the receiver knows the fading coefficients. By employing a theorem for the Clarke subdifferential of the sum of the $k$ largest singular values of the unitary matrix, we present a numerical optimization procedure for finding good constellations of unitary space-time signals and report the best constellations found by this procedure. These constellations of unitary space-time signal improve the performance significantly upon previously best known constellations.

I. INTRODUCTION

Recent information-theoretical work [1], [2] indicate that the capacity of non-coherent multiple-antenna channels, where neither the transmitter nor the receiver knows the fading coefficients of the channel, increase with the number of transmit and receive antennas, although the increase in capacity is somewhat smaller than that in coherent case. They also showed that the capacity-attaining signals had considerable structure. A class of unitary space-time signals that are well-tailored for non-coherent channels was initiated by Hochwald and Marzetta in [3]. Numerous methods have since been proposed to construct unitary space-time signals that operate in the absence of channel state information [4]–[12]. While some extra structure was imposed on the unitary space-time constellation in [5]–[12], the unitary space-time constellation design in [4] is implemented directly on each element with no structure restriction, all using the chordal distance as the objective function. We note that the chordal distance based design criterion is derived from Chernoff bound on the pairwise probability of error under the assumption either for small singular values or for a high SNR. By using this criterion, it is impossible to get the optimal space-time constellation either for a particular SNR or for a system with more than 2 transmit antennas. To guarantee full diversity, the asymptotic union bound (AUB) was proposed in [13], respectively as optimization criteria under the assumption of a high SNR.

The primary purpose of this paper is to design optimal space-time constellations for noncoherent scheme with an arbitrary number of transmit antennas at any given SNR. By employing a theorem for the Clarke subdifferential of the sum of the $k$ largest singular values of the unitary matrix, we present a numerical optimization procedure for finding good constellations of unitary space-time signals and report the best signal constellation found by this procedure. These constellations improve significantly upon previously best known constellations of unitary space-time signals and can be used as a benchmark to assess the optimality of other signal constellations designed for noncoherent Rayleigh block-fading channels.

The rest of the paper is organized as follows. In Section II, we describe our channel model and some preliminaries in unitary space-time modulation. In Section III, unitary space-time constellation design issues are treated, including both objective function and detailed algorithm. The results based on simulation and related discussion are given in Section IV. Section V contains the conclusions.

The following notation is used throughout the paper: $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^\dagger$ denote conjugate, transpose, and conjugate transpose, respectively. $\text{Re}(x)$ is the real part of complex variate $x$; $\| \cdot \|$ represents the Frobenius norm; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. The multi-variate circularly symmetric, complex Gaussian distribution with mean $\mathbf{m}$ and covariance matrix $\mathbf{P}$ is denoted by $\mathcal{CN}(\mathbf{m}, \mathbf{P})$.

II. SYSTEM MODEL AND UNITARY SPACE-TIME MODULATION

Consider a wireless communication system with $M$ transmit antennas and $N$ receive antennas that operates in a Rayleigh flat-fading environment. We assume each receive antenna respond to each transmit antenna through a statistically independent fading coefficient. We also assume that the fading is quasi-static, i.e., the channel coefficients remain constant for a time period $T$, and change to a new independent realization in the next time period. Using complex baseband representation, the channel can be modelled as [3]:

$$X = \sqrt{\frac{\rho}{M}}SH + W,$$

(1)

where $X$ is the $T \times N$ complex matrix of received signals, $S$ is the $T \times M$ complex matrix of transmitted signals, $H$ is the $M \times N$ complex matrix of Rayleigh fading coefficients, and $W$ is the $T \times N$ complex matrix of additive receiver noise. Note that the $j$th column of $S$ represents the signal transmitted over the $j$th transmit antenna as a function of time. Furthermore, we assume each entry in $H$ and $W$ is i.i.d. complex Gaussian distributed $\mathcal{CN}(0, 1)$. The transmitted symbols are normalized to obey

$$\mathbf{E} \sum_{m=1}^{M} |s_{tm}|^2 = M,$$

(2)

t = 1, 2, \cdots, T
where $s_{tm}$ is the signal sent by the $m$th transmit antenna at time $t$. This means that the average expected power over the $M$ transmitted antennas is kept constant for each channel use. Therefore, $\rho$ represents the expected SNR at each receive antenna.

When the realizations of $H$ are not known to the receiver and transmitter, it is shown in [1] and [3] that a constellation of $L$ unitary space-time signals $S_l = \sqrt{T} \Phi_l, l = 1, \ldots, L$, obeying $\Phi_1 \Phi_1^* = \cdots = \Phi_L \Phi_L^* = I$, can approach the capacity. It is also shown in [3] that the maximum-likelihood (ML) demodulator for this constellation of unitary space-time signals is

$$\hat{\Phi}_{ML} = \arg \max_{\Phi_i=\Phi_1,\ldots,\Phi_L} tr\{X^\dagger \Phi_i \Phi_i \dagger X\}. \tag{3}$$

To design a good constellation of $L$ unitary space-time signals, we need to minimize the probability of error for a general constellation of unitary space-time signals. However, this design criterion was cumbersome to evaluate. As discussed in [3], a simplified performance measures based on Chernoff bound is a tractable and sufficiently accurate approximation to the probability of error. The Chernoff bound of the pairwise probability of error of mistaking $\Phi_i$ for $\Phi_i$, or vice versa for the ML demodulator (3) can be derived as follows [3]

$$P_e(l,l') \leq \prod_{m=1}^{M} \left[ 1 + \frac{(\rho T/M)^2(1-\lambda_m^2)}{4(1+\rho T/M)} \right]^{-N}, \tag{4}$$

where $\lambda_m \eqdef \lambda_m(\Phi_1^\dagger \Phi_l)$ is the $m$th singular value of the $M \times M$ correlation matrix $\Phi_1^\dagger \Phi_l$. For the analysis to come, it will be more convenient to assume $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. Due to lack of a way to minimize this Chernoff bound directly, in many papers, such as [3], [4], [5] etc., the Chernoff bound on the pairwise error probability is further loosened by upper-bounding it. They finally use the chordal distance between any two signals, i.e., $\lambda_1^2 + \lambda_2^2 + \cdots + \lambda_M^2$, as the design criterion. Note that the chordal distance based design criterion is derived from Chernoff bound on the pairwise probability of error under the assumption either for small singular values or for a high SNR. By using this criterion, it is impossible to get the optimal space-time constellation either for a particular SNR or for a system with more than two transmit antennas.

In the next section, we shall focus on the design of optimal constellation of unitary space-time signals with no assumption on the singular values and SNR.

### III. UNITARY CONSTELLATION DESIGN

Motivated by the above observation that the chordal distance is not a good figure of merit for unitary space-time constellation design, particularly, when the number of transmit antenna is larger than two or when we need optimal constellation for a given SNR, we develop a new algorithm which uses directly the Chernoff bound on the pairwise probability of error as an optimality criterion. This makes the design of optimal constellation for a given SNR possible. It also lead to the best known constellations both for system with two transmit antennas and system with three antennas as can be seen in the following section. We start this section by problem statement.

We then give a theorem regarding the derivatives of the sum of the $\kappa$ largest singular values of a matrix with respect to each entry in Section III-A. This is followed by constellation optimization algorithm and implementation details in Section III-B.

**Problem Statement:** Given natural numbers $M$, $T$ and $L$, find a complex matrix set $\Phi_1, \Phi_2, \ldots, \Phi_L$ such that the maximum Chernoff bound of the pairwise probability of error between any two distinct matrices given by

$$P_e^{\text{max}} = \max_{1 \leq l < l' \leq L} \prod_{m=1}^{M} \left[ 1 + \frac{(\rho T/M)^2(1-\lambda_m^2)}{4(1+\rho T/M)} \right]^{-N} \tag{5}$$

is minimized under the constraints $\Phi_i^\dagger \Phi_i = I$ for all $i$.

**A. Derivative of the Sum of the Largest Singular Values**

To find the solution for the above problem, a key step is to compute the partial derivatives of the cost function (5) with respect to the parameters of $\Phi_1$ and $\Phi_i$. Here we will give a theorem regarding the derivative of the sum of the $\kappa$ largest singular values of a matrix with respect to each entry. By applying the chain rule for partial differentiation and the following theorem, we finally get the partial derivatives of the cost function with respect to the parameters of $\Phi_1$ and $\Phi_i$.

We start with some definitions. Let the singular values of $m \times n$ complex matrix $A$ be $\lambda_1(A), \lambda_2(A), \ldots, \lambda_p(A)$, where $p = \min\{m, n\}$ and $\lambda_1(A) \geq \lambda_2(A) \geq \cdots \geq \lambda_p(A) \geq 0$. By the singular value decomposition there exist $m \times m$ matrix $U$ and $n \times n$ matrix $V$ such that

$$U^\dagger AV = \text{diag}(\lambda_1(A), \lambda_2(A), \ldots, \lambda_p(A)). \tag{6}$$

The sum of the $\kappa$ largest singular values is a function mapping $\mathbb{C}^{m \times n}$ to $\mathbb{R}$ defined by

$$f_\kappa(A) = \sum_{i=1}^{\kappa} \lambda_i(A), \tag{7}$$

where $1 \leq \kappa \leq p$. With these definitions, we can state the theorem about the derivative of the sum of the largest singular values of a matrix with respect to each entry as follows.

**Theorem 1:** (Sum Derivative) Let $A$ be a matrix-valued function mapping $\mathbb{R}^a$ to $\mathbb{C}^{m \times n}$. Suppose that the singular values of $A$ are $\lambda_1(A) \geq \cdots \geq \lambda_\kappa(A) > \lambda_{\kappa+1}(A) \geq \cdots \geq \lambda_p(A)$, and let

$$g_\kappa(x) = f_\kappa(A(x)) = \sum_{i=1}^{\kappa} \lambda_i(A(x)). \tag{8}$$

If $A$ is locally Lipschitzian and G-differentiable at $x \in \mathbb{R}^a$, then $g_\kappa(x)$ is locally Lipschitzian and semiregular at $x$ with

$$\partial g_\kappa(x) = \begin{cases} u \in \mathbb{R}^a : u_k = tr(U_k^\dagger A_k(x)V_1) + tr(U_k^\dagger A_k(x)V_2G_2), \end{cases}$$

$$G_2 \in \Psi_{\kappa} = \{ G_2 \in \mathbb{R}^{1 \times t} : 0 \leq G_2 \leq I_t, tr(G_2) = \kappa - r \},$$

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where $U_1$ and $V_1$ consist of the first $r$ columns of $U$ and $V$ respectively, $U_2$ and $V_2$ consist of the next $t$ columns of $U$ and $V$ respectively, and

$$A_k(x) = \frac{\partial}{\partial x_k} A(x).$$

If furthermore $\kappa = r + t$, then $g_\kappa$ is G-differentiable at $x$, and

$$\frac{\partial}{\partial x_k} g_\kappa(x) = tr(U_1^\dagger A_k(x)V_1) + tr(U_2^\dagger A_k(x)V_2).$$

If $A$ is smooth at $x$, then $g_\kappa$ is regular at $x$.

Proof: Due to the limited number of pages, here we shall present only the outline of the proof. The complete version will appear in our forthcoming paper [15].

First we note that in the case for real matrices the theorem has been proved in [16]. The extension to complex case utilizes the standard trick in matrix algebra in which there exists a one-to-one correspondence between $m \times n$ complex matrices with $2m \times 2n$ real matrices. The elements of the complex matrix

$$A = \left[ a_{kl} \right]_{k=1, \ldots, m \atop l=1, \ldots, n} = \left[ \alpha_{kl} + j \beta_{kl} \right]_{k=1, \ldots, m \atop l=1, \ldots, n}$$

are replaced by a $2 \times 2$-matrices

$$\tilde{A}_{kl} = \begin{bmatrix} \alpha_{kl} & -\beta_{kl} \\ \alpha_{kl} & \beta_{kl} \end{bmatrix}, \forall k \in \{1, \ldots, m\}, l \in \{1, \ldots, n\}.$$

The matrices $A \in \mathbb{C}^{m \times n}$ and $\tilde{A} = \left[ \tilde{A}_{kl} \right]_{k=1, \ldots, m \atop l=1, \ldots, n}$ have the same singular values. The singular values of $\tilde{A}$ have double the multiplicity of those of $A$. Furthermore, the singular value decomposition of $A$ transfers to the singular value decomposition of matrix $\tilde{A}$ as follows:

$$\tilde{A} = \tilde{U}^T \tilde{\Sigma} \tilde{V},$$

where $\tilde{U}$ and $\tilde{V}$ are special real block orthogonal matrices which consists of $2 \times 2$-orthogonal matrices, i.e. $\tilde{U} = [\tilde{U}_{kl}]_{k,l}$, where every

$$\tilde{U}_{k,l} = \begin{bmatrix} \omega_{kl} & -\delta_{kl} \\ \delta_{kl} & \omega_{kl} \end{bmatrix}$$

is an orthogonal matrix.

Using the result of Qi and Womersley to the matrix $\tilde{A}$ we obtain that $g_\kappa(x)$ is locally Lipschitz and semiregular at $x$. The subdifferential at the point $x$ is given by

$$\partial g_\kappa(x) = \left\{ u \in \mathbb{R}^s : \ u_k = tr(\tilde{U}_1^T \tilde{A}_k(x) \tilde{V}_1 + tr(\tilde{U}_2^T \tilde{A}_k(x) \tilde{V}_2 G_2), \right\},$$

$G_2 \in \tilde{\Psi}_{2t} = \{ G_2 \in \mathbb{R}^{2t \times 2t} : 0 \leq G_2 \leq I_{2t}, tr(G_2) = \kappa - r \},$ where $\tilde{U}_1$ and $\tilde{V}_1$ contains the first $r$ block columns of the special real orthogonal matrices $\tilde{U}$ and $\tilde{V}_1$, and $\tilde{U}_2$ and $\tilde{V}_2$ contains the next $t$ block columns of $\tilde{U}$ and $\tilde{V}$, respectively. Using the relationship between the special real matrices and the complex matrices we obtain the statement. The regularity and the differentiability of $g_\kappa$ follows from the real case.

### B. Optimization Algorithm and Implementation Details

In order to use the steepest decent algorithm to get the optimal unitary space-time constellation, we need to parameterize the matrices $\Phi_1$ and compute the steepest descent direction of the cost function with respect to the parameters of $\Phi_1$.

Let us consider the parameterization of matrix first. There are several ways to parameterize the matrix $\Phi_1$. As discussed in [4], a more practical way to parameterize $\Phi_1$ is to overparameterize the space, which will simplify the derivatives of the objective function with respect to its parameters. This overparameterization is built by expressing any rectangular matrix $\tilde{\Phi} \in \mathbb{C}^{T \times M}$ as $U(\Theta)\Phi_1$, where $U(\Theta)$ is a $T \times T$ unitary matrix, parameterized by $\Theta$. The square unitary matrix $U(\Theta)$ can, in turn, be expressed as the product of a diagonal matrix $S$ and some basic unitary matrices $U^{pq}(\phi_{pq}, \theta_{pq})$, i.e.,

$$U(\Theta) = S \prod_{p=-T}^{1} \prod_{q=-p+1}^{T} U^{pq}(\phi_{pq}, \theta_{pq}), \quad (8)$$

where $S = \text{diag}(\delta_1, \delta_2, \cdots, \delta_T)$ and matrix $U^{pq}(\phi_{pq}, \theta_{pq})$ for $p < q$ and $\phi_{pq}, \theta_{pq} \in [-\pi, \pi)$ is given by

$$U^{pq}_{j,k}(\phi_{pq}, \theta_{pq}) = \begin{cases} 1, & \text{if } j = k \text{ and } j \neq p, q \\ \cos(\phi_{pq}), & \text{if } j = k \text{ and } j = p, q \\ -\sin(\phi_{pq}) \exp(-i\theta_{pq}), & \text{if } j = p \text{ and } k = q \\ \sin(\phi_{pq}) \exp(i\theta_{pq}), & \text{if } j = q \text{ and } k = p \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Note that the parameters involved in the parameterization of $T \times M$ unitary matrix lie in a compact differential manifold, and $P_c^{\text{max}}$ is a continuous function of its parameters. Therefore, it achieves at least one global minimum. However, we also note that $P_c^{\text{max}}$ may have many local minima that are far away from global minimum. In fact, $P_c^{\text{max}}$ is not very smooth and it is not even differentiable everywhere. These facts prevent us from the direct use of gradient-based search algorithms. Consequently, as in [4] following [14], we choose a family of surrogates $f_\alpha$, parameterized by the positive parameter $\alpha$, for $P_c^{\text{max}}$ to overcome the difficulties posed by the undesirable properties of the objective function.

The strategy is to minimize $f_\alpha$ for small $\alpha$, overcoming the difficulties stated above while mimicking $P_c^{\text{max}}$ less well, and then track this minimum while increase $\alpha$. The function $f_\alpha$ closely track $P_c^{\text{max}}$, converging to $P_c^{\text{max}}$ as $\alpha$ tends to infinity. One example of such a family is [4]

$$f_\alpha(\alpha, P_c^{\text{max}}) = \frac{1}{\alpha} \log \left( \sum_{1 \leq i < j \leq L} \exp(\alpha P_c(l, l')) \right), \quad (10)$$

where $P_c(l, l')$ is the Chernoff bound as defined in (4). In this paper we will use the argument of the log in this expression as cost function for constellation optimization. It can be denoted as

$$f_c(\alpha) = \sum_{1 \leq i < l \leq L} \exp(\alpha P_c(l, l')). \quad (11)$$
Now let us consider the problem of computing the partial
derivatives of the cost function $f_c$ with respect to the the
parameters of $\Phi_t$, while assuming that all other matrices
$\Phi_t, l' \neq l$ are fixed. Using overparameterization method
discussed above we can get this partial derivatives through
perturbing the matrix $\Phi_t$ by premultiplying it with matrix
$U(\Theta)$ for $\Theta \approx 0$, since $U(0) = I$. Specifically, for $\theta \in \Theta$, we have

$$
\frac{\partial}{\partial \theta} f_c(\alpha) = \sum_{l < l' \leq L} \alpha \exp(\alpha P_c(l, l')) \frac{\partial}{\partial \theta} P_c(l, l'),
$$

(12)

where $\partial P_c(l, l')/\partial \theta$ can be expressed by applying the chain rule for partial differentiation as

$$
\frac{\partial}{\partial \theta} P_c(l, l') = \frac{\partial}{\partial \theta} \left\{ \Gamma(m) A_m \cdot \frac{\partial}{\partial \theta} (g_m - g_{m-1}) \cdot \prod_{i=1}^{M} (\Gamma(i))^N \right\}
$$

with

$$
A_m = \frac{2N(\rho T/M)^2 \lambda_m}{4(1 + \rho T/M)}
$$

(13)

and

$$
g_m = \sum_{j=1}^{M} \lambda_j, g_0 = 0.
$$

Here, by applying Theorem 1, we can readily obtain $\partial g_m / \partial \theta$.

With the techniques and algorithms detailed above, we can summarize our procedure of constellation optimization as follows [4]. The search procedure starts with a randomly generated set $S$ and a relatively small value of $\alpha$, say $\alpha_0$. We then use the steepest descent algorithm to iteratively update the set $S$, and finally find a set $S_{\alpha_0}$ such that the value of $f_c(\alpha_0)$ is (nearly) locally minimized. At each iteration, we update, in a fixed order, each matrix in $S$ individually, i.e., we calculate the partial derivatives of the cost function with respect to the parameters of only one matrix and update that matrix suitably. After $S_{\alpha_0}$ is found, we slightly increase $\alpha$ to $\alpha_1$ and start from the set $S_{\alpha_0}$, find a new set $S_{\alpha_1}$, such that the value of $f_c(\alpha_1)$ is (nearly) locally minimized. We continue in this manner, each time increasing the value of $\alpha$ slightly and tracking the minimizer of $f_c(\alpha)$. For very large values of $\alpha$, $f_c(\alpha_0)$ would be essentially equivalent to $P_{c_{\max}}$ and minimizing $f_c(\alpha_0)$ will also essentially minimize $P_{c_{\max}}$.

IV. NUMERICAL RESULTS

In this section we use the algorithm developed in the previous section to generate optimal constellations of unitary
space-time signals and examine their performance. We shall see that our method leads to the best known constellations of
unitary space-time signals. We look specifically at $M = 2$ and
3 transmit antennas and consider $N = 1$ receive antenna. We
choose typical parameters of $T \geq 2M$ and $R = 1$ bit/channel
use. Thus we require a constellation of at least $L = 2^{RT}$
signals, each an $M \times T$ matrix, for $M = 2, 3$ and $T = 4, 5, 6$.

Table I shows the results of our search for constellations of
unitary space-time signals that minimize $P_{c_{\max}}$. For comparison,
we also include the previously best known constellations using
diversity sum and/or diversity product as design criterion
[4], [7]. The diversity sum and diversity product of the unitary
space-time signal set $S$ of size $L$ are, respectively, defined as

$$
\delta = \max_{1 \leq l < l' \leq L} \left| \frac{1}{M} \sum_{m=1}^{M} \lambda_m^2 (A(l, l')) \right|^{1/2M}
$$

and

$$
\zeta = \min_{1 \leq l < l' \leq L} \left| \prod_{m=1}^{M} (1 - \lambda_m^2 (A(l, l'))) \right|^{1/2M}.
$$

It can be clearly seen from Table I that our new constellations
outperform the previously best known constellations with the
the lowest $P_{c_{\max}}$ in all cases. It can also be seen from Table
I that our new unitary signal constellations have both larger
diversity product $\zeta$ and larger diversity sum $\delta$ compared with
the previously best known constellations. This means that the
widely used design criterion trying to minimize the maximum
diversity sum $\delta$ is not optimal. On the contrary, we should try
to maximize the minimum diversity product $\zeta$. Actually, it has
been stated in [7] that the diversity product $\zeta$ is crucial for the
performance of the unitary signal constellations at low-SNR
$\rho$, while the diversity sum $\delta$ is at high-SNR $\rho$. However, they
did not give a solution on how to get constellations of unitary
space-time signals with large diversity product $\zeta$. Here we
give an algorithm to solve this problem and make it possible
to directly optimize the constellation using Chernoff bound.

Fig. 1 shows the average bit error rate for the constellations
of space-time signals generated for $M = 2$ transmit antennas
here as well as the same for the previously best known
constellations for a coherence interval of length $T = 4$ and
rate 1 bits/s/Hz. We see that our new constellations has an
improvement in SNR of about 1 dB over previously best
known constellations at BER $= 10^{-3}$. The same relative
performance found with $M = 2$ transmit antennas also extends
to $M = 3$ transmit antennas. Fig. 2 shows the performance of
signal constellations generated for $M = 3$ transmit antennas
here as well as the same for the previously best known
constellations for a coherence interval of length $T = 6$ and rate
1 bits/s/Hz. We gain about 1.5 dB in performance at BER $= 10^{-4}$
through our new constellation relative to the previously
best known constellations.

V. CONCLUSIONS

We propose a novel method for creating constellations of
unitary space-time signals for non-coherent multiple-antenna
communication link, where neither the transmitter nor the
receiver knows the fading coefficients. By employing a theorem
for the Clarke subdifferential of the sum of the $k$ largest
singular values of the unitary matrix, we present a numerical
optimization procedure for finding good constellations of unitary
space-time signals and report the best signal constellations.
TABLE I
THE COMPARISON OF THE DIVERSITY PRODUCT, DIVERSITY SUM AND CHERNOFF BOUND WITH $\rho = 20$ dB FOR OUR NEW CONSTELLATIONS AND PREVIOUSLY BEST KNOWN CONSTELLATIONS

<table>
<thead>
<tr>
<th>M</th>
<th>T</th>
<th>L</th>
<th>$P_{\text{max}}$ ($\rho = 20$dB)</th>
<th>$\zeta_{\text{max}}$</th>
<th>$\delta$</th>
<th>$P_{\text{max}}$ ($\rho = 20$dB)</th>
<th>$\zeta_{\text{max}}$</th>
<th>$\delta$</th>
</tr>
</thead>
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<td>16</td>
<td>9.92e-4</td>
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<td>0.7071</td>
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<td>0.2485</td>
<td>0.7085</td>
</tr>
</tbody>
</table>

found by this procedure. These constellations of unitary space-time signals improve significantly upon previously best known constellations.

REFERENCES


Fig. 1. Comparison of BER performance of our new constellation and the previously best known constellation for $M=2$, $T=4$, and rate 1 bit/s/Hz.

Fig. 2. Comparison of BER performance of our new constellation and the previously best known constellation for $M=3$, $T=6$, and rate 1 bit/s/Hz.