Improving Anchor Position Accuracy for 3-D Localization in Wireless Sensor Networks

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Abstract—Accuracy of ordinary sensor node localization in wireless sensor networks mainly depends on the signal parameter such as time-of-arrival and signal strength estimation errors and the accuracy of the anchor node locations. In this paper a low-complexity but efficient algorithm is derived to improve anchor location accuracy in the presence of both anchor-to-anchor distance and AOA estimates and GPS measurements. Also, a Lenvenberg-Marquardt (LM) optimization based algorithm is developed for accuracy improvement when anchor-to-anchor distance estimates and GPS measurements are provided. Further, we derive the Cramer-Rao lower bound (CRLB) to benchmark the anchor position accuracy. To our knowledge, improving anchor node location accuracy and deriving the CRLB in the presence of both GPS and anchor-to-anchor measurements in 3-D scenarios are not reported in the literature. Simulation results demonstrate that the proposed approaches can improve the anchor position accuracy substantially and that the accuracy of the two developed algorithms approaches the corresponding CRLB.

Index Terms—Anchor location accuracy improvement, least squares estimator, Lenvenberg-Marquardt method, 3-D localization, Cramer-Rao lower bound.

I. INTRODUCTION

Localization in wireless sensor networks has drawn significant attention in the recent years. Different localization schemes and algorithms have been proposed for different scenarios [1–8] (and references therein). To obtain global position information, regardless of the algorithm employed, some nodes (usually a small percentage) are equipped with a GPS (global positioning system) receiver. These nodes are often called anchor nodes, whereas the other nodes are ordinary sensor nodes. The global position of ordinary sensor nodes may be estimated by directly using anchor GPS measurements and a multilateration method under a global coordinate system. Alternatively, sensor position is first estimated under a local coordinate system. Then, sensor global position is produced by transforming the local system into a global system based on the anchor global position information.

Some advanced GPS receivers such as the differential GPS can achieve accuracy ranging from a few meters down to a few decimeters [9, 10]. On the other hand, some low cost commercial GPS receivers may only achieve accuracy that might be as poor as 20 meters [11, 12]. In addition, when a non-line-of-sight (NLOS) signal propagation exists between the satellite and the GPS receiver due to the adverse environments such as in urban canyons, under heavy foliage, or even indoors, the GPS measurement accuracy may degrade significantly. Obviously, the erroneous anchor locations would further decrease ordinary sensor node location accuracy.

In the event that the accuracy of low cost GPS receivers is not satisfactory, is it possible to obtain better accuracy by making use of anchor-to-anchor signal measurements? With the ultra wideband technology and advance signal processing the accuracy of ranging can be sub-meters [13–18]. Also, very accurate angle-of-arrival (AOA) estimates can be obtained by using the advanced signal processing techniques [19–21]. The purpose of the work is to make use of the accurate anchor-to-anchor parameter estimates to enhance the anchor location accuracy. This not only improves anchor position accuracy, but also enhances the ordinary sensor location performance throughout the network.

In this paper we focus on scenarios in which GPS measurement accuracy is not satisfactory due to cost/complexity constraints and more accurate anchor-to-anchor distance and angle-of-arrival (AOA) estimates can be produced. Then, by using both anchor-to-anchor parameter estimates and the GPS measurements, the anchor position accuracy would be improved. A low complexity linear least squares (LS) estimation algorithm is derived when both local distance and AOA estimates are available. In the absence of angle estimates, we propose to exploit the nonlinear optimization method to achieve accuracy improvement. We also derive the Cramer-Rao lower bounds (CRLBs) to benchmark the performance of the two proposed methods. In the literature, anchor locations in wireless sensor networks are usually assumed error free and how to improve anchor location accuracy has not been well studied, to our knowledge.

The remainder of the paper is organized as follows. Section II briefly describes the system and signal model. Section III derives the low complexity LS algorithm in the presence of both distance and AOA estimates, whereas section IV develops an optimization based algorithm in the absence of AOA estimates. Section V derives the Cramer-Rao lower bound (CRLB) to provide an accuracy reference for the developed algorithms. Section VI presents some simulation results to show the effectiveness of the developed methods. Finally, Section VII concludes the paper.

II. SIGNAL MODEL

Consider a sensor network in which there are \( N \) anchors, each of which is equipped with a GPS receiver, and a certain
number of ordinary sensors. Each anchor is able to communicate with some other anchors within radio range to obtain the distance estimates, for example, by measuring the round trip time (RTT). Also, each anchor is able to measure the angle-of-arrival (AOA) (both azimuth and elevation angles) by using a directional antenna or an antenna array. In this paper we do not deal with the time and angle parameter estimation. Instead, we make use of the parameter estimates for anchor position accuracy improvement. As shown in Fig. 1, we define

- \((x_i, y_i, z_i)\) : global position coordinates of anchor \(i\),
- \(d_{ij}\) : line-of-sight distance between anchors \(i\) and \(j\),
- \(\phi_{ij}\) : azimuth angle of the signal received at anchor \(i\) and transmitted from anchor \(j\),
- \(\alpha_{ij}\) : elevation angle of the signal received at anchor \(i\) and transmitted from anchor \(j\).

In the presence of estimation error, the distance and angle estimates are respectively given as:

\[
\hat{d}_{ij} = d_{ij} + n_{d_{ij}}, \quad 1 \leq i < N, \quad i + 1 \leq j \leq N,
\]

\[
\hat{\phi}_{ij} = \phi_{ij} + n_{\phi_{ij}}, \quad \hat{\alpha}_{ij} = \alpha_{ij} + n_{\alpha_{ij}},
\]

where \(n_{d_{ij}}\) is the distance estimation error, \(n_{\phi_{ij}}\) is the azimuth angular estimation error, \(n_{\alpha_{ij}}\) is the elevation angular estimation error, and

\[
d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2},
\]

\[
\phi_{ij} = \tan^{-1}\left(\frac{y_i - y_j}{x_i - x_j}\right), \quad \alpha_{ij} = \cos^{-1}\left(\frac{z_i - z_j}{d_{ij}}\right).
\]

(1)

Also, the GPS coordinate estimates are

\[
\hat{x}_i = x_i + n_{\hat{x}_i}, \quad \hat{y}_i = y_i + n_{\hat{y}_i}, \quad \hat{z}_i = z_i + n_{\hat{z}_i},
\]

(2)

\[
\hat{x}_i = x_i + n_{\hat{x}_i}, \quad \hat{y}_i = y_i + n_{\hat{y}_i}, \quad \hat{z}_i = z_i + n_{\hat{z}_i},
\]

(3)

where \(n_{\hat{x}_i}, n_{\hat{y}_i}, \) and \(n_{\hat{z}_i}\) are the GPS coordinate estimation errors.

The estimation errors primarily come from two sources: channel multipath fading and receiver noise. In particular, multipath fading is often the dominant error source. It increases the estimation error and produces a bias in the presence of NLOS propagation as well. In this paper we restrict our attention to the line-of-sight (LOS) propagation. A number of NLOS mitigation techniques can be found in [22–24] and references therein. The measurement error under LOS conditions is modeled as a zero mean Gaussian random variable with variance dependent on the received signal-to-noise ratio (SNR). The main factors affecting the SNR are the range and transmitted power.

As shown in Fig. 2, the aim is to re-estimate the anchor global positions by making use of local range and angle estimates in (1), and the GPS coordinate estimates in (3) as well. Once the refined anchor position estimates are available, they are employed to determine the ordinary sensor global positions\(^1\).

\(^1\)Localization of ordinary sensors is beyond the scope of this paper.

III. ACCURACY IMPROVEMENT BASED ON LOCAL RANGE AND ANGLE ESTIMATES

From (1) and Fig. 1, we can readily obtain the observation equations related to local distance and angle estimates as:

\[
x_i - x_j \approx \hat{d}_{ij} \sin \hat{\alpha}_{ij} \cos \hat{\phi}_{ij},
\]

\[
y_i - y_j \approx \hat{d}_{ij} \sin \hat{\alpha}_{ij} \sin \hat{\phi}_{ij}, \quad z_i - z_j \approx \hat{d}_{ij} \cos \hat{\alpha}_{ij},
\]

(4)

where the approximations result from dropping the estimation errors. Since (3) and (4) are linear in terms of the anchor coordinates, we can write them in a compact form as:

\[
A \mathbf{p} \approx \mathbf{r},
\]

where \(\mathbf{p}\) is the position vector defined as:

\[
\mathbf{p} = [x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_N, z_1, z_2, \ldots, z_N]^T,
\]

(5)

\[
A = \begin{bmatrix}
C^T & 0 & 0 & \mathbf{e} & 0 & 0 \\
0 & C^T & 0 & 0 & \mathbf{e} & 0 \\
0 & 0 & C^T & 0 & 0 & \mathbf{e}
\end{bmatrix},
\]

\[
\mathbf{r} = [\mathbf{r}_1^T \mathbf{r}_2^T \mathbf{r}_3^T \hat{\mathbf{x}}^T \hat{\mathbf{y}}^T \hat{\mathbf{z}}^T]^T \in \mathbb{R}^{3(N(N-1)/2+3N) \times 3N},
\]

(6)

\[
\mathbf{A} = \begin{bmatrix}
C^T & 0 & 0 & \mathbf{e} & 0 & 0 \\
0 & C^T & 0 & 0 & \mathbf{e} & 0 \\
0 & 0 & C^T & 0 & 0 & \mathbf{e}
\end{bmatrix} \in \mathbb{R}^{3(N(N-1)/2+3N) \times 3N},
\]

\[
\mathbf{r} = [\mathbf{r}_1^T \mathbf{r}_2^T \mathbf{r}_3^T \hat{\mathbf{x}}^T \hat{\mathbf{y}}^T \hat{\mathbf{z}}^T]^T \in \mathbb{R}^{3(N(N-1)/2+3N) \times 1},
\]

(7)

\[
\mathbf{e} \text{ is an identity matrix of dimensions of } N \times N,
\]

\[
\hat{\mathbf{x}} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N]^T, \quad \hat{\mathbf{y}} = [\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_N]^T, \quad \hat{\mathbf{z}} = [\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_N]^T,
\]

(8)
and

\[
C = \begin{bmatrix}
1 & -1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & -1 \\
0 & 1 & -1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots \\
0 & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & -1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & 1 & -1 \\
\end{bmatrix} \in \mathcal{R}^{(N(N-1)/2) \times N},
\]

\[r_1 = [\hat{d}_{12} \sin \hat{\alpha}_{12} \cos \hat{\phi}_{12}, \ldots, \hat{d}_{1N} \sin \hat{\alpha}_{1N} \cos \hat{\phi}_{1N}, \ldots, \hat{d}_{N-1N} \sin \hat{\alpha}_{N-1N} \cos \hat{\phi}_{N-1N}] \in \mathcal{R}^{(N(N-1)/2) \times 1},
\]

\[r_2 = [\hat{d}_{12} \sin \hat{\alpha}_{12} \sin \hat{\phi}_{12}, \ldots, \hat{d}_{1N} \sin \hat{\alpha}_{1N} \sin \hat{\phi}_{1N}, \ldots, \hat{d}_{N-1N} \sin \hat{\alpha}_{N-1N} \sin \hat{\phi}_{N-1N}] \in \mathcal{R}^{(N(N-1)/2) \times 1},
\]

\[r_3 = [\hat{d}_{2N} \sin \hat{\alpha}_{2N}, \ldots, \hat{d}_{N-1N} \sin \hat{\alpha}_{N-1N} \sin \hat{\phi}_{N-1N}] \in \mathcal{R}^{(N(N-1)/2) \times 1},
\]

where it is assumed that each anchor is within radio range of other anchors. In the event that a pair of anchors are out of radio range, the corresponding components in the matrices and vectors are removed. Then, applying the weighted least squares (WLS) estimator, we obtain the refined global position estimates of all anchors as

\[\hat{p} = (A^T W A)^{-1} A^T W r,
\]

where we use double hats to distinguish it from the original GPS coordinate measurements, and \( W \) is the weighting matrix to emphasize the more reliable estimates [25]. For example, the received SNR may be used in choosing the weights. The matrix inverse is assumed to exist and this is usually true for an over-determined system. It would be feasible to run this algorithm at any anchor due to its low complexity, although an anchor with the most computational power might be chosen to run the algorithm. In some circumstances the anchor-to-anchor measurements may be corrupted by non-line-of-sight (NLOS) radio propagation. Investigation of the NLOS effect is beyond the scope of the paper.

IV. ACCURACY IMPROVEMENT BASED ON LOCAL RANGE ESTIMATES

It is often the case that the anchors may not have an antenna array or a directional antenna, so that angle estimates are not available. In this case, the specific algorithm described in Section III is not appropriate. Instead, we propose an iterative optimization method to improve the accuracy of anchor locations [26].

Making use of all the local distance estimates, the cost function is defined as

\[
\epsilon(p) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} w_{ij} (\hat{d}_{ij} - \hat{d}_{ij})^2,
\]

where \( w_{ij} \) are the weights. The anchor global location information is refined by minimizing the above cost function with respect to the anchor coordinates. That is,

\[\hat{p} = \arg \min_p \epsilon(p).
\]

The GPS measurements are used as the initial position coordinate estimates.

A range of optimization methods can be considered and a number of examples of using optimization in positioning can be found in [17, 23, 27, 28]. In the simulation we will use the Levenberg-Marquardt (LM) algorithm [29]. Since the corresponding computational complexity of running an optimization algorithm would be higher than that of the linear LS algorithm described in section III, the algorithm may be run at an anchor that has the most computational power.

V. CRAMER-RAO LOWER BOUND

For clarity, let us redefine the observation vector as

\[\mathbf{r} = [\hat{d}^T, \hat{\phi}^T, \hat{\alpha}^T, \hat{\mathbf{p}}^T]^T \in \mathcal{R}^{(3N(N-1)/2+3N)x1},
\]

where

\[\hat{d} = [\hat{d}_{12}, \ldots, \hat{d}_{1N}, \ldots, \hat{d}_{N-1N}]^T \]

\[\hat{\phi} = [\hat{\phi}_{12}, \ldots, \hat{\phi}_{1N}, \ldots, \hat{\phi}_{N-1N}]^T \]

\[\hat{\alpha} = [\hat{\alpha}_{12}, \ldots, \hat{\alpha}_{1N}, \ldots, \hat{\alpha}_{N-1N}]^T \]

\[\hat{\mathbf{p}} = [\hat{x}^T, \hat{y}^T, \hat{z}^T]^T \in \mathcal{R}^{3Nx1}.
\]

Assume that

- the distance estimation error, \( n_{\hat{d}_{ij}} \), the angle estimation errors, \( n_{\hat{\alpha}_{ij}} \) and \( n_{\hat{\phi}_{ij}} \), are Gaussian distributed with zero mean and variances equal to \( \sigma^2_{\hat{d}_{ij}}, \sigma^2_{\hat{\phi}_{ij}}, \) and \( \sigma^2_{\hat{\alpha}_{ij}}, \) respectively,
- the GPS measurement errors, \( n_{\hat{x}}, n_{\hat{y}}, \) and \( n_{\hat{z}} \) are also Gaussian random variables of zero mean and variances equaling \( \sigma^2_x, \sigma^2_y, \) and \( \sigma^2_z, \) respectively,
- all the GPS measurement errors, distance and angle estimation errors are mutually independent.

It can be shown that after ignoring the irrelevant constants, the log likelihood can be derived as

\[
\Lambda(\mathbf{r}|\mathbf{p}) = \ln p(\mathbf{r}|\mathbf{p}) = -N^{-1} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[ \frac{1}{2 \sigma^2_{\hat{d}_{ij}}} (\hat{d}_{ij} - d_{ij})^2 + \frac{1}{2 \sigma^2_{\hat{\phi}_{ij}}} (\hat{\phi}_{ij} - \tan^{-1} \frac{y_j - y_i}{x_j - x_i})^2 \right] + \frac{1}{2 \sigma^2_{\hat{\alpha}_{ij}}} (\hat{\alpha}_{ij} - \cos^{-1} \frac{z_i - z_j}{d_{ij}})^2 - \sum_{k=1}^{N} \left[ \frac{1}{2 \sigma^2_{\hat{x}_k}} (\hat{x}_k - x_k)^2 + \frac{1}{2 \sigma^2_{\hat{y}_k}} (\hat{y}_k - y_k)^2 + \frac{1}{2 \sigma^2_{\hat{z}_k}} (\hat{z}_k - z_k)^2 \right].
\]
The Cramer-Rao lower bound (CRLB) sets a lower bound on the variance of any unbiased estimator and has been widely used as a performance measure in parameter estimation [25]. The vector parameter CRLB places a bound on the variance of any unbiased estimator and has been widely used. The CRLB results in the CRLB in the absence of the angle estimates when the STD of the AOA estimation error is 2 degrees, and there are either 6 or 10 anchors. The STD of the GPS measurement error is also set at 10 meters. Clearly, the accuracy improves as the number of anchors increases, especially for large GPS measurement errors. This is due to the fact that adding one anchor incurs three more unknown position variables; however, 3N more local estimates and three more global estimates are produced. Generally speaking, more independent estimates could produce a redundancy/diversity gain. We can see that even at five anchors and in the absence of angle estimates, the accuracy improvement is substantial. The accuracy of both the LM algorithm and the LS algorithm approaches the CRLB.

Fig. 3 shows the estimation accuracy with respect to the number of anchors. Results with two different STDs (4 and 10 meters) of the original GPS coordinate measurements are presented. The STDs of the distance and angle (both azimuth and elevation) estimates are 2 meters and 2 degrees, respectively. Clearly, the accuracy improves as the number of anchors increases, especially for large GPS measurement errors. This is due to the fact that adding one anchor incurs three more unknown position variables; however, 3N more local estimates and three more global estimates are produced. Generally speaking, more independent estimates could produce a redundancy/diversity gain. We can see that even at five anchors and in the absence of angle estimates, the accuracy improvement is substantial. The accuracy of both the LM algorithm and the LS algorithm approaches the CRLB.

Then, the FIM can be derived to be
\[
\mathcal{F}(\mathbf{p}) = \begin{bmatrix}
V_{11} & V_{12} & \ldots & V_{1N} \\
V_{21} & V_{22} & \ldots & V_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
V_{N1} & V_{N2} & \ldots & V_{NN}
\end{bmatrix} \in \mathbb{R}^{3N \times 3N}
\]
where
\[
V_{ij} = E \left[ \begin{bmatrix}
\frac{\partial^2 u_1}{\partial x_i \partial x_j} & \frac{\partial^2 u_1}{\partial x_i \partial y_j} & \frac{\partial^2 u_1}{\partial x_i \partial z_j} \\
\frac{\partial^2 u_2}{\partial x_i \partial x_j} & \frac{\partial^2 u_2}{\partial x_i \partial y_j} & \frac{\partial^2 u_2}{\partial x_i \partial z_j} \\
\frac{\partial^2 u_3}{\partial x_i \partial x_j} & \frac{\partial^2 u_3}{\partial x_i \partial y_j} & \frac{\partial^2 u_3}{\partial x_i \partial z_j}
\end{bmatrix} \right] \in \mathbb{R}^{3 \times 3}.
\]
As a result, we obtain the CRLB as:
\[
\text{CRLB}(\hat{x}_i) = \left[ \mathcal{F}^{-1}(\mathbf{p}) \right]_{jj}, \quad j = 3(i - 1) + 1, \\
\text{CRLB}(\hat{y}_i) = \left[ \mathcal{F}^{-1}(\mathbf{p}) \right]_{jj}, \quad j = 3(i - 1) + 2, \\
\text{CRLB}(\hat{z}_i) = \left[ \mathcal{F}^{-1}(\mathbf{p}) \right]_{jj}, \quad j = 3i.
\]
Dropping the angle related terms in the above expressions of the CRLB results in the CRLB in the absence of the angle measurements.

VI. SIMULATION RESULTS

In this section we evaluate the performance of the proposed methods that use both GPS coordinate measurements and local parameter estimates to improve the anchor global position accuracy. A cubic region of dimensions of 100(m)×100(m)×100(m) is considered, where the anchors are randomly deployed. For each simulation run, 1000 anchor position configurations are examined and the performance is then averaged. The performance measure for the two algorithms is the root mean squared error (RMSE) of the global position coordinate estimation. The same anchor position configurations are used to compute the average CRLB that is defined as
\[
\sqrt{\frac{1}{3NL} \sum_{\ell=1}^{L} \sum_{k=1}^{3N} [\mathcal{F}^{-1}(\mathbf{p})]_{kk}.}
\]

The GPS coordinate estimates, the local range and angle estimates are all mutually independent. The weighting matrix is simply chosen as an identity matrix.

Fig. 3 shows the estimation accuracy with respect to the number of anchors. Results with two different STDs (4 and 10 meters) of the original GPS coordinate measurements are presented. The STDs of the distance and angle (both azimuth and elevation) estimates are 2 meters and 2 degrees, respectively. Clearly, the accuracy improves as the number of anchors increases, especially for large GPS measurement errors. This is due to the fact that adding one anchor incurs three more unknown position variables; however, 3N more local estimates and three more global estimates are produced. Generally speaking, more independent estimates could produce a redundancy/diversity gain. We can see that even at five anchors and in the absence of angle estimates, the accuracy improvement is substantial. The accuracy of both the LM algorithm and the LS algorithm approaches the CRLB.

Fig. 4 illustrates the impact of the local distance error when the STD of the AOA estimation error is 2 degrees, and there are either 6 or 10 anchors. The STD of the GPS measurement error is set at 10 meters. Clearly, the results based on local range estimates are more sensitive to the distance error than the results based on both range and angle measurements. This is because the angle estimates provides more reliable information of the anchor locations for the given distance and angle estimation errors. Fig. 5 shows the anchor position accuracy of the LS algorithm versus the STD of the AOA estimates when the STD of the local distance estimates is 2 meters and there are also either 6 or 10 anchors. The STD of the GPS measurement error is also set at 10 meters. Clearly, the performance of the simple LS algorithm approaches the CRLB so that it is not necessary to seek the much more complicated optimization algorithms.

VII. CONCLUSIONS

In the paper, we investigated anchor position accuracy improvement for localization in a 3-D environment. We focused on scenarios where the GPS coordinate measurements are not satisfactory and proposed to employ local parameter estimates to improve the anchor (and hence other ordinary sensor) location accuracy. A simple LS estimation algorithm was derived for scenarios in which both local distance and angle estimates, and GPS coordinate measurements are available. Also, the LM optimization algorithm is developed for accuracy improvement in the absence of angle estimates. Furthermore, we derived the CRLB to benchmark the position estimation
Fig. 3. Impact of number of anchors on estimation accuracy. $\sigma_{d_i}$ = 2 meters, $\sigma_{\alpha_i}$ = 2 degrees. "LM(d)-10m" denotes results based on the LM algorithm using local distance estimates, with the STD of GPS coordinate measurements set at 10 meters. "LS(d+a)-4m" denotes results based on the derived LS algorithm, local distance and angle estimates, with the STD of the GPS coordinate measurements set at 4 meters.

accuracy. It was demonstrated through simulation that the proposed algorithms can improve the anchor global position accuracy substantially and that the accuracy of the proposed algorithms is close to the CRLB.

**Fig. 4.** Impact of distance accuracy given $\sigma_{d_i} = \sigma_{\alpha_i} = 2$ degrees, STD of GPS coordinate estimates equal to 10 meters, and either 6 or 10 anchors. "LM(d)-6" denotes results based on the LM algorithm, local distance estimates, and six anchors. "LS(d+a)-6" denotes the results based on the derived LS algorithm, local distance and angle estimates, and six anchors.

**Fig. 5.** Impact of AOA accuracy when given $\sigma_{\alpha_i} = 2$ meters, STD of GPS coordinate estimates set at 10 meters, and either 6 or 10 anchors.

**REFERENCES**


