Non-Line-of-Sight Detection Based on TOA and Signal Strength

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Abstract—This paper addresses the problem of identifying NLOS propagation by applying the statistical decision theory. A time-of-arrival (TOA) based method is developed under idealized conditions to provide a performance reference. In the presence of both TOA and received signal strength (RSS) measurements, a joint identification method is derived to efficiently exploit both the TOA and RSS measurements. Analytical expressions for the probability of detection (POD) and the probability of false alarm (PFA) are derived. Simulation results demonstrate that the proposed methods perform well and the joint TOA and RSS based method outperforms the TOA based methods considerably. It is also shown that the analytical results agree with the simulated ones.

Index Terms—NLOS identification, Neyman-Pearson Theorem, TOA, signal strength.

I. INTRODUCTION

In many circumstances wireless positioning accuracy is often greatly affected by non-line-of-sight (NLOS) radio propagation. To mitigate the NLOS impact, a variety of techniques and algorithms have been proposed in the literature. For instance, the NLOS mitigation positioning algorithms include the filtering based methods [1–7], the constrained optimization techniques [8–11], the error statistics and pattern matching based methods [12–14]. Another way to deal with the NLOS propagation is to identify the NLOS conditions first and then to eliminate the NLOS corrupted measurements [14–20].

In this paper we apply the Neyman-Pearson (NP) Theorem [21] for identifying the NLOS radio propagation. We first develop a time-of-arrival (TOA) based identification method under idealized conditions to provide a performance reference. Then, we derive a NLOS detection method by jointly using both the received signal strength (RSS) and the TOA measurements. Analytical closed form expressions of the POD and PFA are derived for all the considered scenarios. All the methods do not rely on node location information, so the identification can be performed before carrying out the position estimation.

The remainder of the paper is organized as follows. Section II develops the TOA based approach under idealized conditions to generate a performance reference. Section III derives the joint TOA and RSS based NLOS detection method. Section IV shows simulation results to demonstrate the effectiveness of the proposed methods. Finally, Section V concludes this paper.

II. IDENTIFICATION BASED ON TIME-OF-ARRIVAL MEASUREMENTS

In radio based ranging systems, TOA and received signal strength (RSS) are often employed. The TOA method estimates the distance by determining the propagation time by estimating the round-trip-time (RTT) or by using both a radio-frequency signal and an ultra sound signal [12]. A wide range of high resolution TOA estimation techniques [22–26] can be applied to obtain the TOA estimates.

It is assumed that \( N \) TOA based distance measurements are made for given range and NLOS bias. This requires that the variation of the mobile terminal location and the environmental structure is negligible during the \( N \) measurements. Then, the identification problem becomes:

\[
\mathcal{H}_l : \hat{d}_i = d + w_{los,i}, \quad i = 1, \ldots, N, \text{ LOS condition},
\]

\[
\mathcal{H}_n : \hat{d}_i = d + b + w_{nlos,i}, \quad i = 1, \ldots, N, \text{ NLOS condition},
\]

where \( d \) is the true straight line range between the two nodes such as a mobile station and a base station, \( \hat{d}_i \) is the \( i \)-th measurement of \( d \), \( b \) is the extra distance (positive bias) due to the blockage of the direct path, and \( w_{los,i} \) and \( w_{nlos,i} \) are the measurement noise under the LOS and NLOS condition, respectively. The measurement noise, \( w_{los,i} \) and \( w_{nlos,i} \) are modeled as white Gaussian random variables (RVs) with zero mean, and variances equal to \( \sigma^2_{w_{los}} \) and \( \sigma^2_{w_{nlos}} \), respectively. The NLOS bias, \( b \), in both indoor and outdoor environments, is modeled as an exponential RV with a mean \( \lambda \) and a variance \( \lambda^2 \) [1, 14, 27, 28].

In [15, 17] two different identification methods were proposed based on multiple range measurements. The measurement noise statistics in LOS condition are assumed completely known, whereas the error statistics in NLOS condition are not known. In this section we consider the ideal case in which the true distance and bias are known so that the optimal NP test can be applied. Although the Neyman-Pearson (NP) test is not realizable due to the unknown true distance and bias, the performance of the NP test can be used as a bound for performance comparison. This is analogous to the use of the Cramer-Rao lower bound on unbiased estimator variance.

1Propagation time and time of flight have the same meaning. TOA is also equivalent to propagation time when the starting time of the signal transmission is zero.
The sample mean of the \( N \) measurements is defined as
\[
\hat{d} = \frac{1}{N} \sum_{i=1}^{N} \hat{d}_i = \begin{cases} 
    d + w_{\text{los}}, & \text{LOS condition} \\
    d + b + w_{\text{nlos}}, & \text{NLOS condition}
\end{cases}
\] (2)

where
\[
w_{\text{los}} = \frac{1}{N} \sum_{i=1}^{N} w_{\text{los},i},
\]
(3)
\[
w_{\text{nlos}} = \frac{1}{N} \sum_{i=1}^{N} w_{\text{nlos},i},
\]

are the sample means of the measurement noise in LOS and NLOS condition, respectively. Clearly, \( w_{\text{los}} \) and \( w_{\text{nlos}} \) are Gaussian distributed: \( w_{\text{los}} \sim \mathcal{N}(0, \sigma_{w_{\text{los}}}^2/N) \) and \( w_{\text{nlos}} \sim \mathcal{N}(0, \sigma_{w_{\text{nlos}}}^2/N) \).

Given the true distance and bias, the sample mean \( \hat{d} \) is also Gaussian distributed: \( \hat{d} \sim \mathcal{N}(d, \sigma_{w_{\text{los}}}^2/N) \) in LOS condition and \( \hat{d} \sim \mathcal{N}(d + b, \sigma_{w_{\text{nlos}}}^2/N) \) in NLOS condition. The NP detector decides \( H_n \) if
\[
p(\hat{d} | d, b, H_n) = \frac{\sigma_{w_{\text{los}}}}{\sigma_{w_{\text{nlos}}}} \exp \left( \frac{(\hat{d} - d)^2}{2\sigma_{w_{\text{los}}}^2/N} - \frac{(\hat{d} - (d + b))^2}{2\sigma_{w_{\text{nlos}}}^2/N} \right) > \kappa_0,
\] (4)

where \( \kappa_0 \) is the threshold that is dependent of the predefined PFA. The decision rule in (4) is equivalent to deciding \( H_n \) if
\[
\hat{d} > \gamma_0,
\] (5)

where, giving the PFA \( \varepsilon \), the threshold \( \gamma_0 \) is determined by
\[
\varepsilon = \int_{\gamma_0}^{\infty} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{N} \sigma_{w_{\text{los}}}} \exp \left( - \frac{(x - d)^2}{2\sigma_{w_{\text{los}}}^2/N} \right) \, dx
\]
(6)
\[
= Q \left( \frac{\gamma_0 - d}{\sigma_{w_{\text{los}}}/\sqrt{N}} \right).
\]

Accordingly, we can compute the theoretical probability of detection by
\[
P_D = \int_{\gamma_0}^{\infty} p(x | d, b, H_n) \, dx
\]
(7)
\[
= \int_{\gamma_0}^{\infty} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{N} \sigma_{w_{\text{los}}}} \exp \left( - \frac{(x - (d + b))^2}{2\sigma_{w_{\text{nlos}}}^2/N} \right) \, dx
\]
\[
= Q \left( \frac{\gamma_0 - (d + b)}{\sigma_{w_{\text{nlos}}}/\sqrt{N}} \right).
\]

In the simulation we will compare the performance between the TOA based approaches and the joint TOA and RSS method which will be discussed in the following section.

The main contribution of this section is the developed optimal NP test by using the TOA based distance measurements, which provides a reference for performance comparisons among the different methods.

### III. Identification Based on TOA and RSS Measurements

This section describes a joint TOA and RSS based identification approach by employing the Neyman-Pearson Theorem. The closed form expressions for the probability of detection and the probability of false alarm are derived.

The RSS method uses an empirically developed path loss model to determine the propagation distance between the transmitter and receiver [29, 30]. There are a number of well known path loss models for describing the radio signal propagation in different scenarios. The Walfisch-Ikegami path loss model is suitable for medium city and suburban areas, and metropolitan centers [31], whereas the log-distance model is suited to indoor environments. When using a path loss model for determining the propagation distance, it is crucial to tune the model parameters well so that there is a good match between the model and the field measurements. Due to multipath fading, the path loss computation is based on the mean received signal power of multiple measurements and the known transmitted signal power. Here, we exploit the Walfisch-Ikegami model for study [29, 31]. In this model, the path loss is computed according to
\[
L_p = \begin{cases} 
    A_{\text{los}} + 26 \log_{10} d, & \text{LOS condition} \\
    A_{\text{nlos}} + 38 \log_{10} d, & \text{NLOS condition}
\end{cases}
\]
(8)

where \( A_{\text{los}} \) and \( A_{\text{nlos}} \) are parameters that are dependent on the signal carrier frequency, transmitter and receiver antenna heights, structure of buildings and roads, and street orientation relative to the direct radio path, and \( d \) is the LOS distance between the transmitter and receiver. In the presence of measurement noise and modeling error, the path loss can be expressed as
\[
\hat{L}_p = \begin{cases} 
    A_{\text{los}} + 26 \log_{10} \hat{d} + v_{\text{los}}, & \text{LOS condition} \\
    A_{\text{nlos}} + 38 \log_{10} \hat{d} + v_{\text{nlos}}, & \text{NLOS condition}
\end{cases}
\]
(9)

where \( v_{\text{los}} \) and \( v_{\text{nlos}} \) are the path loss model errors.

Our objective is to identify the NLOS condition by jointly using the TOA based distance estimates and the path loss measurements. The question is how to effectively combine both the measurements/estimates. Our approach is described as follows. Let
\[
\log_{10}(\hat{d} - w_{\text{los}}) = \log_{10} \hat{d} + \rho_{\ell},
\]
\[
\log_{10}(\hat{d} - (b + w_{\text{nlos}})) = \log_{10} \hat{d} + \rho_{n},
\]

where \( \hat{d} \) is the sample mean of the \( N \) TOA based distance estimates and \( w_{\text{los}} \) and \( w_{\text{nlos}} \) are the corresponding sample means of the measurement noise in LOS condition and NLOS condition, respectively. It can be shown that under the assumption of \( w_{\text{los}} \ll \hat{d} \) and \( (w_{\text{nlos}} + b) \ll \hat{d} \), \( \rho_{\ell} \) and \( \rho_{n} \) can be approximated as
\[
\rho_{\ell} \approx \frac{w_{\text{los}}}{2.3\hat{d}},
\]
\[
\rho_{n} \approx \frac{b + w_{\text{nlos}}}{2.3\hat{d}}.
\]
Then, replacing the true distance in (9) by both the distance estimate and distance error in (1), and using the approximations in (10), the joint TOA and RSS based detection problem becomes

\[ H_L : \hat{L}_p = A_{los} + 26 \log_{10} d + v_{los} = A'_{los} + v'_{los}, \]
\[ H_n : \hat{L}_p = A_{nlos} + 38 \log_{10} d + v_{nlos} = A'_{nlos} + v'_{nlos}, \]

(12)

where

\[ A'_{los} = A_{los} + 26 \log_{10} \hat{d}, \]
\[ A'_{nlos} = A_{nlos} + 38 \log_{10} \hat{d}, \]
\[ v'_{los} = v_{los} - \frac{26}{2.3d} w_{los}, \]
\[ v'_{nlos} = \left( v_{nlos} - \frac{38}{2.3d} w_{nlos} \right) - \frac{38}{2.3d} b. \]

Let \( v_{los} \) and \( v_{nlos} \) be Gaussian RVs with means \( \bar{v}_{los} \) and \( \bar{v}_{nlos} \), and variances \( \sigma^2_{v_{los}} \) and \( \sigma^2_{v_{nlos}} \), respectively. Also let \( w_{los} \) and \( w_{nlos} \) be Gaussian random variables with zero means, and variances \( \sigma^2_{w_{los}} \) and \( \sigma^2_{w_{nlos}} \), respectively, and \( b \) is an exponential random variable with mean \( \lambda \) and variance \( \lambda^2 \). Assuming that the five RVs are mutually independent and giving \( \hat{d} \), it is seen that \( v'_{los} \) is a Gaussian RV with mean \( \bar{v}_{los} \) and variance:

\[ \sigma^2_{v'_{los}} = \sigma^2_{v_{los}} + \left( \frac{26}{2.3d} \right)^2 \sigma^2_{w_{los}}. \]

(14)

Let

\[ u = \left( v_{nlos} - \frac{38}{2.3d} w_{nlos} \right), \quad s = \frac{38}{2.3d} b. \]

(15)

Apparently, \( u \) is a Gaussian RV with mean \( \bar{u} = \bar{v}_{nlos} \) and variance

\[ \sigma^2_u = \sigma^2_{v_{nlos}} + \left( \frac{38}{2.3d} \right)^2 \sigma^2_{w_{nlos}}, \]

(16)

and \( s \) is an exponential RV with mean given by

\[ \lambda_s = \frac{38}{2.3d} \lambda. \]

(17)

Based on the fact that the PDF of the sum of two RVs equals the convolution of each of their distributions, the PDF of \( v'_{nlos} \) is determined as follows.

\[ p_{v'_{nlos}}(v'_{nlos}) = \int_{-\infty}^{\infty} p_u(u) p_s(u - v'_{nlos}) du = \int_{v'_{nlos}}^{\infty} \frac{1}{\sqrt{2\pi} \sigma_u} \exp \left( -\frac{(u - \bar{u})^2}{2\sigma_u^2} \right) \times \frac{1}{\lambda_s} \exp \left( -\frac{u - v'_{nlos}}{\lambda_s} \right) du = \frac{1}{\lambda_s} \exp \left( \frac{v'_{nlos}}{\lambda_s} + \beta_1 \right) \times \frac{1}{\sigma_u} \exp \left( \frac{v'_{nlos}}{\sigma_u} + \beta_2 \right). \]

(18)

In some reception conditions some of the RVs may be correlated. The correlations, if known, may be exploited to enhance the NLOS identification accuracy. It would be interesting to investigate this further in the future.

where

\[ \beta_1 = \frac{\sigma_u^2 - \bar{u}}{2\lambda_s^2}, \quad \beta_2 = \frac{\sigma_u}{\lambda_s} - \frac{\bar{u}}{\sigma_u}. \]

(19)

Therefore, the PDFs under \( H_L \) and \( H_n \) are

\[ p(L_p|\hat{d}, H_L) = \frac{1}{\sqrt{2\pi} \sigma_{v'_{los}}} \exp \left( -\frac{(\hat{L}_p - (A'_{los} + \bar{v}_{los}))^2}{2\sigma^2_{v'_{los}}} \right), \]
\[ p(L_p|\hat{d}, H_n) = \frac{1}{\lambda_s} \exp \left( \frac{\hat{L}_p - A'_{nlos} + \bar{u}}{\lambda_s} \right) \times Q \left( \frac{\hat{L}_p - A'_{nlos} + \bar{u}}{\sigma_u} + \beta_2 \right). \]

(20)

According to the NP test, we decide \( H_n \), i.e. the NLOS condition, when the likelihood ratio satisfies

\[ \frac{p(L_p|\hat{d}, H_n)}{p(L_p|\hat{d}, H_L)} > \kappa, \]

(21)

where \( \kappa \) is the threshold that depends on the pre-assigned PFA. If (21) is not satisfied, we decide \( H_L \) is true. It is seen that the inequality in (21) is equivalent to

\[ \hat{L}_p > \gamma, \]

(22)

where \( \gamma \), when assigning a small value (\( \varepsilon \)) to the PFA, is determined by solving

\[ \int_{\gamma}^{\infty} p(L_p|\hat{d}, H_L) d\hat{L}_p = Q \left( \frac{\gamma - A'_{los} + \bar{v}_{los}}{\sigma_{v'_{los}}} \right) = \varepsilon. \]

(23)

When \( \gamma \) is given, we can determine the POD as

\[ P_D = \int_{-\infty}^{\infty} p(L_p|\hat{d}, H_n) d\hat{L}_p = \int_{\gamma}^{\infty} \frac{1}{\lambda_s} \exp \left( \frac{\hat{L}_p - A'_{nlos} + \bar{u}}{\lambda_s} \right) \times Q \left( \frac{\hat{L}_p - A'_{nlos} + \bar{u}}{\sigma_u} \right) d\hat{L}_p = \frac{1}{\lambda_s} e^{\beta_1} \int_{\gamma - A'_{nlos}}^{\infty} \exp \left( \frac{x}{\lambda_s} \right) Q \left( \frac{x + \beta_2}{\lambda_s} \right) dx = Q \left( \frac{\gamma - A'_{nlos} - \bar{u}}{\sigma_u} \right) - \exp \left( \frac{\gamma - A'_{nlos} - \bar{u}}{\sigma_u} + \frac{\sigma_u^2}{2\lambda_s^2} \right) \times Q \left( \frac{\gamma - A'_{nlos} - \bar{u}}{\sigma_u} + \frac{\sigma_u}{\lambda_s} \right), \]

(24)

where the details of deriving the last equation in (24) can be found in Appendix.

In reality there always exist some model parameter errors especially in NLOS conditions. In the following section, the impact of the model errors on the performance of the proposed method will be evaluated through simulation.
IV. Simulation Results

In this section, we evaluate the performance of the NLOS identification methods described in the previous sections through simulations. Also we examine the accuracy of the derived analytical results.

Let us first examine the TOA based identification and the setup is as follows. The distance measurement noise is a Gaussian RV of mean zero and a STD that equals 4% and 1.3 × 4% of the true distance in LOS and NLOS condition, respectively. The NLOS bias is an exponential RV with a mean that equals 8% of the true distance. Ten thousand different distance samples (from 20 to 1000 meters) are examined for each simulation run and the performance is then averaged. Fig. 1 shows the POD versus the PFA of the TOA based methods when four distance samples are used for each decision making. The three curves are produced by using the methods in [15], [17], and the one described in Section II. Clearly, there is a relatively large gap between the performance of the idealized case and that of the methods in [15, 17]. The performance of the idealized case will be compared with that of the joint TOA and RSS based method.

In the joint TOA and RSS based approach, the path loss model parameter \( A_{los} \) is set at 30.2 at frequency 1.9 GHz and \( A_{nlos} \) is set at 31.0 based on some typical building and road parameters. In practice, the parameters should be chosen in accordance with the radio parameters and the realistic environment either indoor or outdoor, so that the RSS based method could be best exploited. This usually requires making field measurements so that the parameters can be finely tuned to precisely match the environmental conditions. Otherwise, inaccurate model parameters would considerably degrade the NLOS identification accuracy. The other parameters are the same as in the TOA based method. Fig. 2 shows the POD versus the PFA under three different STD values (6, 9, and 12 dB) of the path loss error in LOS conditions. The STD of the path loss error in NLOS conditions is set at 30% larger than that in LOS conditions. Each decision is made based on one TOA based distance measurement and one path loss sample. For comparisons the results of the TOA based method in idealized conditions and with \( N = 4 \) samples are also plotted. Both the analytical (denoted by lines) and simulated (denoted by symbols) results are presented. The analytical results are computed using the last equation in (24), while the simulated results are obtained based on (22). To achieve a POD of 90%, the PFA needs to be set at about 0.5, 5, and 17.5% under the three STD values, respectively. Clearly, the method performs well and it outperforms the TOA based methods considerably even in presence of relatively large path loss measurement errors. The analytical results are also in accordance with the simulated ones.

V. Conclusions

In this paper we developed NLOS identification methods based on Neyman-Pearson Theorem. A joint TOA and RSS based NLOS detection method was derived to efficiently make use of both the TOA and RSS measurements that are typically available in wireless communication networks. In addition, the idealized TOA based Neyman-Pearson test was also developed to provide a performance reference. The analytical expressions for the probability of detection and the probability of false alarm were derived for all the scenarios considered. Simulation results demonstrated that the TOA and RSS based method perform well and outperforms the TOA based methods considerably. Good agreement between the derived theoretical results the simulated ones is achieved.

APPENDIX

For notational convenience, let us deal with the integral given by:

\[
I = \int_{\gamma} e^{ax} Q(bx + c) dx,
\]  

(25)
which can be determined as follows.

\[
\mathcal{I} = \int_{\gamma}^\infty Q(bx + c) \, d\left(\frac{1}{a}e^{ax}\right) = \frac{1}{a} e^{ax} Q(bx + c) \bigg|_{\gamma}^{\infty} - \frac{1}{a} \int_{\gamma}^{\infty} e^{ax} \, d(Q(bx + c)).
\]

Making use of the formulae of differentiation under the integral sign:

\[
\frac{d}{dx} F(x) = f(x, g_a(x)) \frac{dg_a(x)}{dx} f(x, g_t(x)) \frac{dg_t(x)}{dx} + \int_{g_a(x)} g_t(x) \frac{\partial}{\partial x} f(x, t) \, dt,
\]

where

\[
F(x) = \int_{g_a(x)}^{g_t(x)} f(x, t) \, dt,
\]

we obtain

\[
d(Q(bx + c)) = -\frac{b}{\sqrt{2\pi}} \exp\left(-\frac{(bx + c)^2}{2}\right) \, dx,
\]

and then

\[
\mathcal{I} = -\frac{1}{a} e^{ax} Q(b \gamma + c) + \frac{b}{a \sqrt{2\pi}} \int_{\gamma}^{\infty} \exp\left(-\frac{(bx + c)^2}{2} + ax\right) \, dx
\]

\[
= \frac{1}{a} \left\{ -e^{ax} Q(b \gamma + c) + \exp\left(-\frac{a}{b} \left(c - \frac{a}{2b}\right)\right) Q\left(b \gamma + c - \frac{a}{b}\right)\right\}.
\]

Let \(a = 1/\lambda_s\), \(b = 1/\sigma_u\), and \(c = \beta_2\), we can readily obtain the fourth equation in (24).

**REFERENCES**


